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Doing Mathematics in Different Places

an Exploration of Young People's Activities as they make Independent Use of a Web-Based Discussion Board

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Doing Mathematics in Different Places: an Exploration of Young People's Activities as they make Independent Use of a Web-Based Discussion Board

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Abstract

This study examined how young people are engaging with and doing mathematics, independently pursuing serious mathematical study, at home away from their classrooms, communicating with like-minded peers from anywhere in the world via the Internet, using the NRICH website and the AskNRICH web-board.

An Initial Study using a mixed methods methodology, including a web-survey, identified the current practice of NRICH problems being undertaken at home and students' perceptions of doing mathematical problem-solving in school. Results revealed a majority of NRICH users, predominantly high-attainers, independently choosing to work on problems, only at home and alone, believing that their teachers were unaware of this.

The Main Study used interpretative methods in an emergent research design to study AskNRICHers' interactions through analysis of some 5000 messages posted in 600 threads from three distinct but interlinked perspectives. Parallel commentaries separating the mathematics and actions in messages were constructed and subsequently coded. A prototype visualisation tool, 'a connection diagram', was developed to portray the complex networks of interactions, categorised by response type, linking participants and messages. Thus this work has resulted in the formation of a set of techniques, including some new elements, that can manage the complexities, size and nature of the task of analysing the AskNRICH web-board.

The findings characterising the AskNRICH environment have led to the proposal of the concept of a Second Learning Place, a specific type of Pupil Learning Place. In the empathetic environment of the AskNRICH Second Learning Place, the AskNRICHers collaborate, cooperate and show consideration and care to each other. Analysis of teaching and learning aspects demonstrates that the AskNRICH virtual world and the AskNRICHers' behaviours strongly promote a transformational pedagogy. The AskNRICH environment provides an exemplar of positive use of Internet-mediated communications leading to a harmonised mathematical experience in which the AskNRICHers are 'independent but not alone'.

Acknowledgements

I have been on quite some journey in 'doing' this thesis, a journey that has involved juggling many balls-in-the-air. Throughout, my supervisor, Dr. Mary Webb, has been there to support me, for which I am very grateful.

Initially the work involved asking a number of schools to allow classes to participate in my research. To those schools, teachers and pupils, I offer my thanks.

The majority of this thesis could never have been written had it not been for NRICH (and AskNRICH). Many members of the NRICH team have been involved and to all I would like to say thank you. I would especially like to thank the three different NRICH directors who have been in post during the gestation of this thesis: Toni Beardon, Jennifer Piggott and Lynne McClure. In addition I need to give special thanks to Emma McCaughan, Moderator and Vicky Neale, Deputy Moderator, of AskNRICH who have acted as sounding boards and a conduit to the anonymous AskNRICHers. It is to the AskNRICHers themselves, especially 'Peter', that I must offer my most heartfelt thanks, for indeed without them there could be no thesis.

I would also like to acknowledge the unstinting support that I have had from the too-many-to-mention colleagues and friends from my Faculty in Cambridge and ITTE who have constantly shown an interest in my work and given time listening to my thoughts and ideas. Equally, throughout this time my many different sets of PGCE mathematics trainees have constantly enquired about my work, displaying an enthusiastic interest that spurred me on to write the next chapter. At long last I can tell them I have indeed submitted my thesis.

Finally, one person has literally been with me every step of the way, except when he has stepped into the kitchen to save my doing so, giving me additional time to concentrate on the enormous task in hand. Thus to Graham I say a simple thank you.

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Glossary

1. Abbreviations

AS	Affinity Space
ATM	Association of Teachers of Mathematics
BMO	British Mathematical Olympiad
CMC	Computer Mediated Communication
CMCs	shorthand for CMC forums
CRB	Criminal Records Bureau
CSCL	Computer Supported Collaborative Learning
DES	Department of Education and Science
DfEE	Department for Education and Employment
DfES	Department for Education and Skills
G&T	Gifted and Talented
GCSE	General Certificate of Secondary Education
HD	Higher Dimension section of AskNRICH Web-Board
IRF	Initiation-Response-Feedback
KS	Key Stage
LRX	Literature Review Chapter no. X in this Thesis (in Roman numerals)
MI	Mathematical Induction
NC	National Curriculum
NNF	National Numeracy Framework
NNS	National Numeracy Strategy
Ofsted	Office for Standards in Education
O&U	Onwards and Upwards section of AskNRICH Web-Board
PE	Please Explain section of AskNRICH Web-Board
PLP	Pupil Learning Place
PQ1/2/3/4	Paper-Based Questionnaires 1 to 4
QCA	Qualification and Curriculum Agency
RG	Research Goal
RIMM	Royal Institution Mathematics Masterclasses

RO	Research Objective
RQ	Research Question
SLP	Second Learning Place
SSS	Semiotic Social Space
UK	United Kingdom
UKMT	United Kingdom Mathematics Trust
US or USA	United States of America
WQ	Web-Survey Questionnaire
WQn	Web-Survey Questionnaire Question n
ZPD	Zone of Proximal Development
4Cs	Cooperation, Collaboration, Consideration and Care

N.B. NRICH, AskNRICH and SPODE are neither acronyms nor abbreviations

2. Naming conventions used to refer to threads and posts within the text

ExThds	[The] Exemplar Threads
ExThd1-Pn	Exemplar Thread 1 – post number n
ExThd2-Pn	Exemplar Thread 2 – post number n
CSThds	[The] Case Study Threads
CS-Pn	Case Study post number n (made by Peter)
T1-Pn	Case Study sample thread 1 – post number n
T2-Pn	Case Study sample thread 2 – post number n
MI-Tn-Pn	Case Study Mathematical Induction thread n – post number n
H-Pn	Case Study Role of Helper Thread – post number n
Peter-Postn	Peter's post number n (used in Chapter Ten)
3Thdn-Pn	Three threads thread n – post number n
3Thds	[The] Three Threads
Pn	Local references to Post number in any thread
AskNRICHer-Postn	nth post of an AskNRICHer not in the list of posting names below

Whole threads are referred to by omitting the **–Pn**, for example **ExThd1**.

3. Posting names for AskNRICHers

Peter	Case Study Subject
Nick	interviewee in Chapter Eight - also Help1 in ExThd2
John	interviewee in Chapter Eight - also HelpC in 3Thd
Julia	a lurker – interviewee in Chapter Eight
Moderator	the AskNRICH Moderator
Deputy Moderator or ***DM***	the AskNRICH Deputy Moderator
O	AskNRICHer who starts Case Study thread [**H]**
R	AskNRICHer who starts 3Thd2
S	AskNRICHer who starts 3Thd3
Plea1	AskNRICHer who starts ExThd1
Plea2	AskNRICHer who starts ExThd2
Plea3	AskNRICHer who joins in ExThd2
Help1, 2 … and ***HelpA, B …***	helpers in any thread
Expert	helper in Case Study thread [**H**]
ANP1 & 2	AskNRICHers who join in late in 3Thd3

Helpn is simply a label given to anonymise helpers in any thread – use of the same label in another thread does not mean it is the same AskNRICHer helping.

4. Naming conventions used in web-survey for location (where NRICH problems are undertaken) groups and respondents

hp	home people – hs and oh combined together
hs	home and school
oh	only home
os	only school
R123-F-oh-KS5	web-survey respondent labelled by respondent number-gender-location group-key stage

Introduction

Chapter One

Introduction to the Study

I feel the atmosphere is very good, and it's great to be able to talk and discuss with other talented mathematicians - an opportunity which I don't really have at school. *[School Student – talking about AskNRICH]*

1.0 Preamble

I begin with a declaration … I confess that this research has evolved from my particular mathematical interests over the last 30 plus years, both within and beyond the classroom door. I was a member of SPODE, a small group of teachers and lecturers who published material [Green & Jared 1992; Jared 1992] promulgating an approach to mathematics teaching using real-life problems. I was involved with the local Royal Institution Mathematics Masterclasses [RIMM], a series of Saturday morning workshops given by university mathematicians to challenge and interest the 'brightest' 13-year-old pupils from the county's schools. I was one of the four founders[1] of NRICH, a project and website[2] to make (problem-solving) mathematics interesting/challenging to a wide audience including RIMM attendees seeking further stimulus to continue their mathematical development.

In contrast to being excited about mathematics, the following two deliberating eye-catching paper titles based on in-school studies: *' "I would rather die": reasons given by 16-year-olds for not continuing their study of mathematics'* [Brown, Brown & Bibby 2008] and *'Is Mathematics T.I.R.E.D?* [**T**edious, **I**solated, **R**ote-Learning, **E**litist, **De**-Personalised]: *A profile of quiet disaffection in the Secondary Classroom'* [Nardi & Steward 2003] imply that this excitement is distinctly absent in many classrooms. Compare this state of affairs with a quotation taken from an out-of-school experience: *'Now I've got the first one* [a trigonometrical equation solved] *I'm motoring through the exercises. Who would have thought trigonometry could be this much fun.*' [Post on AskNRICH Web-board]. This student had a school-based mathematics problem that they were unable to solve

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[1] The four founders were Toni Beardon, School of Education, **C**ambridge University; Dr Roger Bray, Clothworker Fellow, **R**oyal **I**nstitution; Colin Penfold, then Mathematics Adviser for **N**orfolk Local Education Authority and myself, then a lecturer at **H**omerton College, Cambridge.

[2] http://www.nrich.maths.org.uk

unaided. With the encouragement of peers[3] the student was led to finding the solution for themselves and obviously appreciated the experience and demonstrated their enthusiasm for the subject. Perhaps put more correctly: my interpretation of this post is that he/she appreciates the experience and conveys an air of enthusiasm clearly missing in other (classroom) environments as the catchy paper titles cited above imply.

So ... What is it about AskNRICH that leads to posts like this? What is going on within AskNRICH? What are the people posting on AskNRICH up to? Well ...

1.1 Introduction

This thesis investigates how young people[4] are engaging with and doing mathematics via the Internet outside of the school environment/location. There are two connected parts to the research; the first provides contextual background for the second. The first, referred to as the Initial Study, used the NRICH website to replicate my earlier evaluative studies of NRICH [Jared 1997, 1998]. This established the current practice of NRICH problems being undertaken in a home context and students' perceptions of doing mathematical problem-solving in school. The second, substantive part is referred to as the Main Study. It explored how young people used AskNRICH, the web-board forum area of NRICH, to pursue serious mathematical study away from the classroom, with like-minded peers from anywhere in the world. Affectionately known throughout this thesis as *The AskNRICHers*, the quality of work they do and how they do it is, I believe, worthy of sharing. 'The AskNRICHers: an everyday story of virtual folk' is a story worth telling.

1.2 Rationale

Bentley's proposition of school education existing beyond the conventional classroom [Bentley 1998] had led to the phrase "a curriculum without walls" coined by Furlong, Furlong, Facer and Sutherland [2000: 108]. Although speculative, this suggested that the

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[3] The word peer is used here to represent like-minded others who are not necessarily of the same age.

[4] The terms young (people), children, pupils or students are used variously throughout this thesis, depending upon the emphasis of a home or school environment. All are referring to that part of the population aged between 11 and 18. AskNRICH is an open site and thus has the potential to be used by people of all ages. The vast majority of users however are those who are of school and [studying at] university age.

Internet would bring about a change in home/school work patterns. Stahl, Koschmann and Suthers [2006: 410] report that Computer-Supported Collaborative Learning [CSCL] arose in the 1990s, as a means to making learning more interactive and social. Whilst CSCL would include both face-to-face and distance e-learning opportunities, they were not premised on a web-based technology. Moreover, when I started out on this research, as far as I am aware, there had been little (if any) research into individuals using a school curriculum subject, either in the way that the AskNRICHers do or simply choosing to do NRICH problems at home (and alone). Indeed, as few innovations had been developed that expanded students' use of the Internet [Schofield 2006: 529], the idea that the home environment potentially has an increased part to play in curriculum study than previously had remained speculative. I believed it would be worthwhile exploring AskNRICH not only because it is an artefact that spans the dual environments of home and school, but also since exploration would yield data relating to the situation of, and practices within a virtual world. It would provide a concrete example of the practices of a group of young people connecting together in the digital age, who clearly enjoy doing mathematics per se in and/or out of school.

As one of the four founders who designed and set up NRICH (and soon after AskNRICH) and through subsequently maintaining a 'parental' watching brief as it has further developed, I have some insider information and knowledge of the web-board that could be used to facilitate the research. No member of the NRICH team had ever undertaken any systematic research of the AskNRICH site and thus any study of it would begin with a 'blank canvas'.

I have thus set down a rationale for the research. This chapter continues by briefly introducing NRICH and AskNRICH and begins the background to the research by commenting on the outcomes of the earlier evaluative studies, before framing the research by discussing the key areas that became instrumental in shaping the Main Study. Subsequent sections describe the Research Design, give an overview of the research goals and how these were addressed, and discuss ethical considerations. Finally, brief descriptions of the remaining fourteen chapters are provided.

1.3 Introducing NRICH and AskNRICH: A Classroom in the Air

NRICH describes itself as a mathematics 'net-workshop' which offers pupils of all ages who enjoy the challenges of mathematics, the opportunity to participate either with friends in a school mathematics club or individually (via school or home). NRICH problems are based on topics that would be met within the English mathematics curricula and is recognised as a valuable Internet resource that teachers can use [Hodgen & Wiliam 2006; Koshy & Casey 1987]. AskNRICH is a virtual world that allows young people (the AskNRICHers) to ‘meet’ with other interested ‘soulmates’ and engage in doing mathematics. As the opening quotation to this chapter illustrates, it is a place invoking a ‘clubbable’ atmosphere in pursuit of an universally based school and university subject pursued from many centuries past to the present day.

The AskNRICH web-board has three main mathematics sections on open access, differentiated by the level of mathematics under discussion. The research in this thesis examines the first two sections aimed at mathematics study at pre-university level. Participation in AskNRICH is purely voluntary. The level of work that the school-aged AskNRICHers engage with can be beyond that on a ‘normal’ school mathematics syllabus. Hence the work should be viewed as mathematical studies undertaken whilst individuals are still attending school. This might be work met as part of an ordinary mathematics lesson which they wished to pursue further, either as it had been set as homework or as additional practice. However, it is often work undertaken that offers challenge and is at a level of difficulty far in excess of that normally intended for their chronological age and, as such, is more commonly met post school level. In this respect the work the AskNRICHers engage in takes on the attributes of any ‘hobby’ such as stamp collecting, train spotting or playing football which are all done for pleasure or for their own sake. Thus AskNRICH is being used primarily at home and although any content posted could in a sense be connected to a fixed syllabus, a specific curriculum or subject course, the content is only there because it is intrinsically important to the poster.

Thus the AskNRICHers exclusively determine the web-board’s topic and content. There is no teacher/lecturer direction, mathematical problems only appear when someone has started a thread because they need help with the solution, knowing that there will be other

AskNRICHers who ‘know the answer’. A member of the NRICH staff acts as moderator, maintaining a watching eye on the exchanges, and incidentally responding to mathematical problems in precisely the same way as other AskNRICHers. The web-board’s posting protocols require the person seeking help to share their thoughts and any progress made, although having made none is acceptable. Similarly the person offering help is required not to just give the answer even with working-out. This creates a particular type of discursive talk rooted in ‘Inquiry/Socratic Dialogue’ [Collins & Stevens 1982].

The properties of AskNRICH just outlined engender its special, in the sense of singular and distinct, nature, summarised in Table 1.1, that makes it unique[5], as far as could be ascertained, amongst Computer Mediated Communication [CMC] forums for an academic subject, reported in the literature. The implications of this for the research on AskNRICH are introduced later [see Section 1.5].

1	contributors can be (and often are) of secondary school age
2	participants belong voluntarily
3	the web-board is primarily used for learning an academic subject, only at home, for ‘pleasure’ and is thus not institutionally-based or part of any set syllabus, curriculum or subject course
4	topics are only raised if they are of importance to the individual making the initial post
5	there is no teacher/lecturer led element

Table 1.1 The Special Nature of AskNRICH

1.4 Background to the Research

This research was conceived through reflection upon findings of two earlier evaluative studies [Jared 1997, 1998] of the NRICH website. The second evaluation provided a (personal) seminal finding that some children, who were accessing the mathematics problems at home, were only working on them at home. There were young people choosing to do mathematics curriculum work in their free time at home. That is, the earlier studies led to the discovery of ‘home-aloners’ doing curriculum mathematics as a ‘hobby’ and to my subsequent proposals of different *“sites of learning”* [Jared 2005: 135] and the concept of a *“pupil learning place”* [Jared 2004: 66]. These two proposals are directly related to

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[5] Since undertaking this research, The Student Room [nd] has become popular and, whilst there are some similarities, the ethos and etiquette of AskNRICH keep it unique.

Bentley's [1998] proposition mentioned above and might be considered as an example of a "curriculum without walls" [Furlong et al. 2000: 108].

The evaluative studies did not investigate whether mathematics problems had previously been undertaken at home before the arrival of the Internet, but the NRICH website was initiated through the founders' beliefs that establishing such a site would provide greater opportunities for working on mathematics problems outside the school setting [Beardon, Jared & Way 1999]. If the Internet can provide an individual with the choice to continue school and/or subject related work voluntarily, it follows that there is a potential shift in the teaching and learning practices between the dual environments of school and home. Moreover, accompanying such a shift is the opportunity for an individual to have greater control of their education. Put simply, in school a pupil is generally reliant on the teacher to 'dictate' the work to be studied. At home, the Internet with an apparent wealth of educational resources, provides the individual with greater freedom to choose what work they would like to do and when to do it [Henri 1992; Mason & Kaye 1989]. AskNRICH could clearly provide an appropriate vehicle for exploring a teaching and learning 'community'[6] amongst school-aged students away from the school environment.

The next section presents a brief reasoned discussion of three key areas: CMC forums [CMCs]; various types of community and classroom practices, consideration of whose relationship to AskNRICH had consequences in shaping the Main Study at various stages: before, during and even towards the end of it.

1.5 Framing the Research for the Main Study

At the beginning of the Main Study the medium of AskNRICH dictated a consideration of CMC literature. As will become clear in Section 1.5.2 a lengthy examination of different types of community was undertaken until the middle of the exploratory stage of the study. Section 1.5.3 explains the role of theoretical frameworks of classroom practices in the completion of the study.

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[6] At this stage the word community should be considered only in the sense of a group of people sharing experiences and interests and who communicate with each other in pursuing these interests [Mercer 2000: 105].

1.5.1 CMC Forums

There is a plethora of literature on analysis of CMC forum discussions. Reviews [for example, De Wever, Schellens, Valcke & Van Keer 2006; Rourke, Anderson, Garrison & Archer 2003; Steffans & Underwood 2008] reveal the variety of approaches and frameworks used in analysis of CMCs. However, these forums, almost exclusively critical thinking debates in a formal, higher education setting, have little in common with AskNRICH because they share few, if any, of the properties of AskNRICH set out in Table 1.1 above. Moreover, the activities undertaken within AskNRICH are not the type of collaborative, co-constructed or co-operative problem-solving prevalent in the studies reported in the literature, although examples of this might incidentally occur. It is for this reason that, wherever possible in this thesis the term CMC will be used in preference to CSCL, although the two are used interchangeably in the literature [De Wever et al. 2006].

As a consequence of these differences between AskNRICH and other CMCs, the final decision on how to analyse AskNRICH required considerable investigation.

1.5.2 Types of Communities

When I embarked on the exploration of AskNRICH, I imagined that I would be determining the type of learning *community* that it best fitted. Indeed, over the years, in conversations with teachers and other educators about AskNRICH, I have found that it has been implicitly assumed that AskNRICH forms some kind of community (usually with the words 'of practice' appended) where participants can do (and learn) mathematics. Although NRICH was set up with the words 'online Maths Club' forming part of its title which must therefore have implied to some extent a sense of belonging, as the exploration progressed it became increasingly clear that characterising AskNRICH in terms of a community was inadequate and would not capture its true essence.

Theories about learning communities predate widespread use of the Internet having their roots in physical, for example Lave & Wenger [1991] Communities of Practice, and not virtual locations [Sawyer 2006]. Proponents of e-learning have subsequently appropriated these theories into the virtual world [Bruckman 2006a, 2006b].

Some types of communities such as, for example, knowledge building communities [Scardamalia & Bereiter 2006] and Communities of Inquiry [Garrison & Anderson 2003; Garrison, Anderson, & Archer 2000], are predicated on participants building knowledge together. Both of these types of community rely on a collaboration model in which the initiating problem is generally one set by an 'outsider' for all the group, no-one starts out knowing the answer and participants collaborate to build knowledge in order to find an solution/conclusion. AskNRICHers post a problem with the sole purpose of receiving help in solving it for themselves, knowing that other AskNRICHers will have the solution. Thus knowledge is not being built *together*.

In Communities of Practice [Lave & Wenger 1991] the apprenticeship model involves people working with more knowledgeable peers who at least have a lot more expertise and might well know the answer. There is the potential within AskNRICH for such "legitimate peripheral participation" [p29]. However, the situation in AskNRICH is not quite this simple, not least in that at the end of an exchange, individuals are 'isolated' by home location and ultimately left alone to make final sense of meanings.

Bruckman [2006b: 465] appropriates Papert's [1980] proposal of a 'technological samba community', derived from a kind of Brazilian Social Club where participants of all ages, all experiences and disparate skills at different levels travel to a specific location annually to 'dance together', for her own e-learning community. Whilst the spontaneity and egalitarianism at least might resonate with AskNRICH, in AskNRICH the performance is more of a continuous, daily, normal occurrence.

Thus the different type of collaboration that goes on within AskNRICH makes matching it to a specific community type problematical. Furthermore, it became increasingly clear that the fluid nature of the 'membership' of AskNRICH undermines the use of any community model. Consequentially other models had to be considered leading to an examination of Gee's Affinity Spaces [Gee 2004, 2005a] whose model was subsequently appropriated and developed further.

1.5.3 Classroom Practices

As a consequence of the recent nature of technological innovations that enable AskNRICH to exist, there is little directly relevant historical literature about doing ‘school’ mathematics in such an environment to draw on. AskNRICH as a ‘classroom-in-the-air’ should by definition have distinct differences from the bounded school-based classroom. Nevertheless, both have the underlying purpose of learning (and teaching) mathematics and indeed, at the end of the exploration stage of the study, some theoretical frameworks developed within and for the classroom were used to make sense of AskNRICH. In this respect van Lier’s work [1996] on Pedagogical Interactions and Conversations-for-Education and the work of others all rooted in a Vygotskian perspective was central. However, this was preceded by an iterative research process in which various theoretical frameworks reported in the research literature were explored in parallel with the ongoing exploration of AskNRICH, as explained in the next section.

1.6 Research Design

In this section I set out my epistemological view and the theoretical perspective that of necessity is shaped by using AskNRICH as the artefact. Although the earlier section entitled ‘Framing the Research’ discussed key factors that informed the research, this section continues by including further aspects of AskNRICH that impinge on the framework. This is followed by outlining the research goals and questions and describing how these goals and questions were addressed. An overall summary of the research framework and design is then portrayed in a diagrammatic form. The section concludes by reporting on the ethical considerations required to conduct the research.

1.6.1 Epistemology and Theoretical Perspective

I came to this study with an epistemological viewpoint that is, according to Crotty’s [1998: 5] categorisations, one of constructionism, the view that there is no one universal truth [Robson 2002], a common perspective within the field of human sciences. The purpose throughout was to construct meaning, either from the “state-of-the-actual” [Selwyn 2008: 84] established in the Initial Study or, through the exploration in the Main Study, where making meaning of the results of the actions and activities of the

AskNRICHers would be reliant substantially on using written text. Thus the theoretical perspective is within the interpretive paradigm [Brown 2001; Crotty 1998; Denzin 2001; Heywood & Stonach 2006] and as far as the Main Study is concerned, within the field of hermeneutics [Bleicher 1980; Schmidt 2006].

Both the development of an analytical approach and the exploration of AskNRICH proceeded in parallel, although interacting with each other, each traversing a series of multiple, iterative loops combining both inductive and deductive steps [Cohen, Manion & Morrison 2007], with further returns to the literature to seek out theories that could be tested against current findings. The eventual characterisation was established through a further development based on theories put forward by van Lier and Gee [as indicated in Section 1.5 above]. Thus the research process was simultaneously complex, challenging[7], interesting and rewarding.

Working with AskNRICH as the artefact has further consequences, not yet mentioned, in undertaking the research in the way that I did. In this respect, the following section is a return to the discussion of how using AskNRICH as an artefact influenced further developments of the research programme's rationale.

1.6.2 Working with the AskNRICH Artefact

It was inappropriate to act as an active participant of the web-board since that could influence the outcome of the research. Neither was it appropriate to initiate posts that would seek personal details due to child safety policies and the guarantees of non-contact that AskNRICH gives its participants. It was therefore not possible to undertake an ethnographic study in the truest sense of the word. I was one step removed from the participants. However by logging onto the site daily over several months, it was possible to witness, as a non-participant observer, the day-to-day practices of the participants posting their messages. So although not ethnographic, the research clearly did involve substantially more than simply reading 'printed' texts in a detached manner and the interpretations were made with

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[7] Complex and challenging could also be used to describe the writing up process in communicating the 'messy' nature of the exploration.

increasingly developed knowledge of the participants' practices and actions through, and by, their posted messages.

Although the medium brought with it the restrictions just explained, it nevertheless simultaneously brought advantages. AskNRICH provides the conduit for young people to communicate out-of-school, across the globe. Thus the interactivities and activities are unrestricted by both the time imposed on a school timetabled lesson and the confines of a physical classroom. Message exchanges between the AskNRICHers (as with any other virtual CMC) can be made over a longer, contemplative and asynchronous time-frame beyond that available within a normal school lesson where activities are compressed within a time-limited frame. The threads are complete entities: started by a request for help, concluded only when that request for help has been satisfied and, in between, on-going for however long it takes to arrive at that conclusion. In other words there is no school bell ringing to curtail the lesson. Similarly, the medium naturally provides the posters the opportunity (if not indeed a necessity) to write their messages clearly. Such deliberative written exchanges are in marked contrast to transcriptions of verbal exchanges that take place within the bustle of normal daily classroom routine. Thus, as early CMC researchers such as Henri [1992] and Rennie and Mason [2004] point out, in this respect such research brings a different perspective to that provided by classroom research.

Thus the restrictions actually pave the way to gaining an insight into a way of exploring collaborative work, however that eventually comes to be defined, between like-minded peers in a new situation both in terms of location and people involved. Whilst it is impossible to get 'inside the head' of the participants, it is possible to investigate the teaching and learning roles emerging through the exchanges that run through the message threads.

1.6.3 Overview of the Research Goals

Table 1.2 below details the specific research goals (**RG**) and associated research questions (**RQ**). Appendix 1.1 presents: the specific research goals and associated research questions with their subsidiary parts; for each goal a synopsis of the research undertaken to collect the relevant data, and where the major findings are reported within the thesis. Appendix 1.2 provides a synopsis and timeline of the data collection for both the Initial and Main Study.

Initial Study **RG1: To investigate pupils’ general perceptions of doing mathematics in school and of using NRICH type problems in home/school settings**	
Research Questions	**RQ1**: What are the common practices of using NRICH problems in the home context? **RQ2**: What views do students using an on-line mathematics resource (NRICH) have concerning their experience of school mathematics?
Main Study **RG2: To develop an analytical approach appropriate to the nature of AskNRICH**	
Research Questions	**RQ3**: Can existing methods / frameworks for analysing Computer Mediated Communication forums be employed in analysing AskNRICH? **RQ4**: How should the exploration of AskNRICH be organised (planned, structured and executed)?
RG3: To undertake the exploration of the AskNRICH artefact	
Research Questions	**RQ5**: What does AskNRICH offer to participants to enable them to pursue their mathematical practices? **RQ6**: What are participants’ common practices when using the AskNRICH web-board? **RQ7**: What results from participants’ practices when using the AskNRICH web-board?
Overarching Research Objective: **To characterise the network that constitutes AskNRICH, a virtual world that allows people to meet within it and engage in doing mathematics**	

Table 1.2 Research Goals and Research Questions

The overarching research objective (**RO**) was to establish a characterisation of AskNRICH. Three subsidiary goals had to be pursued in order to achieve this overarching goal. The first of these was to collect data from NRICH users on doing mathematics in school and at home as a preliminary to, and to provide valuable background for, the main body of research. The second goal, to develop an analytical approach appropriate to the nature of AskNRICH was a pre-requisite to allow work towards the third goal, the exploration of the web-board itself, to be undertaken.

The first research goal (**RG1**) originated from the wish to establish whether the findings of my decade-old evaluative studies, referred to above, continued to be correct i.e. some school students are doing mathematics ‘in different places’. Thus the Initial Study was designed as a ‘follow-up’ investigation, culminating in a web-survey, into pupils’ general perceptions of doing mathematics in school and using NRICH type problems in home/school environments.

In order to address research questions **RQ1** and **RQ2**, findings from the web-survey, along with a small number of face-to-face and email interviews were analysed using a set of widely accepted and commonly used qualitative and quantitative methods. Thus the "state-of-the-actual" [Selwyn 2008: 84] was established and in the process the then seminal finding of the earlier evaluative studies re-established.

On its own the Initial Study, substantially reliant on the web-survey, could not provide the comprehensive dataset necessary for a complete portrayal of working in different locations due to the very limited contact available with the respondents. It was neither ethically possible to make 'stranger contact' nor could pupils be sought out and then contacted via their teacher since, as re-confirmed in the Initial Study, many pupils apparently do not disclose their out-of-school mathematical activity to them. However the content of the AskNRICH web-board where the mathematics being undertaken is clearly visible would reveal how pupils were working on their mathematics in an out-of-school context. Thus the Main Study was designed to be a systematic and in-depth exploration that offered a view of the how, what and why of the AskNRICHers' doing mathematics away from the classroom. Furthermore, the Initial and Main Studies taken together would contribute to the 'different locations' referred to in the thesis title.

The aim of the Main Study was to 'make sense', of the working practices of these young people who, working on their own, at home and alone, communicate, only in a virtual space, in both a teaching and/or learning sense, with like-minded peers. The Study would necessitate working with non-contactable 'unknown', albeit known to exist, 'actors'. The amount of data available was vast constituting some 50,000 messages in 6,000 text-based threads.

Developing an analytical approach to be able to do this exploration became the second research goal (**RG2**). Its associated questions **RQ3**, looking at whether there were existing methods/frameworks for analysing CMCs that could be adopted or adapted for analysing AskNRICH, and **RQ4**, how the exploration should be organised and executed, are, in essence, methodological [see Table 1.2]. Establishing the process with which to explore was crucial to achieving the exploration.

Addressing **RQ3** was dependent upon an extensive literature review [LRII]. This confirmed the seemingly different nature [see Table 1.1] of AskNRICH and the need for an analytical approach [reported upon at length in Chapter Six] incorporating some new elements. In order to address **RQ4**, two important aspects of the analytical approach had to be determined: how to manage the subject content in order to uncover the activities and nature of the work, and how to impose order on the vast amount of data available to be processed. This was resolved through an evolutionary, iterative process developed alongside the organisation of the exploration of AskNRICH.

The exploration of AskNRICH addressed the third research goal (**RG3**) through three particular research questions: what AskNRICH offers to participants to enable them to pursue their mathematical practices, **RQ5**; what are the common practices, **RQ6**; and what results from these practices, **RQ7** [see Table 1.2]. The goal was achieved through exploration of AskNRICH from three Perspectives[8]: the first Perspective examined two exemplar threads, the second was a case study that followed the postings of one particular AskNRICHer over an eighteen-month period, and the third investigated three different threads that been posted by different AskNRICHers all wishing to solve the same problem.

As previously stated, all three research goals fed into the overarching objective of characterising the virtual world of AskNRICH. All three goals were addressed fully and the overarching objective achieved. Achieving this objective has provided the opportunity to make a contribution to the body of knowledge within both Internet (CMC) related studies and (mathematics) education, as well as the exploration itself making a contribution to the field of educational research processes involving CMCs.

1.6.4 Framework Summary Diagram

Figure 1.1 below illustrates the framework within which the research was conducted. Firstly it encapsulates the background and framework of the study as presented above; the text within the circles indicates the relevant areas of literature used for the study. Secondly it

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[8] The word Perspective is used here in the sense of an orientation of a view of AskNRICH and should not be confused with the use of the word to describe a methodological theoretical standpoint.

shows the interconnections (linked by arrows) between the different research goals (placed within rectangles).

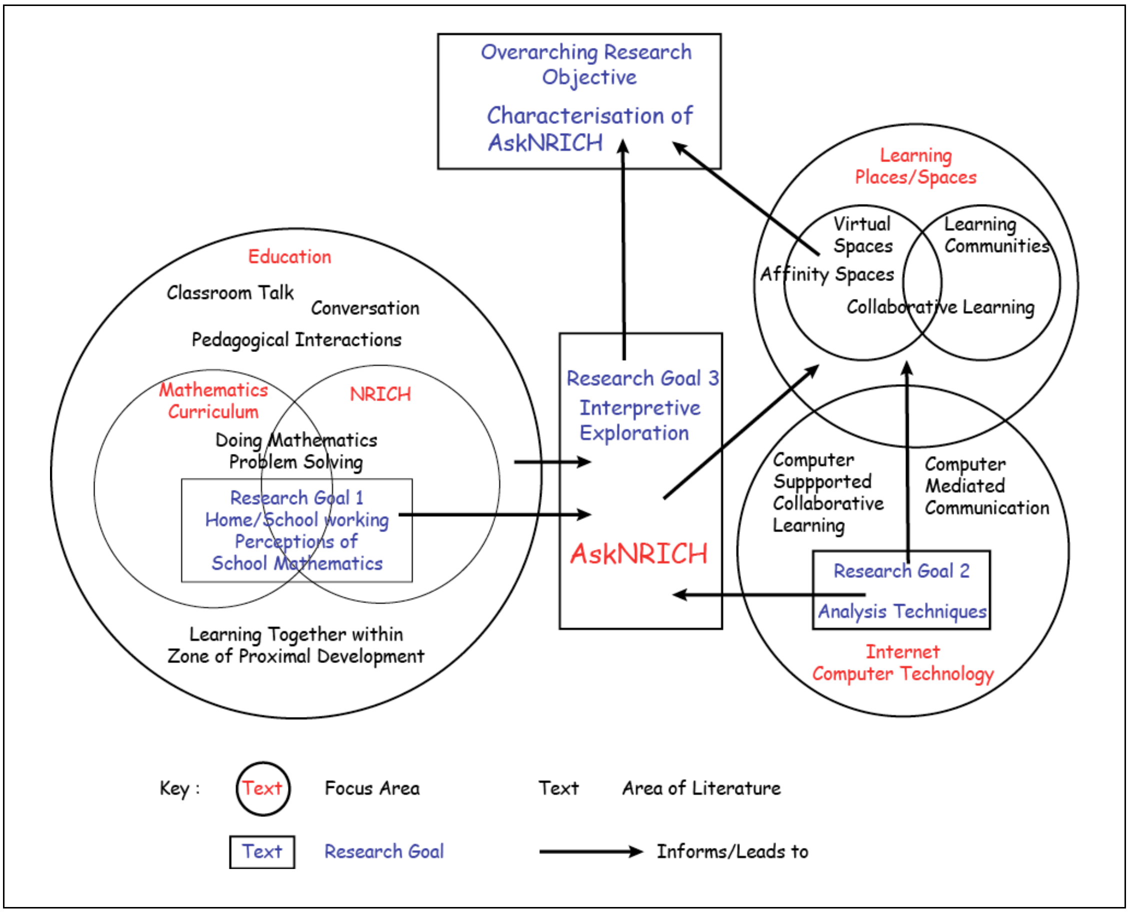

Figure 1.1 Interconnections between Focus Areas, Research Goals and Selected Literature

1.6.5 Ethical Considerations

All of the research was carried out following the revised ethical guidelines for educational research issued by the British Educational Research Association [BERA 2004] and whilst holding a valid CRB certificate[9]. However, the great care necessary when electronic communications are involved, requires special mention in the context of this study. Thus the following measures were adopted. No email addresses were sought or obtained directly from respondents to the web-survey. Where email communication was entered into with members of AskNRICH, this was first initiated between the AskNRICH Moderator and the individual and, for those aged under 16, no correspondence was entered into until telephone contact

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[9] Anyone wishing to have contact with young people in then UK has to have been vetted and hold valid Criminal Records Bureau documentation.

had been made with parents. All face-to-face interviewees gave written agreement before any interview took place. All material taken from the AskNRICH web-board and quoted within this thesis was taken from the openly accessible sections and, as such, is in the public domain. At all stages of the research, the Director and other members of NRICH (and AskNRICH) have been aware of this work and were in full agreement with the measures adopted. Garrison & Anderson [2003: 146-149] provides an account of ethical issues within an e-learning context. Samples copies of the full text of relevant communications used within this research study can be found in Appendix 1.3.

1.7 Outline of Thesis Chapters

This thesis is constructed of four parts, corresponding to each of the Research Goals, with the related supporting literature introduced and reviewed cumulatively through the thesis. The first part starts with a review of literature on the state of school mathematics [Chapter Two] and then describes the mixed methods methodology [Chapter Three] used in the Initial Study, whose findings are set out [Chapter Four]. The second part presents a study, supported by a review of a range of studies on the analysis of CMCs [Chapter Five], of the methodological requirements for the exploration of AskNRICH, and the decisions leading to the derivation of a new analytical approach developed for the exploration [Chapter Six]. The third part describes the Main Study starting with a review of literature on peer interactions [Chapter Seven]. Chapter Eight provides background and contextual information about the web-board and the AskNRICHers. The three following Chapters [Nine, Ten and Eleven] report the findings of analysis of AskNRICH from each of three Perspectives: a detailed study of two exemplar threads; a case study of all of one AskNRICHer’s posts; and an examination of how AskNRICHers’ learning together could be considered as emulating the work of professional mathematicians. Chapter Twelve takes stock of the findings from the preceding chapters. In the fourth part, literature on types of collaboration is reviewed and the concept of an Affinity Space is set out [Chapter Thirteen], feeding into the final Characterisation of AskNRICH [Chapter Fourteen]. The thesis’ conclusions, claims and limitations are set out in Chapter Fifteen: Conclusions and Reflections.

Part One

Chapter Two

Literature Review I
School Mathematics

> *Too often, pupils are expected to remember methods, rules and facts without grasping the underpinning concepts, making connections with earlier learning and other topics, making sense of the mathematics … pupils rarely investigate open-ended problems which might offer them opportunities to choose which approach to adopt or to reason and generalise.* *[Ofsted [1] 2008: 5]*

2.1 Introduction

The review of literature presented in this chapter primarily covers aspects of mathematics in the school context in England and thus shapes the focus of the inquiry in the Initial Study. The review also provides a picture of the school situation that the AskNRICHers, who are the subject of the Main Study, experience. The chapter begins with a brief review of literature on the state of school mathematics teaching, including a series of UK governmental reports since the 1970s, the start of a period of intensive political scrutiny [Batteson 1997; Phillips & Harper-Jones 2002]. This is followed by a review of recent studies that examine school pupils' perceptions of mathematics. Working on mathematical problems is a key component of the NRICH ethos and thus the chapter continues with sections on mathematical problem-solving and pupils' perceptions of it. The chapter then explores what it might mean to 'do', i.e. engage with and work on, mathematics. Finally, examination of the topic of 'doing mathematics' is pursued further with a discussion of mathematical understanding and a review of literature on what it is to work as a mathematician, however one might be defined.

2.2 The State of School Mathematics Teaching

For centuries the 'best way' to teach mathematics has been keenly debated as is illustrated by Mason and Johnston-Wilder's [2004] survey spanning, inter-alia, Plato's Laws [c. third century BC], Spencer's writings of 1878, finishing with Nunes in 1999. The debate remains relevant to the present day where concerns are still expressed about the general unpopularity

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[1] The Office for Standards in Education [Ofsted] is the UK government's own inspection agency.

of, and poor achievement in, school mathematics [see e.g. Brown 1999; Howson 1996 for a UK perspective and Schoenfeld 1987 for the USA]. However, knowing that there is a problem and being in a position to do something about it are two quite different things [Mason & Johnston-Wilder 2004: 35].

Within recent history (1976) the then UK Prime Minister, James Callaghan, delivered a speech[2] at Ruskin College Oxford where he voiced "concern about the standards of numeracy of school-leavers". One consequence of the speech was the setting up of a committee of inquiry to report on the teaching of mathematics in schools that subsequently published *'Mathematics Counts*[3]*'* [DES 1982], recommending a series of changes in mathematics teaching. The 'legendary' paragraph 243[4] [p71] listing six different teaching styles that pupils should experience in their lessons: from exposition, consolidation and practice through to problem-solving and more open-ended investigational work is still frequently quoted.

The recommendations made in *'Mathematics Counts'* are not only still fundamental to the mathematics curriculum, but also became the starting point of other reforms that took place during the 1980s [Tanner & Jones 2000a: 2]. These included a reform in the national examination system and in 1988 the inception of a "direct state prescription of curriculum structure and content" [Beck 2003: 16], i.e. The National Curriculum [NC]. The subsequent NC reviews [DES 1989; DfEE/QCA 1999; DfES/QCA 2007] and the implementation of the National Numeracy Strategy[5] [NNS] [DfEE 1999] and the National Numeracy Framework [NNF] [DfEE 2001], have kept mathematics teaching in a constant state of change. A similar story applies in the USA [for an overview see Klein 2003].

'Mathematics Counts' was respectfully 'celebrated' some two decades after its publication by the Smith Report, *'Making Mathematics Count'* [DfES 2004], which cites the "perceived quality of the teaching and learning experiences" and the "failure of the curriculum to excite interest and provide motivation" as two of the four factors contributing to the decline in

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[2] A full copy of this speech is still available on the Internet e.g.: http://education.guardian.co.uk/thegreatdebate/story/0,9860,574645,00.html last accessed on 08.08.13.

[3] Commonly known as The Cockcroft Report after the committee's chairman, Sir William H Cockcroft.

[4] 'Interestingly' this also happens to be 3^5.

[5] An announcement that the strategy was to be disbanded was made on June 24th 2009.

pupils opting to study mathematics beyond aged 16 [ibid: 12-13]. Further confirmation of the disheartening inference that little had apparently changed since the 1970s was provided by a study by Brown, Brown and Bibby [2007, 2008] discussed below in Section 2.3.

Furthermore, a key finding from an Ofsted report on 26 institutions teaching the 14-19 age group was that, amongst the factors which acted against effective achievement, motivation and participation, was: "teaching which presents mathematics as a collection of arbitrary rules and procedures, allied to a narrow range of learning activities which do not engage students in real mathematical thinking" [Ofsted 2006: 2]. Moreover, two years later a second Ofsted report [2008] based on evidence from 192 school (primary and secondary) inspections of mathematics over nearly a three-year period, (depressingly) reported in the Executive Summary that " evidence suggests that strategies to improve test and examination performance … coupled with a heavy emphasis on 'teaching to the test', succeed in preparing pupils to gain qualifications but are not equipping them well enough mathematically for their futures" [ibid: 4]. The prevalence of 'teaching to the test' and its negative consequences is echoed in the findings of the Initial Study and provides a stark contrast with the engagement with mathematics revealed in the Main Study.

Similarly, Schoenfeld [1989] in the USA, bemoans the fact that students reiterated the teachers' rhetoric that the most important thing to do to pass exams is to memorise. However, it appears that it may have been forever thus. In 1938, giving a national address to UK mathematics teachers, Hogben shared his belief that teachers who stimulated their pupils by working beyond a tight syllabus would obtain better examination results than "the teacher who keeps one eye glued on the syllabus" [Hogben 1938: 113]. Hogben's view recurs in Boaler's [1997] longitudinal study based in two pedagogically contrasting schools. Thus, although in England changes to the mathematics curriculum [see e.g. the national documents listed above] embrace the broadening of the mathematical processes and experiences as proposed by *'Mathematics Counts'*, this is not always seen in practice.

A pursuit of good results within an examination system in an overcrowded curriculum often leads to lessons being reduced to exposition followed by examples practice, with all too few teachers feeling frustrated that they cannot afford to give time to 'play' with the subject [e.g. see Hatch 2002; Watson 1994]. These authors argue that, although expanding mathematical activities to have a more open nature might mean that they take longer to execute, such activities are likely to increase interest and ‘fun’. Moreover, such activities will ultimately better situate pupils to perform equally well, if not better, within the constraints of the examination system [Boaler 1997; Romberg & Kaput 1999: 3]. This point is further supported by key findings from Ofsted quoted at the start of this chapter and at the beginning of the following section on pupils’ perceptions of their mathematics lessons.

2.3 Pupils’ Perceptions of School Mathematics

> Pupils wanted to do well in mathematics. They knew it was important, but were rarely excited by it … [Ofsted 2008: 6]

One of the concerns highlighted in ‘*Mathematics Counts’* was the general public’s negative reaction to the subject [DES 1982: 2]. Mathematics remains an unpopular subject, one that adults do much to avoid. Unlike other subjects, adults often openly (and often proudly) profess to be poor at mathematics [Sam 2002]. Mathematics’ poor image is also apparent in its general unpopularity amongst many school pupils [Brown et al. 2007, 2008; Miller, Parkhouse, Eagle & Evans 1999; Nardi & Steward 2003].

Miller et al. [1999] undertook a survey involving over 6000 pupils in nine secondary schools responding to a sixty item questionnaire in one of the three English NC core subjects: English, mathematics or science. Out of the three subjects, mathematics had both the lowest enjoyment and fun factor; only a third looked forward to their mathematics whilst a quarter wished they did not have to go to mathematics lessons at all. Only just over a quarter thought that mathematics was fun. Of equal concern are the approximately one in three pupils who never found mathematics interesting. Admittedly there were some who felt the same in the other two subjects, although the ratios for these had dropped to around one in four. In a one-year study of year 9 (13-to-14 year-olds) and involving 74 interviews, Nardi and Steward found that there was an overall feeling that mathematics was a “tedious”, “isolated”, “elitist” subject, involving “rote-learning” and “rule-and-cue following” activities [Nardi & Steward 2003: 361-362]. “I would rather die” [Brown et al. 2007: 18, 2008: 3] were the first few words used for the title of a paper which reported on an analysis of responses to a questionnaire about future participation (or not) in further mathematical studies from a sample of over fifteen hundred GCSE (15 to 16 years old) students from seventeen schools[6]. The authors suggested that the results supported previous studies in finding that lack of confidence and perceived difficulty were the two major reasons for not continuing mathematical studies. The major reason for continuing was enjoyment of the subject and, as such, the factor that differentiated the schools with high/low participation rates. In a reduced sample size of 400 students in five schools, 37% of the respondents chose to tick the word 'bored', also the most frequent selection, from a multiple-choice of ten words. Such findings add evidence to the claims made in UK governmental reports, referenced above, that there is a failure to excite and motivate pupils. Although these three separate studies reported in this paragraph span different ages, sample sizes, durations and research methods, taken together they show current students' perceptions of mathematics as far from positive.

The subject's unpopularity is not exclusive to the domain of the English education system. Some ten years earlier than the Miller study referred to above, Schoenfeld's research in the US shows the same difficulties. Using a questionnaire of seventy closed and eleven open questions administered to 230 mathematics students enrolled in Grades 10 to 12, comparisons were made between students' perceptions of mathematics, English and social studies, again with mathematics scoring poorly [Schoenfeld 1989].

The findings of Hodgen, Küchemann, Brown and Coe's [2009] research differ from other studies, possibly due to a younger age group being involved. In the analysis of data from 1422 Key Stage 3 students (11 to 14 year-olds) it was found that for years 7 and 8 (11 to 13 year-olds), just over 60% gave a positive response to enjoying Mathematics lessons, though this dropped to 50% for the girls in the following school year. Other differences, unrelated to the focus here, led the authors to state that they were surprised that results seemed at odds with other research and this would require further investigation. These results were proposed as preliminary findings and used only data from one section of a larger four-year project and could therefore be considered subsidiary and a by-product to the main thrust of the research.

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[6] Both Nardi & Steward [2003] and Brown et al. [2008] were selected for the Preamble to the thesis.

Nevertheless, all of the studies discussed above agree substantially with Ofsted findings that most secondary pupils were "relatively ambivalent about mathematics … most said they 'quite enjoy it' but few cited mathematics as their favourite subject, even those who were doing well" [Ofsted 2008: 53]. NRICH website materials are intended to allow children to enjoy mathematics, to experience challenging and interesting problems, and to find and maintain a love for the subject.

Furthermore, [Ofsted 2006: 1] claims "The best teaching gave a strong sense of the coherence of mathematics ideas; it focused on understanding mathematical concepts and developed critical thinking and reasoning". The NRICH website supports teachers wishing to follow this ethos by publishing problem-solving, investigational activities that can be incorporated into their classroom teaching. The next section contains a discussion on what a mathematical problem is and what it means to do problem-solving.

2.4 Mathematical Problem-Solving

> [Problem-solving is] the joy of confronting a novel situation and trying to make sense of it - the joy of banging your head against a mathematical wall, and then discovering that there may be ways of either going around or over that wall. [Olkin & Schoenfeld 1994: 43]

The phrase "the ability to solve problems is at the heart of mathematics" appears unattributed in *'Mathematics Counts'* [DES 1982: 73] though others [Ewing 2007: 1141; Larson 1994: 30; Stacey 2005: 341] correctly attribute the sentiment to Halmos [1980] who also considers "the mathematician's main reason for existence is to solve problems" [ibid: 519]. Polya is widely acknowledged as the 'father' of problem-solving [Mason & Johnston-Wilder [2004: 186] and his book *'How to Solve it'* first published in 1945 and re-issued [Polya 1957] remains a seminal text. Polya [1962: v] defines solving a problem as a means of "finding a way out of a difficulty, a way round an obstacle, attaining an aim that was not immediately attainable" but this is only one of many descriptions that have been offered over the intervening years [Mayer 2002; Mayer & Wittrock 2006; Olkin & Schoenfeld 1994; Piggott 2005]. Indeed, Schoenfeld [1994: 50-51] implies that obtaining a definition for problem-solving may not be a straightforward task. He found that a Universal Dictionary published in 1979 gave two definitions of the term problem; Definition 1: A question that is perplexing or difficult and Definition 2: In mathematics, anything required to be done. Thus defining precisely what is meant by both a mathematical problem and the process of problem-solving in mathematics is itself clearly problematical. Piggott suggests that, at the most fundamental level, a mathematical problem is when:

> you have to start with a question or context which needs to be resolved but it becomes a problem only to those who do not immediately know the answer, that is, it is only a problem for a particular individual at a particular time. Piggott [2005: 77]

and the goal for that individual is finding the solution. Mayer [2002: 70] makes a similar point saying that a problem exists when there is a goal that cannot immediately be reached. This definition therefore covers any mathematics studied, since taken to an extreme, a routine, repetitive exercise might be classed as problem-solving in the eyes of the 'worker'. Similarly, Wilson, Fernandez and Hadaway [nd] point out, anybody "not enthralled with mathematics may describe any mathematics activity as problem-solving". However, such a view [and also Dictionary Definition 2 above] does not take into account the joy in obtaining a solution expressed in the quotation at the beginning of this section that Olkin and Schoenfeld [1994], more in line with Dictionary Definition 1, imply accompanies a good problem.

Mayer and Wittrock [2006] considered all mathematics questions to be problems, but then typify a problem as either well-defined (it is clear what and how to do) or ill-defined (open), and also as either routine or non-routine (depending on the knowledge of the learner). They stress that "most real problems are ill-defined" and "important real-world problems are genuinely non-routine" [p288]. However, the literature cited earlier in this chapter implies that the problems forming much of the mathematical experiences within the core of school education would be classed as well-defined and routine and thus, in the widest sense, not genuine problems. Indeed, Schoenfeld [1994] had earlier suggested that people differentiate between *memory-school* mathematics and *creative-outside* mathematics, where memory gives way to the pursuit of the problem.

However, whilst emphasising important differences, Mayer and Wittrock's categorisation above appears to over-stress the exclusive, high value of 'real-life' problems and too easily dismiss the value of ill-defined, non-routine *pure mathematical* problems. Both real-life and pure problems fit the category of '*rich tasks*' such as those found on the NRICH website that: have multiple methods of solution; illustrate important mathematical ideas; and have an accessible entry point commonly referred to as *low threshold*, but with the ability to stretch the most able, *high ceiling* [see Hewson nd; Piggott nd]. The quandary of whether one or both of these types are 'proper' problems continues to be debated. For example, Blum and Niss [1991] separate problems into 'mathematical' and 'real world' [p38]; Haylock and Cockburn [2008] have similar terminology placed on 2-D axes (abstract mathematics / real-life context on one axis, open/closed on the other) and Orton [2004: 84] suggests that one simple classification would be routine practice problems, word problems, real-life applications and novel situations.

The literature contains many examples of studies of students participating in problem-solving activities and approaches within a classroom context [Bernardo 2001; Francisco & Maher 2005; Goos, Galbraith & Renshaw 2000, 2002; Nunokawa 2005] that treat problem-solving as a method/process rather than an end in itself [Stacey 2005]. Since problem-solving became a more explicit part of the curriculum, attempts have been made to distinguish between the role and purpose of problem-solving within the curriculum. This has evolved as teaching: *for* problem-solving (an activity in its own right); *about* problem-solving (what skills are required and how they can be used) and *through* problem-solving (using it to learning new mathematical concepts) [Piggott 2005: 80; Stacey 2005: 345]. All three are very much present in Polya's texts [1957, 1962] and Mason, Burton and Stacey [1982], drawing on Polya's work, have these as a central focus throughout their text '*Developing Mathematical Thinking*'. More recently McClure and Piggott[7] [2007] added a fourth *teaching to motivate* as "it [problem-solving] can provide a context or incentive for doing mathematics" [p72].

2.5 Pupils' Perceptions of Problem-Solving

Nardi and Steward [2003], having reported their findings of "quiet disaffection" [p345] discussed above, go on to stress that the negative attitudes found should not "be confused with an overall unwillingness to engage with mathematics as such" [p363], the same sentiment reported by Ofsted [see Section 2.2 above]. Students involved in Nardi and

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[7] Coincidentally later becoming the current and previous Directors of NRICH.

Steward's study were willing to suggest aspects that they felt were personally effective in their mathematics learning. Nardi and Steward [2003] grouped these under themes, the most prominent of which they labelled "Nature of Classroom Activities – the notion of 'Fun'", and further reported that there was "strong evidence in the data that students perceive enjoyment (relevance, excitement, variety) to be central to learning" [p363]. Although students had differing ideas as to what precisely constituted fun, this could usually be found in lessons which had "a format that was varied or dynamic and a context or content that was practical or relevant" and specific activities that could be described as "games, puzzles or investigations" [ibid].

Ofsted further reports that there was strong agreement (by both primary and secondary pupils) on the features of mathematics lessons they did/did not enjoy:

> 'It's fun working in groups' and 'Working with someone else helps you understand, especially if they ask you questions' … many pupils, especially in secondary schools, described a lack of variety, which they found dull … 'Every lesson, you have to answer questions from the textbook. It gets boring'. [Ofsted 2008: 53]

The 'dull' and 'boring' experiences can be contrasted with "occasional lessons they enjoyed where they did investigations, tackled puzzles, sometimes working in groups, and used ICT independently. Often such lessons happened at the end of term and were regarded as end-of-term-activities rather than being 'real maths' " [ibid]. Thus it would appears that these enjoyable lessons could only be 'special treats'.

2.6 'Doing Mathematics'

> If I'm having to remember … then I'm not working on mathematics.
> [Hewitt 2002: 47]

In a chapter beginning with this apparently provocative statement, Hewitt makes a distinction between two types of facts – *Arbitrary* (facts that need to be given, such as the decision to call a square, a square) and *Necessary* (things one can work out for oneself by making connections, such as a quarter turn) [ibid].

Less provocatively, Mayer and Wittrock [2006: 289], using mathematical problem-solving to illustrate their ideas, suggest different permutations of two classic constructs used to measure learning outcomes, *retention* (ability to remember which can be assessed using recall and recognition items) and *transfer* (ability to use what was learned in new situations). If from a set of test items students perform well only on retention type questions then rote-learning has taken place; if students perform well on both types then meaningful learning, which they do not define, has taken place; if neither, then no learning has taken place. [See Orton 2004: 184-191 for Ausubel's [1968] general theory of meaningful learning, annotated with mathematical examples].

In a similar vein Greeno, Collins and Resnick [1996] state:

> Most students who learn to recite definitions and formulas that express meanings of concepts in general terms, or to carry out procedures with numbers or formulas, show limited proficiency in solving problems and understanding other situations in which those concepts or procedures could be used. [p29]

that not only echoes sentiments implied or explicitly expressed in several papers cited in earlier sections of this review, but can be used to exemplify the prevailing view of mathematics educators. It is hard to find any dissenting view in the literature.

Hewitt's *Arbitrary* facts would map onto retention/rote-learning, and *Necessary* facts to transfer/meaningful learning. I describe Hewitt's statement as provocative; it is an example of a statement expressing an apparently, on first reading, extreme view in order to 'provoke' the reader into serious consideration of the issue. In this study, I shall interpret the statement as also making a crucial connection between 'learning' and ways of 'working'.

Different ways of working are closely examined in Anthony's [1996] paper reporting on her one-year study of a class of twelve Year 12 students (17-years-old) in a New Zealand school. She presents two cases, 'Gareth' and 'Adam' to illustrate contrasting extremes of mathematical learning behaviours. One she considers *passive* the other *active*. Passive learning is receiving, recording and repetition of knowledge: "listening to the teacher's exposition, being asked a series of closed questions and practice and application of information already presented" [p350]. Active learning could be defined as the direct opposite: "denoting learning activities in which students are given autonomy and control of the direction of the learning activities … investigational work, problem-solving, small group work, collaborative learning and experiential learning" [ibid]. However, as Anthony points out, there is a second, and somewhat different definition that shifts the focus to the individual's mindset: "a quality of the pupil's *mental* experience in which there is active intellectual involvement in the learning experience characterised by increased insight" [Kyriacou and Marshall 1989: 2]. This 'mind' definition is independent of the passive/active classroom activities as an individual may experience a passive learning, but mentally construct their own knowledge, though the construction in these circumstances is likely to be weaker than in active classroom activities [Noddings 1990]. Orton [2004: 134] makes a similar point when he describes an *intentional* learner as someone who builds strong connections with the subject, ultimately more successful than someone who is weak in transferring that knowledge, content just to pass the examination. The defining distinction between Gareth, the passive learner and Adam, an active learner, is Gareth's reliance on rote memory, in essence only able to reproduce identical solutions that had been offered to him in class. On the other hand, Adam[8] moved away from such restrictions, he was: proactive in establishing connections to previous knowledge; self-questioning to construct new knowledge; completing exercises of his own choosing and frequently undertaking additional reading: "I think it is good to read ahead" [Adam as quoted in Anthony 1996: 359].

The '*Enhancing Teaching-Learning Environments in Undergraduate Courses Project*' based at Edinburgh University co-directed by Entwhistle and Hounsell [ETL nd] presents the same categorisation of ways of working/learning but with different terminology. The project used a range of subjects/courses though none were specifically mathematical. One aspect of the project focuses on different approaches to learning [Entwhistle 1998; Marton & Booth 1997; Marton & Saljö 1997]. Two approaches to learning, reminiscent of the definitions already given above in a school context, are presented as opposites: Deep (seeking meaning by relating ideas) and Surface (reproducing using routine memorising and a narrow focus on task). A third, the Strategic Approach (achieving success through organised, self-regulated effort) is then introduced, which can be directly related to the second definition of active learning of Kyriacou and Marshall [1989] above and illustrated by Adam in Anthony's 1996 study. Sawyer [2006] presents a similar overview, summarising a Learning Sciences standpoint reproduced in Figure 2.1 below, where the left-hand column

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[8] By all accounts Adam would have made a fine AskNRICHer.

contains a list of descriptors that match well to a combination of ‘Deep’ and ‘Strategic’ as outlined above, although headed with the term Knowledge juxtaposed with Learning. The descriptors in right-hand column, labelled traditional classroom practices, match with ‘Surface’ and also have similarities with van Lier’s exposition of Transmission [see LRIII later]. It will be seen later that several of the AskNRICHer’s appear to exhibit ‘Deep’ and ‘Strategic’ learning approaches in their activities.

Learning Knowledge Deeply (Findings from Cognitive Science)	*Traditional Classroom Practices (Instructionism)*
Deep learning requires that learners relate new ideas and concepts to previous knowledge and experience.	Learners treat course material as unrelated to what they already know.
Deep learning requires that learners integrate their knowledge into interrelated conceptual systems.	Learners treat course material as disconnected bits of knowledge.
Deep learning requires that learners look for patterns and underlying principles.	Learners memorize facts and carry out procedures without understanding how or why.
Deep learning requires that learners evaluate new ideas, and relate them to conclusions.	Learners have difficulty making sense of new ideas that are different from what they encountered in the textbook.
Deep learning requires that learners understand the process of dialogue through which knowledge is created, and they examine the logic of an argument critically.	Learners treat facts and procedures as static knowledge handed down from an all-knowing authority.
Deep learning requires that learners reflect on their own understanding and their own process of learning.	Learners memorize without reflecting on the purpose or on their own learning strategies.

Figure 2.1 Deep Learning Versus Traditional Classroom Practices [reproduced from Sawyer 2006: 4]

The discussion so far has focused on learning rather than on understanding. Understanding is a concept “difficult to capture in words” [Mason & Johnstone-Wilder 2004: 310] but “closes a cycle, for deciding what it means to understand is very close to deciding what it means to learn” [ibid: Epilogue]. The discussion now turns to examining definitions of understanding and starts with Skemp’s (simple) distinction.

2.7 Mathematical Understanding

In Mathematics Education, Skemp's work of the 1970's on understanding is still much used (and debated) within school mathematics [Goulding 2011] and remains an important reference in classroom research: e.g. failing to connect fractions and irrationals as numbers [Schmittau 2003: 228]. Although Skemp had originally published *'The Pyschology of Learning Mathematics'* in 1971 [re-published Skemp 1987], it is a short seven page article [Skemp 1976] in '*Mathematics Teaching*', the relatively lowly professional magazine of the Association of Teachers of Mathematics (ATM) that had the impact and gained the status of a seminal and universally cited work. In the article Skemp acknowledges the ideas and definitions that Mellen-Olsen had some years earlier brought to his notice and goes on to distinguish *instrumental* understanding, knowing the rule but not the reason (a collection of rules), and *relational* understanding, knowing both what to do and why (an integrated system of knowledge) [ibid]. Although these may seem to be two disparate definitions, it is not the case that it should be all of one and none of the other. Skemp critically points out the advantages of each, making clear the importance of developing relational understanding rather than remaining with the limitations of instrumental. That is, Skemp implies a continuum of understanding [Duffin & Simpson 2000]. However limiting the definitions of understanding to two 'different types' keeps the construct simple enough for investigating teaching practices and pupils' perceptions, i.e. appropriate for the Initial Study. Indeed the Ofsted reports cited above arrange their discussions and reporting with implicit reference to Skemp's ideas. Furthermore this division of instrumental/relational understanding relates directly to work on learning already cited: Marton and Booth [1997] and Marton and Saljö [1984] works on surface and deep approaches; Mayer and Wittrock [2006] retention and transfer; Anthony's [1996] passive and the second form of active learning, in the sense of 'closing the cycle' referred to in the previous paragraph. Finally, an interpretation of Hewitt's [2002] provocative statement above is that without some relational understanding one cannot be truly learning mathematics.

Nonetheless, there are more developed and finer distinctions that add increasingly complex layers of categorising understanding that go beyond Skemp's bipartite division. These provide a more sophisticated perspective from which to consider the detail of the content of some of the AskNRICHers' exchanges in the Main Study.

Byers and Herscovics [1977: 26], for example, initially added *intuitive* (obtain a solution without prior analysis) and *formal* (combine integrated ideas into chains of logical reasoning), but they later dropped Skemp's nomenclature to devise, within a constructivist framework: *intuitive* (informal knowledge), *initial conceptualisation* (procedure to first construction of a concept), *abstraction* (concept gains precision and detached from procedure) and *formalisation* (of content) [Herscovics 1996: 356]. This division was refined even further to produce a two-tiered model for conceptual understanding: understanding of preliminary physical concept and moving to an understanding of emerging mathematical concept [ibid: 361]. When explained in this way, these models again evoke the sense that these 'listed' forms could be considered to lie on a continuum line.

On a more abstract level, Duffin and Simpson [2000] describe their search to come to a personal understanding of the term understanding in the context of their on-going work developing a personal theory of learning. They were nudged into this by mathematical incidences that they came across in their professional lives working with undergraduates and trainee teachers. Sierpinska's [1994] book *'Understanding in Mathematics'*, again focusing substantially on advanced, undergraduate mathematics takes a similar track in as far as the underlying theme was to gain "a better understanding of how real people understand mathematics in real life" [pxv]. The motivation for both of these studies was to find out, for want of better words, 'How to teach so that students understand'. Duffin and Simpson [2000] first considered what "internal characteristics" [p417] and "external manifestations" [p418] might be discerned when someone had some understanding. This led to a three-component definition of understanding: *building,* a formation of connections using understanding that an individual already has at hand ready to use; *having*, the state of connections that could be used at any particular time; *enacting*, use of available connections in the moment [ibid: 420]. A 'tool-box'[9] seems a good analogy here, equating the state of connections with tools. *Building* is therefore adding tools to the tool-box following further experiences which are subsequently kept; *having* is the tools currently in the 'tool-box' and *enacting* is using what tools are in the 'tool-box'. At the very least "understanding is not static" [Watson 2002: 162].

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[9] Coincidentally, 'tool-box' is a term used by some AskNRICHers in their discussions on problem-solving [see Chapter Nine], though it too serves a similar analogy.

This chapter has so far focused on areas that primarily contextualised current thinking and practices in school mathematics teaching and learning. The theme of 'doing mathematics' is now pursued further with a review of literature which deliberately avoids the difficult issue of attempting to define a mathematician, merely considering traits associated with a person who may be regarded as a mathematician.

2.8 Mathematicians

> Of course, by mathematicians, we mean more than just the members of AMS [American Mathematical Society]; we mean the ***people who do mathematics***. Some mathematicians are children; some would never call themselves mathematicians. [Cuoco, Goldenberg & Mark 1996: 384 my emphasis]

A Google search for the exact phrase 'definition of a mathematician' yielded only 31 results and no serious, widely cited definition. It produced such 'gems' as: "[the person] you don't want to meet at a party" [Barrow 1992: 5] and "a blind man in a dark room looking for a black cat which isn't there" attributed to Charles Darwin [various sources including Nahin 2006: 352] and thus no helpful results to reproduce here. Dictionary definitions such as, for example, "an expert in or student of mathematics" [OED online nd] are similarly unhelpful.

Phillips found that asking both children and adults to describe a mathematician elicited a myriad of simple descriptions [Upitis, Phillips & Higginson 1997]. She gives a list of responses [see Table 2.1] that I have re-ordered so that the first eight are descriptors that can also directly apply to school pupils[10].

Although these descriptors are not overly informative, some of these simple descriptions, such as "a person who studies patterns" recur in other literature examined later. However, Phillips, who refers to herself as a primary school teacher, researcher, and mathematician, goes on to develop her own definition:

> My belief is that a mathematician is someone with a special way of viewing his or her world … has the ability to stop and enjoy mathematical beauty … can generalise from specifics and who can generate specifics from generalisations … can create a multitude of strategies to solve a problem … Hence, professional life is an open-ended adventure for a mathematician. [p140]

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[10] The remaining four may well be present amongst the AskNRICHers either now or in the future!

What exactly defines a mathematician?
When I have asked this question to children and adults alike,
I routinely get answers that include

someone who is good at maths
not a person who just does computations
someone who likes to solve problems
a person who studies patterns of numbers
a person who see maths in everything
a person who forms hypotheses and works on answers using numbers
a very logical, ordered person
a person who uses the creative strategies to solve the logic problems

someone who works with numbers
a genius or a brain
like an accountant or a banker
a person who teaches maths at university

Table 2.1 Defining a mathematician [Upitis, Phillips & Higginson 1997: 139]

In addition to substantiating the views expressed in the literature reviewed in previous sections, Phillips' definition concurs with Burton's work [1999] in which the world of professional (paid to undertake research) mathematicians is described as: "a world[11] of uncertainties and explorations and the feelings of excitement, frustration and satisfaction associated with these journeys, but above all a world of connections, relationships and linkages" [p138]. In other work Burton [1995, 2004] identified the aesthetic feeling gained from doing mathematics and the nurturing of intuition and insight as important parts of the process of coming to know mathematics.

Whereas Burton's focus is on Professional Mathematicians, the ultimate goal of Cuoco et al. [1996], a group of academic mathematics educators, was developing a school mathematics curriculum. The quotation that appears at the start of this section is a footnote in their paper discussing the "habits of mind" of people who create mathematics. The authors highlight the gap between a subject that is studied in school called mathematics, and the way that mathematics is created and applied outside of the school environment [ibid: 375]. School mathematics is disjointed in that it tends to consist of a set of discrete skills studied independently, as illustrated by the original analogy of the apocryphal footballer who learns to dribble but never plays the game[12], attributed to Wiggins

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[11] Peter's opening quotation in Chapter Eight illustrates just such a world.

[12] a concept pursued in Chapter Eleven.

[cited in Greeno et al. 1996] or the woodworker who practices the joints but never makes a [coffee] table [Schoenfeld 1989].

Cuoco et al.'s [1996] proposal intended to help pupils to learn and adopt some of the ways that "mathematicians *think* about problems" [p376] … "mental habits that allow students to develop a repertoire of general heuristics and approaches that can be applied in many different situations" [p378]. Habits of mind that students should acquire are those of: *Pattern Sniffers, Experimenters, Describers, Tinkerers, Inventors, Visualisers, Conjecturers and Guessers* [pp378-384]. Although each habit is illustrated by a mathematical example, the authors claim that these habits are generic. From this they progress to listing habits that are not so common outside of mathematics, i.e. ways that mathematicians approach things: *Talk Big & Think Smal*l (i.e. consider special cases), *Talk Small & Think Big* (simple problems can give rise to deep and/or general theories), *Use Functions* (studying the mechanism of change), *Use Multiple Points of View, Mix Deduction and Experiment, Push the Language, Use Intellectual Chants* (meditating or reflecting on ruminations that occur when working away at a problem)[13]. There are some striking similarities between this list and Phillips' prose account of what she feels is being a mathematician, for example Phillips' 'generalise / specifics …' [Upitis et al. 1997: 140] parallels 'Talk / Think Big / Small …' here.

In the preface of *'How to Think Like a Mathematician'* Houston [2009], a university lecturer in mathematics[14], presents some friendly, practical advice [see Table 2.2] aimed at undergraduates on 'doing' and 'working' on mathematics. The motivation for publication Houston gives is similar to that of Cuoco et al. in wishing students to experience exciting mathematics: "I aim to make [students] free to explore, give them the tools to climb the mountains and give them their own compasses so that they can explore other mathematical lands" [pxi]. Houston too recognises the limitations of school mathematics in not promoting such habits of mind. Although expressed in a different way to Cuoco et al.'s listing of habits of mind, Houston's friendly advice is suggestive of how to acquire such habits.

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[13] Not content to stop here, Cuoco et al. assemble further lists for a Geometers' special stash of tricks of the trade [p389] and a special collection of habits of mind for algebraists - people in 'algebra mode' [p393].

[14] This makes Houston a mathematician according to one criteria on Phillip's list and a candidate for Burton's study.

Some Friendly Advice

It's up to you – Your actions are likely to be the greatest determiner of the outcome of your studies. Consider the ancient proverb: The teacher can open the door but you must enter by yourself
Be active – Read the book. Do the exercises set
Think for yourself – Always good advice
Question everything – Be sceptical of all results presented to you. Don't accept them until you are sure you believe them
Observe – The power of Sherlock Holmes came not from his deductions but from observations
Prepare to be wrong – You will often be told you are wrong when doing mathematics. Don't despair; mathematics is hard, but the rewards are great. Use it to spur yourself on
Develop your intuition – But don't trust it completely
Collaborate – Work with others if you can to understand the mathematics. There is no competition. Don't merely copy from them though
Reflect – Look back and see what you have learned. Ask yourself how you could have done better

Table 2.2 Extract from How to Think Like a Mathematician [Houston 2009]

Houston's inclusion of 'intuition' makes an immediate connection to Burton's work above. Equally the advantageous reasons that he gives for 'prepare to be wrong' reflect Burton's difficult but rewarding journey. Moreover, collaboration, the penultimate item in Houston's list, was a main focus of Burton's research and is addressed in a later literature review chapter [LRIV].

Indeed, the situations and people perspectives in /from which the authors present their arguments are all different: Phillips (general public and teacher); Cuoco et al. (curriculum development); Burton (professional research mathematicians); and Houston (teaching undergraduates mathematics). Nevertheless, they all portray traits which form overlapping and consistent pieces of an indistinct, incomplete picture[15] of that imprecisely defined creature: a mathematician.

2.9 Conclusions

This chapter has presented a literature review with a primary focus on school mathematics, giving a historical overview of teaching practice, a discussion of different views of problem-solving in teaching mathematics together with pupils' perceptions of mathematics and problem-solving. The concept of 'doing mathematics' has been explored by examining mathematical learning and understanding and considering traits of mathematicians. The topics addressed in the review are inter-related. For example, the identification of (apparent)

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[15] one might say Turneresque.

deficiencies in school mathematics is substantiated in the findings reporting pupils' perceptions. This agreement in findings reflects the type of mathematical experience that is considered to hinder mathematical study beyond the over-riding goal of passing tests/examinations. As a further example, a richer and more interesting mathematics curriculum, inclusive of problem-solving activities, requires a different, more motivating way of working advocated in the literature.

The NRICH website, with its kernel of pure mathematical problems, some of which are clearly portrayed as (mathematical) games, puzzles and investigations, has been recognised as a valuable resource for any school mathematics curriculum [Hodgen & Wiliam 2006; Koshy & Casey 1987]. NRICH problems match the 'out-of-the-routine' and 'away-from-the-textbook' type of mathematical problem-solving promoted in the literature. NRICH problems are 'rich tasks', far less reliant on exposition and following routine exercise, thus inherently providing pupils with opportunities to be creative and experience the much advocated high energy [Hatch 2002] and playful [Watson 2006] lessons.

As will be seen in the two chapters that follow describing methodology and findings, the literature reviewed here was influential in establishing the focus of the Initial Study expressed in **RG1**: *to investigate pupils' perceptions of doing mathematics in school and of using NRICH type problems in home/school settings*. Similarly this review led to specific lines of inquiry, in particular, perceptions of and engagement with problem-solving and desire for understanding, within the series of questionnaires, web-survey and interviews that address **RQ2**. In addition the predetermined codes used to assign a series of rudimentary characterisations to web-survey respondents were derived from a synthesis of inferences from the literature reviewed here. The focus on the home setting of **RQ1** means that the influence of this review on it is less direct. Nevertheless, the topics reviewed, especially in Sections 2.6 to 2.8, provide a context allowing comparison of what the Initial Study's participant population, and later the AskNRICHers, may achieve and what is commonly found in school.

Chapter Three

Overview of Methodology for Initial Study

> *We hope the field will move beyond quantitative versus qualitative research arguments because […] both […] are important and useful. The goal of mixed methods research is not to replace either of these approaches but rather to draw from the strengths and minimise the weaknesses of both ...* *[Burke-Johnson & Onwuegbuzie 2004: 14]*

3.1 Introduction

The differences between the processes of inquiry in the Initial and Main Studies necessitated the adoption of separate, epistemologically compatible, methodological approaches. This chapter describes the mixed methods [Teddlie & Tashakkori 2009] methodology, methods and instruments adopted to undertake the Initial Study: an investigation of pupils' general perceptions of doing mathematics in school and using NRICH type problems at home. This provided purposeful contextual background for the Main Study.

RG1: To investigate pupils' general perceptions of doing mathematics in school and of using NRICH type problems in home/school settings	
Research Questions	**RQ1: What are the common practices of using NRICH problems in the home context?** *Why and with whom do these students do NRICH problems at home? To what extent do these students perceive their teacher knowing that they do mathematics problems at home? Why do these students not tell their teacher?* **RQ2: What views do students using an on-line mathematics resource (NRICH) have concerning their experience of school mathematics?** *What are students' perceptions about doing mathematics puzzles and problems in lessons? What are students' perceptions about the relative merits of rules, methods and understanding? How do these students seek help with school mathematics?*

Table 3.1 Research Goal and Research Questions for the Initial Study

As stated in Chapter One findings from two small, earlier evaluative studies [Jared 1997, 1998] shaped the research goal **RG1** and associated questions (**RQ1&2**) of the Initial Study [see Table 3.1[1]]. These were addressed through an analysis of qualitative and quantitative

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[1] This Table and Table 3.2 were first presented in the appendices accompanying Chapter One.

data derived primarily from a web-survey with additional material from interviews and questionnaires.

The purpose of this chapter is to:

i. present a summary of the research and the rationale and timing of each step
ii. indicate the types of participants involved at different times
iii. describe the methodology adopted
iv. detail the research design for the questionnaires, web-survey and interviews
v. explain the management of the data obtained, the system of coding and the characteristic codes derived

The remaining three sections of this chapter parallel the division used in setting out its purpose above. Section 3.2 provides a research summary incorporating purposes (i) and (ii) with the methodology adopted, purpose (iii), reported in Section 3.3. Section 3.4 encompasses both the research design (purpose iv) and the management of data and derivation of coding (purpose v).

3.2 Research Summary

The research is summarised via its timeline in Table 3.2 below. This also tabulates the instruments used, the number and description of participants involved, the rationale for each step and its contribution to later stages of the Initial Study.

The kernel of the Initial Study was a web-survey posted on the publicly accessible NRICH website and therefore open to respondents of any age, worldwide.

A series of paper-based questionnaires were trialled during a developmental period in four schools [see 1, 2 and 5 in Table 3.2] and further data was collected through interviews with two pupil-groups and two teachers [3 and 4]. Although the results of these School-based Investigations are not reported, the responses and findings of these preparatory questionnaires and interviews informed Initial Study Stage Two [Primarily Online Investigations]. Crucially, it determined the final content of a web-survey [6].

Concurrently with analysis of the web-survey responses, two individuals known to enjoy doing mathematics problems out of school were interviewed separately [7 and 8] and a further school visit made to conduct interviews with three pupil-groups, selected by their questionnaire responses [9 and 10].

Date	Instrument	Participants	Rationale/Use
Initial Study Stage One: School-based Investigations			
1 July 2004	Questionnaire PQ1	31 pupils across ability range in years 7 to 9 in local comprehensive school	Pilot Study
2 July 2005	Questionnaire PQ2	124 pupils in years 7 (63) and 8 (61) across ability range in two local schools	Adaptation of pilot study and used as basis for interviews
3 July 2005	Tape recorded Group Interviews	Two groups of four pupils, two girls and two boys, one group from each of year 7 and year 8 in one of the schools used in 2 above	Gaining additional responses to open questions on questionnaire
4 July 2005	Tape recorded Interviews	Two teachers, one from of each of the schools used in 2 above	Gaining background information from teacher perspective of problem-solving opportunities within the curriculum
5 July 2006	Part of Questionnaire PQ3	51 pupils in years 7 (27) and 10 top set (24) in local comprehensive school	Pilot re-design prior to completing web-survey design
Data analysis of above is not reported within this thesis. Evaluation undertaken of questionnaire design and quality of responses used to inform next stage of research is reported within this chapter.			
Initial Study Stage Two: Primarily Online Investigations			
6 January to May 2007	Web-Survey WQ	Public access – open to respondents (with Internet Access) of any age from any country. 117 replies used	Obtain general perceptions of school mathematics across a wide range of schools
7 May 2007	Tape recorded Individual Interview	17 year old male participant in NRICH for five years (from EU country)	Long time user of NRICH monthly problems
8 September 2007	Tape recorded Individual Interview	11 year old male	Known to do problem-solving at home without teacher knowing
9 October 2007	Questionnaire PQ4	15 A level Mathematics students (final year) in Boys Comprehensive	Additional information from high-achievers at school level
10 October 2007	Tape recorded Group Interviews	Three groups of 3/4 students from group used in 9 above	Gaining information from known users of Mathematics Internet Sites

Table 3.2 Timeline of Research Undertaken for the Initial Study

3.3 Mixed Methods Methodology

A combination of quantitative and qualitative methods was adopted throughout in order to obtain a broad view of the data as well as allowing in-depth analysis, as explained below. The research maintained a cyclical structure of collection, analysis and evaluation. The adoption of mixed methods for this Initial Study could be seen as working within a pragmatist paradigm “interested in both narrative and numeric data and their analyses” [Teddlie & Tashakkori 2009: 4]. However, this is compatible with my constructionist epistemological viewpoint [Crotty 1998: 5], introduced in Chapter One, since although I start by testing an earlier finding, my intention is to then *construct meaning for its existence*. The constructionist stance is set out in Chapter Six [Section 6.2.1 p107].

The advantages and disadvantages of quantitative compared with qualitative methods are extensively debated in the research literature [e.g. see Gall, Gall & Borg 2003; Miles & Huberman 1994; Wengraf 2001]. Pring [2004] presents some strong philosophical arguments as to the “false dualism” [p64] and the danger of drawing too sharp an oppositional divide between each type. Along with practical examples of how the two have been combined [e.g. see Burke-Johnson & Onwuegbuzie 2004], by 2010, with books [e.g. Teddlie & Tashakkori 2009] dedicated to the integration of the two research paradigms, the mixed methods research started some twenty years earlier [ibid: 4] appears far more widely accepted [Symonds & Gorard 2008]. A central argument is that the use of a mixed methods approach can be exploited to allow the limitations of one method to be balanced by the advantages of the other, as exemplified in this chapter’s opening quotation. Thus, for example, whereas quantitative data from *questionnaires* could provide a broad view, qualitative data could provide in-depth analysis of the issues raised from the quantitative data. In principle, study of longer, *interview* responses can reveal more depth [Cohen, Manion & Morrison 2000].

Thus in this study the early in-school paper-based questionnaires[2] [1 and 2 in Table 3.2] provided the opportunity to collect some basic quantitative data and, from open responses, add to emerging themes that, in turn, became the focus of further study through interviews

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[2] Sections 3.4.1.and 3.4.2 contain details relating to the questionnaires and interviews.

[3 and 4]. Equally, issues emerging from qualitative data could be further investigated via the collection of quantitative data involving a larger population sample. So, for example, the responses obtained from the interviews influenced the design and content of the web-survey [6]. The web-survey in turn provided further quantitative data that thus enabled the findings of the in-depth qualitative analysis of those earlier interviews to be seen in a wider context. Moreover, the analysis of findings from the web-survey prompted further qualitative investigations [7-10]. Thus the themes emerging from the web-survey could be studied in greater depth by accessing a smaller number of target groups and individuals to allow exploration through "the unique words of the respondents" [Gall et al. 2003: 223].

Hence, qualitative methods were advantageous in enabling a more detailed analysis of discourse. This added to the more general findings elicited from the questionnaires to provide the opportunity to expand on areas related to pupils' views expressed therein. Drawing on these multiple data sources and methods, with short written responses and an, albeit small, number of interviews enabled triangulation [Cohen et al. 2000: 112-115], enhancing the validity of both the process and the study [ibid]. Each method therefore complemented the other through an iterative process.

In addition, coding techniques [Strauss & Corbin 1997, 1998] were used to consider and categorise characteristics of web-survey respondents from three different viewpoints [see Section 3.4.4]. This was also, in effect, a means of experimenting with the coding techniques that would be employed in a more sophisticated way in the Main Study [see Sections 6.3.4.3 & 6.3.4.4 pp124-128].

This section has explained the rationale for adopting a mixture of quantitative and qualitative methods. The next section reports on the various research instruments used and the methods employed in analysing the resulting data.

3.4 Research Methods and Instruments

General surveying methods were adopted to maximise the sample size and to allow valid comparisons with the results of the two earlier studies that had used questionnaires as their research instrument. The rationale for using interview alongside the questionnaire has already been explained in the previous section. Both are commonplace research instruments with extensive associated texts, cited where appropriate in the following sections, supporting their design and use.

3.4.1 Survey Questionnaires

Although the findings reported in Chapter Four result from the web-survey, the strength of its foundations lies in three earlier paper-based questionnaires trialled over a two-year developmental period in four schools to achieve the best possible design. Decisions important in determining the final form of the web-survey and embodied in these precursors are consequently reported in Section 3.4.1.1 below.

All the questionnaires were devised for broadly descriptive purposes [Oppenheim 1992: 12] in other words, determining *what is* rather than *the why* [Gall et al. 2003] which would be left to be more substantially explored through interviews. Particular attention was paid to careful question design [Cohen et al. 2000] and devising effective questions [Munn & Drever 2004] which are both key to survey analysis and of extreme importance in avoiding either asking superfluous questions or ones that cannot be analysed [Gorard 2001: 92]. The two types of questionnaires, school/paper based [PQ1, PQ2, PQ3, PQ4[3]] and web-based [WQ] are described separately below.

3.4.1.1 In-school Paper-Based Questionnaires

The timeline of the four paper-based questionnaires' trialling and development is given in Table 3.2 with full texts presented in Appendices 3.1 to 3.4. Some of the questions of PQ1 were modified in PQ2 and an invitation to volunteer for a follow-up interview was added. PQ3 was trialled to evaluate a re-design of the question format as described below.

The construction of PQ1 and the development of questions in subsequent versions was influenced by consideration of a variety of items and formats from others' questionnaires [Boaler 1996; Dillman, Tortora & Bowker 1998; Miller et al. 1999; Op't Eynde &

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[3] PQ4 was implemented after the web-survey as indicated in Table 3.2.

De Corte 2003; Schoenfeld 1994]. The layout followed the instrument design lines detailed by Gorard [2001: 87-89]. The wording throughout all the questionnaires was designed to be 'pupil-friendly' by, for example, using: *Definitely, I mainly think so, Not sure, No not really* and *Certainly not* to select from a five-point Likert-type agreement scale rather than the formal ones more often given as exemplars [Cohen et al. 2000].

Analysing the results from PQ1 and PQ2 highlighted three recurring difficulties that needed consideration [Oppenheim 1992: 53]. The first concerned the poor quality of responses to open-ended questions. This was addressed by replacing such questions with a list of statements, partly devised through studying previous responses, each accompanied with a five-point Likert-type attitudinal scale. However, the scale itself led to a second difficulty in that any selection of the '*Not sure'* option tended to reduce the usefulness of the datasets; a potential problem highlighted by Gorard [2001: 15]. The '*Not sure'* option, essentially implying 'I-do-not-know-the-answer-to-this', had been included to make the questionnaire appear non-threatening. However, it was reasonable to assume that each pupil should have a view on the statements leaning towards one side of the spectrum. The *'Not sure'* option was thus omitted, hence maximising the relevant data quantity for each analysis [Cohen et al. 2000: 253].

The third difficulty stems from the well-known and intractable problem [highlighted in LRI Section 2.4 pp44-45] of establishing a clear understanding of the type of mathematical work the questionnaire was probing, in essence, what is understood by the terms: problem-solving, games and puzzles within a mathematical context. Arriving at a tight, universally understood definition 'in pupil-speak' of these terms remains difficult. It became clear from the administration of PQ2 and the subsequent pupil interviews, that some (who found mathematics per se difficult) were considering *any* type of mathematics as problem-solving. As a result, Section B was re-titled 'Doing Puzzles and Games in Maths' with an additional explanation [see Appendix 3.3]. Even then, there was no absolute guarantee that respondents would all have a clear understanding although when administering the in-school paper-based versions, it was possible to discuss the intended types. The potential for misunderstanding in the web-survey was much reduced given that firstly, it would be accessed from the NRICH site that has only problems of the type intended for the research and, secondly, there was a clear instruction that the questions referred to an NRICH type of problem.

PQ3 was trialled specifically to test the effects of the change of nomenclature in Section B and the reduction to a four-point Likert-type scale. The latter did produce the required result in that a larger number of pupils did present an opinion, although as with any response, caution is needed as to the truthfulness of respondents [Cohen et al. 2000: 254]. Furthermore, it was apparent when administering the questions that pupils experienced far less difficulty in understanding the definition of the type of mathematics being targeted than in the two earlier versions.

A year later, and after the web-survey had been completed, a short questionnaire [PQ4] [Appendix 3.4] guided by web-survey responses was undertaken by a small group of sixth formers in their second year of A-level mathematics study, in preparation for group interviews [9 and 10 in Table 3.2]. This provided additional supporting data on mathematics students recognised to be higher achieving as opposed to the mixed-ability respondents involved with PQ1, 2 and 3.

3.4.1.2 Web-Survey

The web-survey [WQ] was devised and finalised following the trialling of PQ3. The on-screen version is pictured in Appendix 3.5. Where, during trialling, analysis showed that questions would not contribute to the final focus, they were omitted from subsequent versions. However, the number of WQ questions was increased to 32, requiring a total of 65 responses, by the process used to address the paucity of written responses in earlier versions described above. Nevertheless, the time taken to select the statements was likely to still be at most equivalent to that taken when attempting to answer by writing prose. Table 3.3 below lists the six sections of WQ and shows the type of response required for each question. Appendix 3.6 presents, in table form, the design rationale for each of the sections and the questions asked, together with an indication of how each would be analysed – predominantly quantitatively. Thus, for all the question types, other than the one labelled ‘Open-ended written response’ in Table 3.3, initial analysis was quantitative, but as the later Section 3.5.4 on coding data explains, a further, qualitative analysis of the responses was undertaken.

Question type	Multi-choice, selecting one option	Multi-choice, selecting (all) appropriate options	Selecting agreement level to a (number) of statements	Open-ended, written response	Personal data
Section Heading Question Numbers	Question Number				
A. Using the Internet at home[4] WQ1 - WQ4	3		1 (5), 4 (3)	2	
B. Our future with the Internet[5] WQ5 - WQ6			5 (5)	6	
C. Using the NRICH website WQ7 - WQ17	7, 8, 9, 12, 13, 15, 17	11 (5)		10, 14, 16	
D. General information WQ18 - WQ26	24, 25, 26		23 (3)		18, 19, 20, 21, 22
E. Doing puzzles and games in mathematics lessons WQ27 - WQ28			27 (9)	28	
F. Understanding and Learning mathematics WQ29 - WQ32	30		29 (5), 31 (6)	32	
Total number of responses per type	**12**	**5**	**36**	**7**	**5**

Table 3.3 Breakdown of Web-Survey Questions by Type

For ethical reasons, the respondents were not obliged to answer all of the questions; in other words incomplete sets of responses could be submitted and would be recorded automatically. For the same reason, there was no request for any email address or name, in order to ensure no possibility of unsolicited contact to potentially young people. Even though none of the responses would ever enter the public domain, respondents remained totally anonymous and still are. The implications of this imperative ethical position for the Study are addressed in discussion of limitations [see Chapter Fifteen].

A hyperlink to the web-survey was placed on the NRICH home page for the first three months of 2007. Information about the survey was included in the NRICH site's Newsletter, but no further advertisement of its existence was made.

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[4] & [5] Not reported in Chapter Four.

A small number of interviews were conducted to provide additional related material for greater in-depth exploration and triangulation purposes. The next section briefly sets out and discusses the interview processes adopted.

3.4.2 Student Interviews

The purpose of interviews during Stage One [see Table 3.2] was to delve deeper into areas raised by responses [Drever 2003; Robson 2002] to PQ2. In Stage Two the interviews were more targeted on the person [see details below]. Thus in both stages, although there were specific issues to be addressed, it was intended that there should be flexibility to respond to the points being made by the interviewees. The method adopted for the face-to-face interviews was therefore semi-structured, someway along on the continuum between structured (pre-planned and tightly maintained interview schedule) to unstructured (flexible in gaining responses to a general set of ideas) [Cohen et al. 2000: 273; Wengraf 2001: 5]. Thus the interviews were, as intended, "conversational encounters to a purpose" [Powney & Watts 1987: vii]. The interview design consisted of a planned sequence of investigations of particular areas, each starting with a specified question, but the sequence and selection of subsidiary questions was dependent upon responses.

Table 3.2 (earlier) indicates that the first two interviews took place concurrently with the administration of PQ2. The decision to conduct any in-school interviews with pupils as group interviews was made on two grounds. Firstly, the advantage that opportunities arise for discussion between the group members as well as with the interviewer thus permitting the collection of a wider range of responses. Secondly, group interviews can also provide the pupils some security in talking alongside peers rather than individually with an unknown person [Cohen et al. 2000: 273]. Although it might be argued that one-to-one interviews are easier to manage since only a single person's response are dealt with at any one time, these are more time consuming for the interviewer and possibly more disruptive to the school routine.

After initial analysis of the web-survey, interviews targeting high-achievers were conducted. Three were group interviews undertaken concurrently with administration of PQ4 in an all-boys' school with second year A-level students known to use Internet sites. In, addition, two individual interviews with keen problem-solvers were conducted: one with a male (aged 17 from an European Union Country) who has been a long time user (5 years) of NRICH monthly problems; a second with a younger boy (aged 11 from the United Kingdom), moving from primary to secondary education, who was known to do problem-solving a home without his teacher knowing. All these interviews were then transcribed into Nvivo in preparation for coding and subsequent analysis. [See Appendix 3.7 for sample interview schedules].

These two sub-sections above have discussed the two instruments, questionnaires and interviews, used in both Stages of the Initial Study. The remaining two sub-sections report on managing the data from WQ and the open coding undertaken on data produced by both instruments.

3.4.3 Managing Automated Web-Survey Data

This sub-section describes how the data were collected, cleaned and re-organised into an appropriate structure ready for a systematic analysis. The first of these tasks was unproblematic since the web-survey software allowed the data collected to be exported in an Excel spreadsheet. The other two tasks could only be achieved through much more time consuming manual processes.

Each visit to the web-survey site created a new respondent identifier (a number) regardless of whether any responses to questions were made. Any resulting blank rows in the spreadsheet were removed, but the remaining identifier numbers were left unchanged. Likewise responses that were either clearly 'silly' or 'offensive' (only a few) were immediately discarded. Further decisions needed to be made about the accuracy and validity of the responses remaining. For example, in comparing home and school working, responses giving ages 19+, i.e. beyond school age, were eliminated. Next, for each remaining respondent careful consistency checks were made between responses to dependent questions as in, for example, question WQ8 which only those opting for '*only home*' or '*home and school*' in question WQ7 were required to answer. In clear cases of inconsistency, the respondent was eliminated. In other cases, the relevant data was omitted from the analysis. Also, from reading the free text responses it became clear that a whole class had been ‘persuaded’ by a stand-in teacher to complete the questionnaire in the lesson. Unfortunately some of these replies showed clear inconsistencies and the relevant data had to be omitted.

The 117 valid responses from WQ were then grouped into three distinct sets according to location choice (*only home*, *home and school* and *only school*) to form the first series of workbooks. Subsequently, the data was further sub-divided into two other series of workbooks, based on age and gender, each containing all the data relating to each respondent [see Appendix 3.8 for sample pages of workbooks]. Separate sheets within workbooks held responses from different sections of the questionnaire, but data held in each sheet was always separated according to the three location groups. This simple manual approach was appropriate for two reasons. Firstly the size of the dataset made the manual task manageable. Secondly, in this way, it was possible to gain an intimate and detailed knowledge of the data, not available when using a more sophisticated, technological tool, that would aid later coding and classifications.

3.4.4 Coding Data

This section is further subdivided to report separately on how the data from the web-survey and the interviews was open coded [Harry, Sturges & Klingner 2005; Strauss & Corbin 1997, 1998].

3.4.4.1 Web-Survey

Several sets of data selected from the spreadsheets [see above] containing subsets of the WQ responses were constructed and imported into Nvivo for coding. Table 3.4 gives a breakdown by location group of the number and content of these Nvivo files. The files divide into three groups that were each processed differently.

Location group	1. Written responses to seven open questions (file per question)	2. Synopsis of respondent (file per individual)	3. Vignette (file per individual)
Only Home	52	37	3
Home and School	30	14	1
Only School	14	4	0
Total	**96 entries in 7 files**	**55**	**4**

Table 3.4 Datafiles Imported into Nvivo

i. The scheme of coding used for the seven files containing all responses for each open-ended question is exemplified by the open coding of WQ16 presented in Appendix 3.9 with the accompanying memo recording the coding process and explaining the derivation of codes in Appendix 3.10.

ii. The synopses of responses to all questions for each of 55 individuals, selected from each of the location groups, were used to assign rudimentary characteristics using predetermined codes as reported below.

iii. Narrative vignettes were constructed from synopses for four individuals, who were amongst those considered to be likely potential AskNRICHers. The limitation to four was imposed by time constraints. Appendix 3.11 contains an exemplar synopsis and accompanying vignette.

Table 3.5 lists the predetermined, literature-based codes used for three rudimentary characterisations of respondents and an explanation of the underpinning criteria for that code.

WQ question(s)	Characterisation Code	Explanatory Comment
Puzzles and Problems WQ27	Using responses to WQ27, count of correct responses to hypothesised agreements	
	Keen	7-8 agreements
	Positive	5-6 agreements
	Neutral	4 agreements
	Negative	3 or less agreements
Understanding WQs29&30	Using selection of understanding or remembering to be of more importance (WQ30), and responses to statement 2 of WQ29 (likes to be told rule without explanation)	
	Relational [Skemp 1976]	Selected understanding as more important and placed likes to be told about the rules and methods as either *'not really'* or *'certainly not'*
	Instrumental [Skemp 1976]	Selected remembering as more important and placed likes to be told about the rules and methods as either *'definitely'* or *'I mainly think so'*
	Hybrid	Selected understanding as more important and placed likes to be told about the rules and methods as either *'definitely'* or *'I mainly think so'* **or** selected remembering as more important and placed likes to be told about the rules and methods as either *'not really'* or *'certainly not'*
Working Practices WQ31	**Self-sufficient**	Selecting *'often'* for statement (e): quietly work it out for self
	Collaborative	Selecting *'often'* or *'quite often'* for statement (c): talk about it with others in class
	Explainer	Selecting either *'often'* or *'quite often'* for statement (f): explain to others in class
	Seeking Help	Selecting *'often'* for any one of the statements (a): asking teacher (b): asks friend to show (d): help at home or '*quite often'* for at least two of these three statements

Table 3.5 Characterisation Codes for Web-Survey

The characterisations were assigned from individuals' patterns of responses to: WQ27, Puzzles and Problems; WQs29&30, Understanding and WQ31, Working Practices. Whereas for the first two the characterisations are discrete i.e. each respondent can be assigned to one and only one, for the third each respondent could be assigned from zero up to all four characteristics.

WQ27 had nine statements about doing Puzzles and Problems in mathematics each with four levels of (dis)agreement. Four characteristic codes were derived by a 'synthesis of inferences' from the literature [including Boaler 1997; Haggarty 2002a, 2002b; Sutherland 2006; Tanner & Jones 2000a] as to what the expected choices would be for 8 of the 9 statements. Statement (e), *harder to learn*, was excluded as an expected choice was more difficult to determine. Positive agreement was expected to all except statement (c) where disagreement was deliberately included as a validity check. Each individual was then characterised according to the number of agreements to expected choices: 8 or 7 was labeled **Keen**, 6 or 5 **Positive**, 4 **Neutral** and less than 4 **Negative**.

Responses to WQs29&30 were used to assign three codes under the heading of Understanding: **relational**, **instrumental** [Skemp 1976] and **hybrid**. Finally, four codes for Working Practices based on Anthony [1996]: **Self-sufficient**, **Collaborative**, **Explainer** and **Seeking Help**, were determined by the combinations of selections of '*often*' or '*quite often*' for the five statements in WQ31.

3.4.4.2 Interviews

All interviews listed in Table 3.2 were transcribed and entered into Nvivo. Open coding was used to establish preliminary themes and the results compared with the WQ findings for triangulation. Nevertheless, time limitations and the quality and quantity of written responses in WQ were insufficient to combine with interview data to complete a detailed and complex analysis. However, as Chapter Four reports, the coding undertaken did make a useful contribution. It served as an *overview* of both practices and beliefs of a specific population and thus helped to shape the in-depth exploration of AskNRICH in the Main Study.

3.5 Summary Comments

This chapter has presented a summary of the research and the rationale, timing and the types of participants involved, in each step of the Initial Study. The adoption of a mixed methods approach has been explained and justified. The design of a series of questionnaires, culminating in a web-survey published worldwide on NRICH; the method of collection of additional interview data, and the management of data, have all been reported.

The application of these methods aims to obtain a general overview of the "state-of-the-actual" [Selwyn 2008: 84] seen through the eyes of a self-selecting group of mathematics students. The next chapter summarises the findings of WQ on pupils' general perceptions of school mathematics and using NRICH type problems in home/school settings.

Chapter Four

Findings from Initial Study

> *A teacher can never teach you something from all angles. It's good to have different viewpoints, and different methods of approaching problems. By learning some maths before lessons, it makes it easier to understand it when it's finally taught in school. Also, I don't get the opportunity to do Further Maths at my school, so I enjoy learning new things on my own. It gives me a feeling of satisfaction when I can grasp something, and I know I've done it on my own.*
>
> *[Web-Survey Respondent]*

4. 1 Introduction

The previous chapter provided an overview of the methodology and methods used in the Initial Study. This chapter presents a summary of a selection of its findings pertinent to the Main Study based on the results of the web-survey; additional supporting material generated by other instruments and extended reporting and discussion of the web-survey findings are presented in this chapter's appendices. The summary here provides an overview of the practices and beliefs of a population that overlaps with that of potential participants in the Main Study and can be considered likely to be interested in 'doing mathematics' in a similar manner to the AskNRICHers. Indeed, the web-survey findings reveal that some respondents are AskNRICHers. The findings presented in this chapter address **RG1**: *to investigate pupils' general perceptions of doing mathematics in school and using NRICH type problems in home/school settings*. The objectives of **RG1** are expressed in two Research Questions: **RQ1**: *What are the common practices of using NRICH problems in the home context?* and **RQ2**: *What views do students using an on-line mathematics resource (NRICH) have concerning their experience of school mathematics?*

Thus the purpose of this chapter is to:

i. give an overview of the dataset
ii. to address **RQ1** which will also demonstrate that there still exists a group of 'home-aloners'

iii. to present a summary of a series of findings on respondents' experiences and general perceptions of school mathematics relating to: doing puzzles and games; ways of working in lessons, and the desire for understanding mathematical content

The remaining part of this chapter is in three main sections that parallel the division used in setting out its purpose above. Each provides a summary overview of work undertaken in the Initial Study. Full presentations and detailed discussions of findings are contained in Appendices, 4.1, 4.2 & 4.3.

4. 2 Overview of Dataset

The web-survey data was cleaned [as explained in Section 3.4.3 p68] leaving 117 entries that form the statistical basis of the findings reported in this chapter. Table 4.1 shows the breakdown by age group based on English schools' Key Stages [KS]. Only seven responses came from pupils of primary school age. Although the KS3 group appears to be more strongly represented with 20 more replies than either KS4 or KS5, this is the result of a stand-in teacher 'persuading' a class to spend the lesson replying to the questionnaire. Thus it can be argued that there is a more even division of 'free choice' respondents between these three secondary-aged Key Stages. 68 (58%) respondents were female and, of the 115 who disclosed their 'Country', 107 (91%) were British.

Age Key Stage	Under 11 KS1&2	11-13 KS3	14-16 KS4	17-18 KS5
Total (117)	7	50	30	30
Percentage of total	6%	42.7%	25.6%	25.6%

Table 4.1 Web-Survey Respondents by School Age-Group and Key Stage

The entries were separated into three discrete groups, according to the locations where NRICH problems were used: (i) only home [**oh**], (ii) home and school [**hs**] and (iii) only school [**os**]. Table 4.2 below gives the percentages of respondents from each location and proportions represented as a pie chart [Figure 4.1]. These three distinct *location groups* form the basis for all subsequent data analysis in this chapter.

Location where NRICH used	
Location (number)	%
Only Home (55)	47%
Home & School (34)	29%
Only School (28)	24%

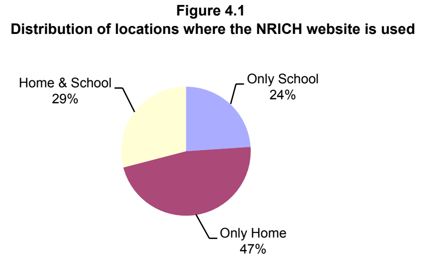


Table 4.2 Breakdown of Web-Survey Responses by Location

Table 4.3 and Figure 4.2 present the results of respondents' self-assessment of their mathematical performance in relation to other school subjects (WQ23).

	Home Only	Home & School	School Only
Good at all subjects	94%	88%	93%
Mathematics is stronger than other subjects	85%	62%	30%
Mathematics is weaker than other subjects	6%	15%	43%

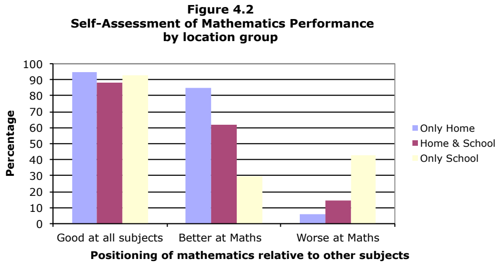


Table 4.3 Respondents' Self-Assessment in Mathematics in relation to other Subjects

The results show that the respondents considered that they were high performers overall, with the **oh** and **hs** groups considering that although good at all subjects, mathematics was one of their strongest. The **os** group did not score themselves so highly in ranking mathematics alongside other subjects. Thus by inference those in the **os** group might not be so keen, interested and motivated to pursue this subject as those in the other two. Such apparent absence of interest supports the further inferences that those in the **os** group do not pursue additional study and thus those in the home groups will have greater knowledge and experience than the majority of their classroom peers. These findings are key to describing the nature of the population from which the AskNRICHers of the Main Study are likely to be drawn.

4. 3 Findings and Discussion Part One: The Home Context

This section uses results from the web-survey to address in a wholistic manner the three sub-questions of **RQ1**:

i. *Why and with whom do these students do NRICH problems at home?*
ii. *To what extent do these students perceive their teacher knowing that they do mathematics problems at home?*
iii. *Why do these students not tell their teacher?*

This is achieved by firstly establishing that there are respondents who do mathematics problems at home and that some of them do not tell their teacher about it. Secondly, by examining respondents' reasons for not telling their teacher and, finally, by investigating the reasons why respondents do mathematics at home. These are again key findings in relation to the Main Study since the AskNRICHers will predominantly be working at home, alone and away from the classroom.

4.3.1 Home, Alone and (Possibly) Not Telling the Teacher

Table 4.2 above immediately corroborates the findings of earlier studies [Jared 1997, 1998] that some people only work on NRICH problems at home. Just under half of web-survey respondents, 55 (47%), are in this group (**oh**). No claim is made that this result is replicated across the whole school population since the survey was only posted on the NRICH website. On the contrary, the purpose is simply to establish whether there are school-aged pupils working on NRICH problems only at home, as indeed there are.

Table 4.4 below shows the perceptions of the 'home' respondents (groups **oh** and **hs**) of their teacher's awareness of their undertaking NRICH problems at home. Strikingly, a large majority (75%) of **oh** respondents believe that their teacher is unaware. Of the five who are sure that their teacher is aware, three had told their teacher and two were directed to NRICH by their teacher. There is no inference intended that the teacher should be aware; the result simply shows the number of respondents who have not informed their teacher. Moreover, since the data only records the pupils' beliefs it does not definitively indicate whether the teacher actually knows. Possible reasons as to why they decided not to tell are discussed at the end of this section.

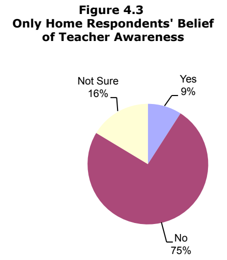
<table>
<tr><td colspan="3">Does your teacher know?</td><td rowspan="6">
</td></tr>
<tr><td></td><td>Only Home</td><td>Home & School</td></tr>
<tr><td>No</td><td>41 (75%)</td><td>9 (26.5%)</td></tr>
<tr><td>Not sure</td><td>9 (16%)</td><td>12 (35.3%)</td></tr>
<tr><td>Yes</td><td>* 5 (9%)</td><td>** 13 (38.2%)</td></tr>
<tr><td colspan="3">* 2 teacher-directed, 3 told teacher
** 8 teacher-directed, 4 told teacher, 1nr</td></tr>
</table>

Table 4.4 Breakdown by Location of Respondents' Belief of their Teacher's Awareness

Table 4.5 presents a more conservative analysis of the same data as Table 4.4 in which the **oh** and **hs** groups have been combined into a 'home people' [**hp**] group of 89. If all the 'not-sures' were actually 'does-not-know' then 80% of the teachers were unaware that NRICH problems are being undertaken at home. Alternatively, if the 'not-sures' were actually 'does-know', then 56% of teachers would still be unaware. Whatever the correct figure is, Table 4.5 shows that less than one in two respondents' teachers are apparently aware of this out-of-school though curriculum based activity.

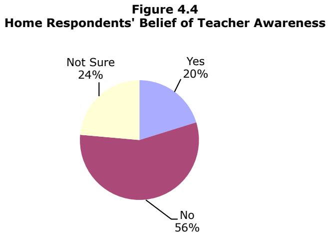
<table>
<tr><td colspan="2">Do you think your teacher knows that you do maths problems at home?</td><td rowspan="4">
</td></tr>
<tr><td>No</td><td>50 (56%)</td></tr>
<tr><td>Not sure</td><td>21 (24%)</td></tr>
<tr><td>Yes</td><td>18 (20%)</td></tr>
</table>

Table 4.5 Teacher Awareness: Combined data (oh & hs) for Home People (hp)

WQ11 directly addressed '*with whom do these students do NRICH problems at home*' by asking respondents to select one or more of the five options listed in the first column of Table 4.6 below. Multiple selections were allowed in order to capture combinations of working patterns. The data provides evidence that some respondents work with others; around a quarter of the **oh** group work 'with adults' and a similar proportion of the **hs** group work ‘with friends’, although the latter group may have meant working together in the classroom. Nonetheless the dominant 90%+ score for ‘*alone*’ in both groups, can at least be interpreted as the majority of respondents working ‘*sometimes alone*’.

Solving NRICH problems*	Only Home [**oh**]	Home and School [**hs**]
Alone	49 (92%)	29 (91%)
With a friend	6	9
With a group of friends	2	1
With siblings	1	5
With adults	14	5

*multiple options allowed

Table 4.6. Group Working when Solving NRICH Problems within the Home Setting

Table 4.7 presents a more detailed analysis of those responses in which the only selected option is ‘*alone*’ i.e. respondents who work ‘*always alone*’.

Only Home [**oh**]	(53 responses)
Mention alone	49 (92% of responses)
Always alone	34 (64% of responses)
Teacher knows	1
Not sure if teacher knows	4
Teacher does not know	29 (85% of ‘always alone’)
Home and School [**hs**]	(32 responses)
Mention alone	29 (91% of responses)
Always alone	17 (53% of responses)
Teacher knows	5
Not sure if teacher knows	8
Teacher does not know	4 (24% of ‘always alone’)
Home People [**hp**] combined	(85 responses)
Mention alone	78 (92% of responses)
Always alone	51 (60% of responses)
Teacher knows	6
Not sure if teacher knows	12
Teacher does not know	33 (65% of ‘always alone’)

Table 4.7 ‘Home Respondents’ for whom the Only Selected Option for Way of Working is ‘Alone’

The three parts of Table 4.7 provide separate data relating to the two home groups and their combination. In the first part of the table, for example, 34 of the 53 (64%) **oh** respondents who replied to this question are '*always alone*'. Likewise this was 17 out of 32 (53%) for the **hs** group and 51 out of 85 (60%) for the combined **hp** group. An alternative calculation would be to use only the 49 responses indicating '*alone*' instead of the full dataset of 53, in which case the percentage of '*always alone*' for the **oh** grouping would increase to 69%. Similarly this would produce 59% and 65% respectively for the other two groupings. Thus, whichever calculation is used, there is a majority of *'home-and-always-alone'* respondents. This provides further corroboration of findings [Jared 1997, 1998], some fifteen years earlier.

Table 4.7 also incorporates the responses that the '*always alone*' group gave about their teacher's awareness of their working on NRICH problems at home. Just one of the 34 *home-only-and-always-working-alone* respondents indicated that the teacher knew, a further four were unsure. Thus the vast majority, 29 (85%), indicated that their teacher did not know.

The results for the **hs** group are more evenly distributed which should not be unexpected as, in this case, assuming that the teacher will have worked with the class on the problems in a lesson, it can be inferred that teacher and pupil talking about a problem would be a more natural situation. Nevertheless, the percentages scores on respondents' beliefs that their teachers are unaware are high and indeed, in the next sub-section, evidence is shown that some pupils neither wish nor expect their teachers to be aware.

Three other web-survey questions [WQ15-17 see Appendix 3.5], albeit with a different wording and format, that were about work being studied at home (i.e. not necessarily confined to NRICH problems) that was not teacher instigated (i.e. independent working) can also provide further evidence on teacher awareness. Table 4.8 below presents data on the extent of **oh**, **hs** and **hp** groups' self-learning and the frequency of talking to their teachers about it, from which the level of teacher awareness may be inferred. The smallest percentage of independent working is still high at 71% (**hs**), although in this scenario as few as 34% of **hp** *never* discuss the work with their teacher.

	Decide to self-learn without teacher knowing			If yes, frequency of occasions talk to teacher about this		
	No	Yes		Always	Sometimes	Never
Only Home (55)	11 20%	44 80%		1	26	17
Home & School (34)	10 29%	24 71%		2	16	6
Combined (89)	21 24%	68 76%		3 4%	42 62%	23 34%

Table 4.8 Choices in Self-Learning Mathematics and Frequency of Talking to Teacher

Thus, even in this situation, one in three of the respondents from the **hp** group never talk about such independent work to their teacher, leaving the teacher unaware of home practice. Establishing the existence of such a culture of independent study provides a further indication of the way in which AskNRICHers may work.

4.3.2 Reasons for not Telling (Teacher)

As stressed earlier, there is no certainty that teachers are aware or not of such home practice. Moreover, teachers, respectful of privacy, may 'not feel it is their business' to know. However, open-ended responses to WQ16 [see Appendix 3.9 & 3.10 for coding] provide evidence that there is intentionality by some respondents in keeping the work secret. Although, minimally, one young respondent stated they were '*too shy*' [R215-F-oh-KS2][1], analysis shows the importance that some other respondents give to privacy. Across all age-ranges respondents were adamant that their teacher did not need to know, for example: *'I need to learn but my teacher does not need to know'* [R196-F-hs-KS2]. Others gave pragmatic, if somewhat disheartening, reasons linked to different, necessary priorities in the classroom: *'[I do puzzles] because I'm curious! and my teachers don't need to know, they know I love mathematics but are not so interested in discussing anything not on the A-level syllabus'* [R139-F-hs-KS5] and *'It isn't stuff covered on syllabus, so they do not need to know'* [R128-M-oh-KS5]. Such comments are an indicative portrayal of points made in LRI [Section 2.2 p41] concerning students' acceptance of, and teachers' reaction to, an examination-dominated curriculum, further supported by: *'I usually don't feel the need to tell my teacher...we're too busy getting through M3*[2] *at the minute!'* [R123-F-oh-KS5]. The pervasive effect of examination-dominated curricula and related target setting is further illustrated in an interview with Scott aged 11 [see Appendix 4.4].

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[1] convention for web-survey respondent labeling is respondent number-gender-locationgroup-KeyStage.

[2] third and final Mechanics module taken at A level.

4.3.3 Why do Mathematics (NRICH) Problems at Home?

Further analysis of the 56 open responses used in the previous sub-section provides evidence to address why respondents study NRICH problems and/or other more general mathematics topics at home. Respondents' choice to undertake such 'extra work' not only implies inherent motivation and interest in it, but also an inclination and/or openness to independent learning: *'I do it so that I can learn maths outside of the classroom and away from the school environment as well as in lessons'* [R97-F-oh-KS4] and *'I like to broaden my horizons in a way that does not rely on input from my teacher'* [R135-M-hs-KS5]. Statements such as: *'I'm self teaching certain modules'* [R134-F-oh-KS5] only explicitly imply openness to independent learning, whether or not this work was 'forced' upon them is not determined. However, eighteen respondents did indicate that they were self-teaching/learning to expand their experience in some way such as study beyond the syllabus: *'I want to try to do well in the BMO[3], for which I need to know some things which I'm not taught at school'* [R143-F-oh-KS5]; or broadening the work in school: *'It's good to have different viewpoints, and different methods of approaching problems'* [R22-M-hs-KS5], *'I like to know all of the maths behind a problem, not just what the exam specification wants you to know'* [R119-F-oh-KS3]. Another respondent simply states *'I like trying to find different ways to learn'* [R75-F-oh-KS4]. Eight respondents mentioned a desire to improve, five wanted to 'get ahead' or prepare in advance for new topics: *'So I can understand the next topic we cover more easily'* [R121-F-oh-KS4]; *'i can find things which i may come up to later on at school but would like to know some knowledge of them before i do them'* [R75-F-oh-KS4]. Tellingly, all the respondents' comments in this paragraph mirror the proactive characteristics attributed to Adam, the active learner, presented in LRI [Section 2.6 p48].

Eleven respondents made critical comments about teaching in school not meeting individual needs, being restricted to examination syllabi or, for example, wrong pace: *'Because more often then not my teachers take too long going over the same topics for weeks on end'* [R18-M-oh-KS5], *'Mathematics is taught very slowly at my school and in a way that generally confuses me (i.e. 'learn this formula' when you have no idea where it comes*

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[3] British Mathematical Olympiad [BMO]. As Chapter Eight indicates, working on BMO problems is a common usage of AskNRICH.

from)' [R124-F-oh-KS5]. The second of these comments both succinctly demonstrates the point made by Greeno et al. [1996] in LRI [Section 2.6 p48] and highlights the issue of perceptions of understanding that is the focus of a later section of this chapter. In contrast, some respondents were tolerant of school situations that did not meet their needs because, for example, the work that interested them was not on the examination syllabus or they were the odd-ones-out ahead of the majority: '*My teachers know that, while it is obviously nice to broaden the syllabus, it is foolish to do so at the expense of the basic understanding of key concepts by other members of the class'* [R135-M-hs-KS5].

This section has established the existence of the '*home-and-alone-and not-telling-the-teacher*' learner and explored reasons why they might keep work a secret from the teacher. This chapter now presents a summary of a series of findings on respondents' experiences and general perceptions of school mathematics relating to: doing puzzles and games; ways of working in lessons, and the desire for understanding mathematical content.

4.4 Findings and Discussion Part Two: Perceptions of Mathematics

This section considers **RQ2**: *What views do students using an on-line mathematics resource (NRICH) have concerning their experience of school mathematics?* with sub-questions:

i. *What are students' perceptions about doing mathematics puzzles and problems in lessons?*
ii. *How do these students seek help with school mathematics?*
iii. *What are students' perceptions about the relative merits of rules, methods and understanding*

The sub-questions focus on three areas of mathematics. The first two relate specifically to school mathematics and the respondents' ways of working in mathematics lessons. The third investigates a broader issue, extending beyond the classroom, by examining respondents' perceptions of relational and instrumental understanding of mathematical concepts [Skemp 1976]. The findings presented in this section add to the purposeful contextual background for the Main Study.

This section presents the findings for each of the sub-questions in the form of three corresponding summary sub-sections. In addition, each sub-question is addressed and findings presented and discussed in full in separate Appendices [4.1, 4.2 & 4.3]. Each Appendix has the same structure of an introductory presentation and discussion of responses, a quantitative comparative analysis of the location groups' responses, and a complementary analysis of rudimentary characterisation [see Table 3.5 p70 for criteria] of individuals through assigning codes to specific combinations of responses.

4.4.1 Doing Puzzles and Games in Mathematics Lessons

WQ27 elicited respondents' perceptions of doing problem-solving in mathematics lessons with a focus on undertaking puzzles and games in class through their level of agreement with nine statements. Six statements had high levels (more than two-thirds) of agreement in keeping with research reported throughout LRI. Respondents felt that were doing mathematics in a different way (86%), the work was more interesting (77%) and more fun (90%), they worked more collaboratively (68%), and in a more problem-solving way (80%) than otherwise and it was not harder to learn mathematics (77%).

This last result calls into question teachers' often expressed opinion that the examination system requires 'normal' work [see LRI p41], although respondents are by and large high-attaining and may not find mathematics 'difficult'. Similarly, for groups such as this, the work might not necessarily be 'more challenging', nor might they 'have a need to think more' or 'work any harder' to find the solution. Indeed, agreement levels for all respondents for the related statements of WQ27 were lower than might be expected. The general view expressed in the literature is that working in a problem-solving manner is beneficial in requiring deeper thinking skills, offering greater mathematical challenge and, furthermore, the level of challenge generates greater enthusiasm for the subject, bringing with it greater perseverance [Fennema & Romberg 1999; Schoenfeld 1994]. The **os** group's results are consistent with this view, but the 'home' groups are not [see Table C Appendix 4.1]. Thus it could be inferred that the problem-solving mindset is natural to, and embraced by, the respondents in the home groups and therefore they will not perceive problem-solving as something special/unusual.

Further supporting evidence for this inference emerges from the analysis of the rudimentary characterisation of individual respondents into **Keen/Positive/Neutral/Negative** in their attitudes to puzzles and games. The proportion of **oh** respondents in the Negative category (21%) is consistent with the inference just made. Taken together with often discerning/ambivalent open text responses on the nature of 'fun' this inference is also consistent with Nardi and Steward's [2003] finding that perception of what might constitute fun related to an individual's view, liking and attainment in mathematics.

4.4.2 Ways of Working in Mathematics Lessons

WQ31 probed working practices in the classroom situation, a setting in which no-one is ostensibly alone and it might be assumed that the teacher would be the main/frequent source of help. However, the results show teachers to be one of the less popular sources of help or, given the self-perception of high performance of the respondents reported earlier, it might be inferred that their teachers' help was frequently not required. This self-sufficiency may also underlie the small percentage of respondents who sought help at home. The results also show that 'home' respondents are more likely to work things out for themselves and more likely to provide explanations for others, a further indicator of high performance. These results also give support to the inferences made earlier [Section 4.2] that 'home' respondents' inherent motivation and interest leads to greater knowledge and experience relative to classroom peers.

The rudimentary characterisation of respondents used the codes **Self-sufficient**, **Collaborative**, **Explainer** and **Seeking Help**, but each respondent could be assigned none, some or even all four codes. Indeed, three-quarters were assigned more than one. This together with an examination of the respondents assigned one of six possible combinations of two codes leads to the inference that respondents can work in different ways at different times according to the classroom activities. So, for example, although home respondents work substantially at home on their own [see Section 4.3] and score highly on the Self-sufficient code, this does not preclude them from collaborating at other times [see Tables C and E Appendix 4.2]. The results can also be viewed as exemplifying scenarios of the individual's mindset discussed in LRI [Section 2.6 p49]. The analyses for the home groups is of particular importance in positioning the type of people who use the AskNRICH web-board intensively, who are the focus of the Main Study, from how they work in other contexts. Indeed, characteristics that might be described as ‘self-sufficiency’, ‘explaining’, ‘seeking help’ and ‘collaborating’, are all essential to full participation in AskNRICH.

4.4.3 Perceptions of Understanding Mathematical Content

This sub-section is based on responses to WQ29 and WQ30 that investigated, in a relatively simplistic way, respondents’ desire to veer towards relational or instrumental understanding [Skemp 1976]. Results for all location groups [see Appendix 4.3] are again consistent with studies reported in LRI and open responses quoted at the end of Section 4.3.3, for example respondents:

i. overwhelmingly want to know why a rule or method works (92%, 77% and 79%)[4]
ii. believe that they would understand the rule or method if it was explained (98%, 97% and 93%)
iii. dislike just being told what the rule is (83%, 84% and 68%)
iv. opted for understanding as more important than remembering (89%, 84% and 64%)

Even though the **os** group were less positive in (iii) and (iv) above, there was still a majority of 2:1 in favour of (relational) understanding. Yet again the high-performing nature of the whole group of respondents can be inferred from the results for (ii) above.

The rudimentary characterisation divided the respondents into **relational, instrumental** and **hybrid** types. In the results of this analysis [Table C Appendix 4.3] the most noticeable feature remains the small percentages assigned the Instrumental code in *all* location groups; the largest, belonging to the **os** group, is only 11%.

The analyses above and in Section 4.4.1 show that the majority of respondents in the Initial Study across the locations would prefer to have opportunities in mathematics lessons that deepen understanding and go beyond the transmission style of teaching that the literature portrays as currently prevalent [Boaler 1997; Hatch 2002; Ofsted 2006, 2008; Schoenfeld 1989].

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[4] for **oh**, **hs** and **os** groups respectively.

This completes the summary of selected findings from the Initial Study that portrays a specific population that includes potential AskNRICHers.

4.5 Conclusions

This chapter has summarised findings from the Initial Study on the ways that young people work on mathematics in both home and school settings, and their general perceptions of doing mathematics in school. These findings are drawn from a mixed methods analysis of the web-survey results. The responses were divided into three groups according to where NRICH problems were undertaken: only home [**oh**], home and school [**hs**] and only school [**os**]. A quantitative comparative analysis between and within location groups was complemented by assigning codes to specific combinations of responses to produce rudimentary chararacterisations of individual respondents.

The findings present a telling account of the characteristics of a population that intersects with likely AskNRICHers. Crucially the findings conjure up the vision of the '*home-and-alone*' problem-solver and their motivations. Just under half of all respondents indicated that they worked on NRICH problems only at home, 75% of these reported that their teacher did not know. Open text comments illuminate the independent nature of such people working on openly accessed NRICH mathematical problems. Moreover, 64% of the **oh** group indicated they always worked on the problems alone. Importantly, both the characterisations of individual respondents and their perception of their attainment in mathematics show that the web-survey population is dominated by high-attainers. In the **oh** group, 94% were 'good at all subjects' and 85% stated that they were strongest in mathematics.

Respondents were overwhelming positive about undertaking problem-solving in mathematics lessons, placing an emphasis on the 'fun' nature that doing this type of work can bring, even if some more discerning respondents were ambivalent as to what this really meant in terms of serious mathematics studies. Results relating to ways of working in the classroom demonstrate that respondents worked in different ways at different times. In particular, those in the **oh** group working independently at home were often also 'collaborators' and/or 'explainers' in the classroom situation. Open text comments support educationists' views of the restrictive nature of school mathematics that focuses on examinations. Respondents contrasted this adversely with the stimulation of challenging mathematics and their desire for teaching that allows relational understanding.

Many respondents made reference to the Internet providing new opportunities for being in touch with like-minded, high-performing others who may live far away. Such respondents perceived AskNRICH as enabling them to be in-touch and pursue their interest in mathematics. An exploration of how young people used AskNRICH to pursue serious mathematical study away from the classroom, with like-minded peers, is the focus of the Main Study's research, presented in the following chapters.

Part Two

Chapter Five

Literature Review II
Analysing Computer Mediated Communication

> *Practitioners and researchers must be able to describe on-line action more than impressionistically and measure more effectively than anecdotally.* *[Fahy, Crawford and Ally 2001: 2]*

5.1 Introduction

This is the first chapter concerned with the Main Study. It presents a literature review necessary to underpin the development of appropriate methods and instruments to carry it out. The Main Study primarily uses the message threads of the AskNRICH web-board as its data source to obtain a view of the how, what and why of the involvement in doing mathematics of the young people using it. Such a view requires a systematic and in-depth exploration that, in turn, requires an analytical approach appropriate to the nature of AskNRICH. This is expressed by **RG2** and **RQ3** [see Table 5.1] that are addressed through a review of literature reporting previous studies' analyses of CMC Forums [CMCs]. The way in which **RQ4** is addressed is inherently dependent upon the result of this review and is reported in the subsequent methodological chapter.

RG2: To formulate an analytical approach appropriate to the nature of AskNRICH		
RQ3: Can existing methods / frameworks for analysing Computer Mediated Communication forums be employed in analysing AskNRICH?		Reported in
Sub-questions:	*1. What different types of frameworks have been reported?* *2. What different methods/approaches already exist?*	Sections 5.2 to 5.5
	3. What are the key methodological issues?	Section 5.6
RQ4: How should the exploration of AskNRICH threads be organised (planned, structured and executed)?		Chapter Six

Table 5.1 Methodological Research Goal and Questions

Thus the literature review that follows aims to establish whether any of the frameworks and/or methods employed for different situations to AskNRICH could be related to, and be appropriated for, its systematic exploration and analysis. The resulting effects on methodology are reported in the next chapter.

An initial search revealed a plethora of literature documenting a variety of theoretical stances, frameworks and instruments used in analysing specific CMCs, that continued to expand during the period of this study. In addition to the traditional critiquing process of a literature review I could use both my prior, general knowledge of AskNRICH and that gained in the scoping and reconnaissance stage of exploration [see Figure 6.1 p105] to constantly compare it, in terms of its features and rationales, with what the literature was reporting. This not only facilitated inferences made at the time of reviewing, but also has enabled the choice of material reported within this chapter to be more selective.

At the heart of this review there are two recent, comprehensive surveys of a number of reputable CSCL studies [De Wever et al. 2006; Rourke et al. 2003]. The review initially built upon these surveys using their primary sources before expanding further, critiquing other CMC studies in the general literature. The review has been kept up to date by incorporating more recently published sources.

There are five remaining sections in this chapter. The first describes how the two surveys are used to initiate the review. The next section, based on a number of papers from the first survey, examines frameworks and methods for CSCL analysis and their potential usefulness for this study. This is followed by a discussion of the size and setting of studies in comparison with AskNRICH. This leads into a further section describing two studies, postdating the surveys that had a more explicit impact on shaping the methodology used in this study. Finally, the second survey is used as a template for the discussion of key methodological issues that were considered germane to this study.

5.2 Initiating the review with two CSCL surveys

Rourke et al. [2003: 133] surveyed nineteen studies, sixteen published within a five-year period 1995-2000. Six reappear in De Wever et al. [2006] analysis of fifteen studies, eight published post 2000, that they claim to be representative of the field of CSCL. The short timespan of publications used in both surveys implies high levels of activity in what was then a new field brought into being by the same advances in IT that also enabled the creation of NRICH in 1996.

Henri's [1992] detailed and theoretical study is the earliest reported and referenced in both surveys. Henri underpins her work by arguing that it was timely to develop a methodology for analysing CMCs that moved away from a predominantly quantitative approach that had initially accompanied early CMC analysis [Marra, Moore & Klimczak 2004]. At that time it was common to merely count participants, messages, interactive messages, conferences, length, or to quantify indirect indicators such as user perception, level of satisfaction, attitudes etc.. Thus Henri proposed that there should be a shift to employing methods that capture in-depth qualitative knowledge of the pedagogy and learning taking place within the CMC. Henri's widely cited study has been subjected to extensive analysis and attempts at replications earning the description "seminal" [Rourke et al. 2003: 138] and "landmark" [Schrire 2006: 51]. As will become clear Henri's study forms part of the reporting within this chapter.

The two surveys have two radically different presentational structures. De Wever et al. present a succession of individual studies, providing, for each study, an insight into its theoretical underpinning and details of implementation of the instrument described, in addition to specific, related methodological issues. Where appropriate they report on any practical implementation and further studies that have replicated the original. By contrast, Rourke et al.'s survey is organised as a series of discussions of methodological issues and considers techniques that have been used to address them, listing the studies by authors without the in-depth descriptions that De Wever et al. provide. Thus in addressing **RQ3**, the two surveys are advantageously complementary. The first relates the theoretical frameworks, what instruments are available and how they function. As such it provides the means to consider whether any of these aspects can be be employed or adapted and appropriated. An evaluation of the frameworks and categorisations reported by De Wever et al. is presented in the next section. The second survey sets out the issues that will need to be addressed in any analytical approach and, as such, is used as the template for Section 5.6: Methodological Issues.

5.3 Frameworks and Methods I

In order to ensure a systematic evaluation of how each study might provide a basis for a methodology for analysing AskNRICH, a table was created as preparation [see Appendix 5.1], listing the theoretical backgrounds, together with a précis description of categories/dimensions/indicators, of the studies surveyed by De Wever et al. [2006: 13-23]. In addition, to gain greater insight than provided by the survey's overviews, selected primary sources were examined. Using these and De Wever et al.'s paper, comments were added to each study's entry outlining perceived compatibility with, and potential for, exploring AskNRICH.

Although the studies reported have diverse theoretical backgrounds, they all fall within the boundaries of social or cognitive constructivism and all employ a model of collaborative learning that is oriented to a 'hot-topic', critical thinking [Lipman 1991] discussion CMC. In these, collaboration invariably involves participants working/building-knowledge together to 'solve a problem' in the wider sense, or coming to an agreement/resolution. By contrast, in AskNRICH there may be such types of discussion, but these are an occasional, peripheral by-product rather than the essential, central function of obtaining the correct solution to a mathematical problem. As already outlined in Chapter One, the purpose of AskNRICH is to allow an individual to pose a problem, to which they themselves require a correct solution, knowing that others, already knowing the solution, will lead them to a means to obtain it. The group collaboration models used to describe 'hot-topic' discussion cannot describe the collaboration that is within AskNRICH [see LRIV Section 13.2 pp272-276].

This difference in purpose and thus collaboration model made it likely that the research frameworks described in these papers could not be adopted wholesale for this study as some constructs would appear differently. Nevertheless it might be said that paradoxically AskNRICH simultaneously 'fits all and none' of the frameworks. So for example, the labels attached to themes that are part of the theoretical framework or empirically emerge from coding, sound plausible in terms of exchanges within AskNRICH. Thus in this sense AskNRICH could be considered to fit whereas the reality is the opposite. The descriptions, examples/indicators given in the CMC literature bear no resemblance to the way that AskNRICH functions. The apparent matching of terms is perhaps unsurprising as the CMCs reported and AskNRICH are both within the educational domain. By way of an example, Henri lists five skills within the cognitive dimension of her framework: elementary clarification, in-depth clarification, inference, judgment and strategies which are "connected to reasoning which uses critical thought" [Henri 1992: 129]. AskNRICHers certainly do reasoning and these elements might be considered to exist but would require re-interpretation to fit with the mathematical focus that replaces the purposes behind critical thinking. Furthermore, the definitions and indicators attached to Henri's participative and social dimensions demonstrate that these dimensions would also have to be re-interpreted to fit with AskNRICH where, for example, all participation is related to learning and the social can still be related to formal subject matter content. Appendix 5.1 contains many similar examples of such mismatches as well as noting the differences in group collaboration models discussed in the previous paragraph.

Nevertheless, examination at a more detailed level of the frameworks of the studies i.e. methods and instruments, showed that some of the rationales and choices of concepts to investigate, particularly interactivity and sociability, were potentially helpful in devising an appropriate framework and methods to be applied in the exploration of AskNRICH. Examples include: Zhu's [1996: 284] division of social interactions into vertical (looking for those more capable to contribute) and horizontal (all contribute as no-one has more knowledge), a direct and cited adoption of Hatano and Inagaki's theory; Henri's [1992: 127] interactivity that includes direct and indirect responses/commentaries, and the focus on social presence [Rourke, Anderson, Garrison & Archer 1999]. In addition, the examination provided insight into methodological issues and related advice, for example, the advantages of undertaking CMC analysis with sound knowledge of the subject domain [Newman, Webb & Cochrane 1995], a key aspect of my study. Later sections of this chapter and the next (Methodology) will reveal adoptions and adaptions of ideas and advice from these studies.

Two remaining aspects, size and setting, are the focus of the next section which starts to draw in additional studies reported after those of De Wever et al. [2006].

5.4 Size and Setting

The general CMC literature appears dominated by studies whose setting is university based, either for undergraduate and or postgraduate learning [Anderson et al. 2001; De Smet, Van Keer & Valcke 2006; Garrison, Anderson & Archer 2001; Kanuka, Rourke & Laflamme 2007; LaPointe & Gunawardena 2004; Liu & Tsai 2008; McAllister, Ravenscroft & Scanlon 2004; Oh & Jonassen 2007; Pena-Shaff & Nicholls 2004; Veerman, Andriessen & Kanselaar 1999] or professional development [De Laat & Lally 2004; Duphorne & Gunawardena 2005; Thorpe, McCormick, Kubiak & Carmichael 2007] or for interested adults [Guldberg & Pilkington 2006; Mazzolini & Madison 2003]. Furthermore, participation in the CMC is facilitated/guided by instructor(s) and often a compulsory, assessed element of a module or course. Finally, only one of the studies just cited [Duphorne & Gunawardena 2005] goes beyond the small scale in terms of the duration or the number threads/messages analysed.

In the studies surveyed by De Wever et al. [2006], the largest number of messages was just over two thousand, many had two hundred or less and the smallest forty four; generally the content studied was posted over a timespan of between 3 and 12 weeks; the numbers of participants were small and confined to a group and its tutor/teacher, and the ages were at undergraduate or graduate level [see Appendix 5.1]. For example, Zhu's [1996] empirical study had a formal setting in that it was teacher/syllabus driven and related to a graduate distance learning course with a 'hot-topic', essentially topic-of-the-week, which took place over 16 weeks. Zhu had a dataset totalling 408 messages. She selected two sample weeks for analysis, containing 55 messages, 21% of which were from the instructor. Moreover, the course was mandatory and assessed: "participation was worth 25% of the final class grade" [ibid: 824]. AskNRICH, by contrast, contains some six thousand threads and fifty thousand accessible messages, archived and live, covering a timespan starting around 2002. There is no restriction on participation in AskNRICH which is completely voluntary, and the sections studied in this thesis target school-age pupils. Furthermore, my research study took place over approximately two years.

There are, however, more recent examples of studies with similarities in size of the CMC or in setting to AskNRICH. Ryberg and Christiansen [2008] report on a Danish Social networking site whose rationale is linking like-minded enthusiasts for peer-to-peer learning, but primarily for ages 20+. If AskNRICH is large compared to other studies, then this site is *huge* with 126,240 participants in 3,200 different groups and up to hundreds of postings per day. However, the site is not open access and the consequent ethical constraints severely reduced the data available for their study to become, in effect, similarly small scale to others surveyed above.

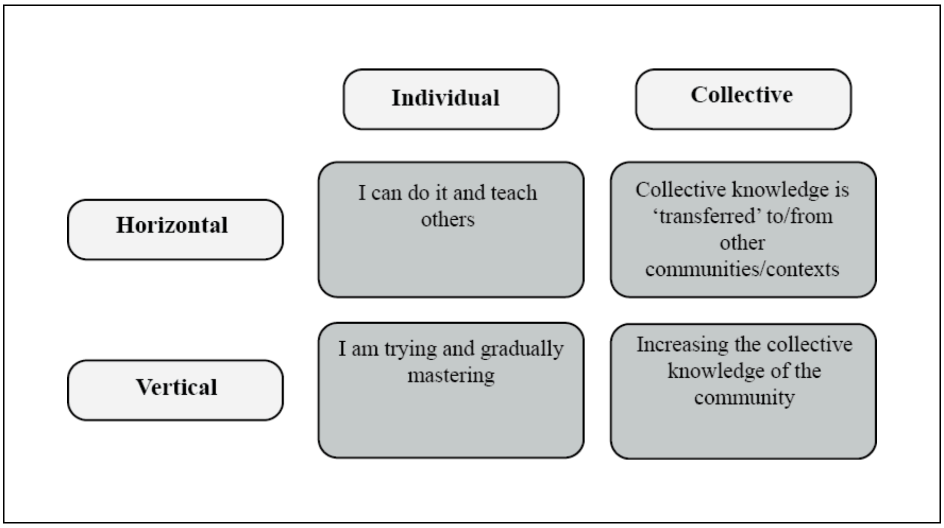
Thus the one study of a comparable or larger size CMC than AskNRICH that might have reported sampling methods that could be adopted was in the event restricted to analysis of a tiny pragmatically chosen sub-group. Hence, although sampling has been employed in several of the studies reviewed, no rationales or strategies for sampling of CMCs of a comparable size to AskNRICH are reported.

Chen and Chiu [2008] study involved high achieving Chinese mathematics graduate and undergraduate students making use of an, essentially open, web-board to conduct voluntary, independent academic discussions with no instructor. Moreover, the average length of thread is comparable to many found in AskNRICH. Thus the setting of their study has much more in common with that of AskNRICH than others reviewed, although the mathematical level is equivalent to that of the section of AskNRICH addressing mathematics beyond school study. However, it is once again a limited, small-scale study.

Despite the differences set out above, elements of the framework of Ryberg and Christiansen [2008] and the methods of Chen and Chiu [2008] influenced the methodology adopted for the exploration of AskNRICH, as evidenced in the next section.

5.5 Framework and Methods II

The framework aspect considered in this section concerns the role of participants as both an individual and part of a whole group/collective. Early in the development of collaborative CMC learning models, Henri [1992] argued that “not only does the work of the group improve but the individuals involved also learn more than those of comparable skills working alone"[1] [p120]. More recently, and in a setting more akin to AskNRICH, a similar viewpoint is expressed by Ryberg and Christiansen [2008] in presenting a 'social networking learning model', using the slogan "from COP to ZOP[2]" [p209], in terms of a "ladder of participation and mastering" [p210], which they explain as moving from the apprenticeship learning model of a Community of Practice [Lave & Wenger 1991] to being helped through their own Zone of Proximal Development [ZPD] [Vygotsky 1978] to eventually helping others through their ZPD. Making the distinction between COP and ZOP appears superfluous as the apprenticeship model of the COP itself initiates movement in any individual's ZPD. However, the need for the distinction becomes more apparent with the authors' intentions to consider not only the individual but also the collective group and how both become more proficient in learning and sharing knowledge. This is illustrated in the diagram of Figure 5.1 with a 2x2 matrix of cells containing statements representing modes of learning and development whose rows distinguish vertical/horizontal learning[3] and development and columns differentiate the individual from the collective. Such distinctions have parallels with van Lier's [1996] four-part model of types of teaching/learning activities within a ZPD presented in LRIII [Section 7.6.2 pp158-160].



	Individual	Collective
Horizontal	I can do it and teach others	Collective knowledge is 'transferred' to/from other communities/contexts
Vertical	I am trying and gradually mastering	Increasing the collective knowledge of the community

Figure 5.1 Horizontal and Vertical relations and movements in learning and development [reproduced from Ryberg & Christiansen 2008: 211]

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[1] Words echoed in Gee [2004], see Table 13.1 p281.

[2] The authors have presumably used the acronym ZOP instead of ZPD for Vygotsky's Zone of Proximal Development, apparently in order to 'rhyme' with COP - **C**ommunity **O**f **P**ractice.

[3] again see Gee [2004] different forms of knowledge in Table 13.1 p281.

As explained in Section 5.3, Zhu [1996] used the terms vertical and horizontal to categorise social interactions and her definitions encompass both the individual and collective. Thus some connection might be envisaged between Zhu's categories of interactions [see p93 above] and the modes of learning and development depicted in Figure 5.1. Indeed there appears to be some compatibility between Zhu's *vertical* interactions "looking to capable peers" [p824], a situation that predominates in AskNRICH, and the individual/*vertical* learning "trying and gradually mastering" of Ryberg and Christiansen's matrix. However, no such 'simple' connection can be made with Zhu's *horizontal* interactions "all contributing with equal knowledge" [p824] that appear to be closer to the collective/*vertical* cell of the matrix or Henri's statement on collective learning cited at the beginning of this section. This is perhaps unsurprising given that Henri and Zhu share a critical thinking collaborative learning model rather than Ryberg and Christiansen's model that is predicated on social networking.

The impact of Ryberg and Christiansen's [2008] work on shaping the methodology for exploration of AskNRICH stems from their learning model that explicitly combines consideration of both the individual and collective. The exploration of AskNRICH and, by implication, the subsequent analysis of findings, integrates three Perspectives: one a Case Study of an *individual*, the other two more oriented to the *collective*.

The second aspect to be considered in this section relates to methods and concerns the depiction of interactivity. As part of her theoretical model for the analysis of interactivity and interactions, Henri [1992] proposes a categorisation of statements/messages as *explicit* or *implicit* in their reference to others, or *independent* where they make no reference and are not referred to. The explicit and implicit categories are further subdivided into *response* and *commentary* sub-categories. Henri proposes the categorisation of messages as a means of inferring/exposing the relationship, if any, between each of them. From this she envisages the derivation of interpretations of the interactions between participants.

Such a proposal is implemented by Chen and Chiu [2008] in an empirical study in which messages are categorised by their individual properties in order to examine the relationship between them. Although their research objective differs from the purpose of exploring

AskNRICH and their learning model is critical thinking, some of the five dimensions they specify to characterise messages are relevant to consideration of interactions between AskNRICHers. In particular, Chen and Chiu's focus on the differences between face-to-face and online discussions leads them to select social cues and personal information, important aspects of AskNRICH, as two of the dimensions of characterising interactions. In addition to an intricate (opaque), computer-managed statistical analysis Chen and Chiu devise a tree-mapping diagram showing a hierarchy of messages, numbered chronologically, on a topic. It is, however, unclear through what process the link between specific messages was made and, as the authors acknowledge, there is a strong limitation in restricting each message to appear only once in the tree.

Similarly, Schrire [2004, 2006], in a study whose setting, size and theoretical framework have little in common with AskNRICH, proposes a more sophisticated '*visual mapping*' of what she infers are the connections between messages, based on a network rather than a tree structure [2004: 483]. She then uses such a map as a means of uncovering different, common interaction pattern types which are subsequently fed into a scheme for assessing knowledge building in CSCL [2006: 49]. However, in AskNRICH seeking pattern types is superfluous as the exchanges are essentially uniform in this respect, continuing until the original question(s) posted by an individual have been solved to their satisfaction.

The impact on the shaping of methodology of Chen and Chiu [2008] and Schrire [2004, 2006] stems from their empirical demonstration of the value of using a visual mapping to enhance the depiction of interactivity as a potentially powerful tool for unpicking the complexity of interactions. As they acknowledge, the expressive power of both schemes is limited. Chen and Chiu's [2008] tree model with only a single connection between messages cannot represent the complexity of interactivity, and neither scheme associates response types with the connections. The map developed for the exploration of AskNRICH remedies both these deficiencies as shown in Section 6.3.4.5 [pp129-131].

This section concludes the part of this review in which existing frameworks and methods for analysing CMCs are examined in order to determine whether they may be employed in exploring AskNRICH. However, in the process of finalising a methodology for exploring

AskNRICH it is necessary to ensure that the germane, key methodological issues, set out in the next section, are considered and addressed. These issues and their potential influence on the creation of a methodology for the exploration of AskNRICH are discussed below. The practical response embodied in the actual methods employed is elucidated in the methodology chapter that follows.

5.6 Methodological Issues

Cook and Ralston [2003] called for the development of effective analytic techniques for CMC analysis [p361] adroitly repeating Fahy et al.'s [2001] earlier plea quoted at the start of this chapter. Rourke et al. [2003] describe the objective of their survey as "provid[ing] subsequent researchers with a privileged starting point …" [p134]. As already stated, this survey is organised according to a series of methodological topics, explaining what are the issues and associated difficulties and how they have been addressed. Those topics considered germane to the exploration of AskNRICH are: (i) nature of content; (ii) unit of analysis; (iii) validity.

5.6.1 Nature of Content

Rourke et al. [2003] explain the categorisation of content into *manifest* (surface) and easily observable or *latent* (hidden within and thus inferred from the message(s)), encapsulated elsewhere as "reading between the lines in texts" [Henri 1992: 118]. Rourke et al. [2003] report Potter and Levine-Donnerstein's [1999] division of latent variables into *pattern* (coder looks for explicit clues in the content to relate to the variable) or *projective* (coder interprets the meaning of the content). The whole tenor of Potter and Levine-Donnerstein's [1999] paper is to enable researchers to make informed choices of appropriate strategies for analysing content by providing insight into the nature of content through a fine-grained, precise, in-depth exposition of the methodological implications of a process of which aspects are considered by experts to be "inherently subjective and interpretative" [Rourke et al. 2003: 141]. The intended aim of the exploration of AskNRICH was to interpret content, facilitated by both extensive mathematical and pedagogical subject knowledge, in order to generate constructs to be subsequently employed in extending current theory. Using Potter and Levine-Donnerstein's [1999] definitions, the combination of subject knowledge types, in effect, transforms some of both types of the latent content of AskNRICH posts into manifest. In other words, what would be latent content for the non-expert is easily observable by the knowledgeable coder. This is a key factor affecting the consideration of validity, the subject of Section 5.6.3 below.

5.6.2 Unit of Analysis

Rourke et al. [2003] conclude discussion of selecting the unit of analysis by observing that the process is “complex and challenging” [p145] and cite Krippendorf ‘s [1980] observation that it “involves considerable compromise” [p64]. Their preceding discussion had explored the trade-off involved in attempting a segmentation of content into fixed, discrete units that “multiple coders can identify reliably” [Rourke et al. 2003: 142] (favoured by shorter units) whilst still “encompass[ing] the construct under investigation” [ibid] (favoured by longer units) i.e. reliability versus meaningfulness.

However, Rourke et al. [2003] point out that mitigating the trade-off by dynamically adjusting the unitisation, as exemplified by the *units of meaning* strongly advocated by Henri [1992: 126, 134], may completely capture meaning, but delineating the unit can itself be unclear or subjective and thus inconsistent across multiple coders. However, Schrire [2006: 56] in agreement with Henri, citing [Chi 1997], argues that capture of meaning is improved and subjectivity reduced by using a dynamic process in which content is analysed in multiple, sequential passes each with unitisation of different grain size.

In the exploration of AskNRICH, interpretation is used to generate new constructs rather than seeking to determine the presence of pre-defined constructs. Thus flexibility in the process of segmentation of content is essential since the whole purpose of the study is to derive meaning from the exchanges in AskNRICH, though with the desire that reliability, one of the issues discussed in the next section, will accompany it.

5.6.3 Validity

There is a preconception that validity of subjective studies is particularly questionable i.e. subjectivity undermines validity. Social science research has been criticised due to its frequent reliance on subjective analyses [Crotty 1998, Evans 2009a, Walford 2001]. Validity, i.e. the factual or logical soundness of a study depends on a number of inter-related qualities described by Rourke et al. [2003] as “objectivity, reliability, replicability and systematic coherence” [p135]. De Wever et al. [2006] labels the last of these as “theoretical base” [p9] and cite concerns about validity as one of the primary motivations for undertaking their survey, calling for “replication studies that focus on the validation of existing instruments …” [p25].

Rourke et al. [2003] propose a continuum that starts from *intra-rater* reliability, moves through *inter-rater* reliability and ends with replicability, which they define as “the ability of multiple and distinct groups of researchers to apply a coding scheme reliably” [p138]. This implies that high reliability defines replicability. However, neither survey attempts the detailed consideration of the relationship between validity and reliability given by [Potter & Levine-Donnerstein 1999], centered on their differentiation of the forms of content reported above. Potter and Levine-Donnerstein [1999] discuss reliability, in terms of stability, reproducibility and accuracy, and then explain that it has a different relationship with validity according to the type of content. Although for manifest content, reliability is the precursor of validity [see also Section 6.5 p134], for latent pattern content it is much more complex so that, for example, coders may be in agreement but the coding misses the essence of the phenomenon of interest, thus producing high reliability but low validity. For latent projective content they argue reliability and validity are the same thing. In other words consistency of interpretations from multiple coders creates a criterion that brings both reliability and validity simultaneously. Figure 5.2 below was drawn up in order to show the interconnections implied by the model of validity just described.

The discussion above has relevance for the exploration of AskNRICH in which all three forms of content defined by Potter and Levine-Donnerstein [ibid] are used in an interpretive paradigm, even though it could be argued that only issues relating to intra-rater reliability are relevant for a sole researcher. However, the aim of the study was to develop an analytical approach in order to carry out *an* exploration of this particular web-board, but to do so with the desire that at least the approach and techniques developed to examine the exchanges could be replicated in similar circumstances[4].

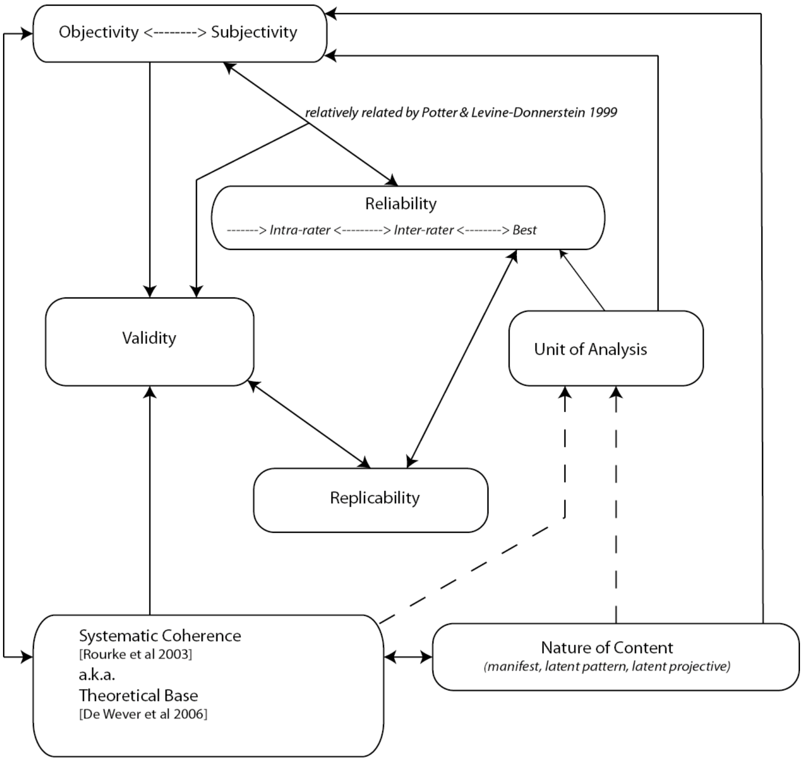

Figure 5.2 Connections between Influences on Validity

Moreover, the methodology could still build on a ‘privileged starting point’ through consideration of the three methodological issues discussed in this section as well as the overarching issue of maintaining an open mind, striving for objectivity within an interpretative paradigm.

5.7 Conclusions

Two surveys, their primary sources and other papers have been used to review frameworks for CMC analysis, to examine existing methods and approaches and to consider key methodological concerns. This systematic assessment of frameworks and methods has found no off-the-shelf solution. Crucially and conclusively, all of the CMC analyses studied had a

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[4] Indeed, after completion of this study my approach was successfully used in further research on AskNRICH by an additional researcher, thus providing evidence of inter-rater reliability and implying objectivity in the original analytical approach.

different setting, purpose and rationale, for example a mandatory, taught, ‘hot-topic’ web-board, that produced a model of collaboration and knowledge construction, quintessentially different and fundamentally incompatible with AskNRICH. In addition, none of the studies reviewed addressed an important issue for the analysis of AskNRICH of strategies to ensure adequate coverage of very large CMCs.

However, the review did provide substantial advice, a “privileged starting point” [Rourke et al. 2003: 134], and revealed some elements/ideas that could be adapted or adopted in devising a methodology for exploring AskNRICH. These can be summarised as:

- incorporating a focus on interactivity and sociability as concepts to investigate, augmenting this with visual mapping
- an integrated examination of the effects of both the individual and collective
- transformative insights into effect of methods chosen and expertise of coders on analysis of content

The next chapter, in elaborating the methodology used in the exploration of AskNRICH, relates how these elements from existing studies are incorporated and how key methodological issues are addressed.

Chapter Six

Methodology and Methods for the Main Study

Exploration of AskNRICH

> *There is a difference between an open mind and an empty head ... We need to use accumulated knowledge, not dispense with it. The issue is not whether to use existing knowledge, but how ... the danger lies not in having assumptions but not being aware of them.*
>
> *[Dey 1993: 63-64 also reproduced in Walford 2001: 9]*

6.1 Introduction

The central purpose of this chapter is to explain and justify the methodology adopted to undertake an interpretative, systematic, in-depth exploration of AskNRICH. The conclusion of the previous chapter made it clear that substantial parts of the methodology and methods for this Study would require some degree of innovation, although the use of some elements of previous work examined in the literature review was possible. A skeletal overview of the complete methodological design is presented in Figure 6.1 [next page]. It also indicates the correspondence between the phases of the methodology and the research goals and questions set out in Table 6.1.

<table>
<tr><td colspan="2">RG2: To formulate an analytical approach appropriate to the nature of AskNRICH</td></tr>
<tr><td colspan="2">RQ3: Can existing methods / frameworks for analysing Computer Mediated Communication forums be employed in analysing AskNRICH?</td></tr>
<tr><td colspan="2">RQ4: How should the exploration of AskNRICH threads be organised (planned, structured and executed)?</td></tr>
<tr><td>Sub-questions:</td><td><i>Which threads should be selected for analysis?</i>
<i>How should individual threads be analysed?</i></td></tr>
<tr><td colspan="2"></td></tr>
<tr><td colspan="2">RG3: To undertake the exploration of the AskNRICH artefact</td></tr>
<tr><td colspan="2">RQ5: What does AskNRICH offer to participants to enable them to pursue their mathematical practices?</td></tr>
<tr><td colspan="2">RQ6: What are participants' common practices when using the AskNRICH web-board?</td></tr>
<tr><td colspan="2">RQ7: What results from participants' practices when using the AskNRICH web-board?</td></tr>
<tr><td colspan="2"></td></tr>
<tr><td colspan="2">Overarching Research Objective: To characterise the network that constitutes AskNRICH, a virtual world that allows young people to meet within it and engage in doing mathematics</td></tr>
</table>

Table 6.1 Research Goals and Questions for the Main Study

The methodological innovations contribute to the claims of this thesis. As will become clear, the resulting emergent methodological design comprised of two phases each involving a number of steps and complex interactions.

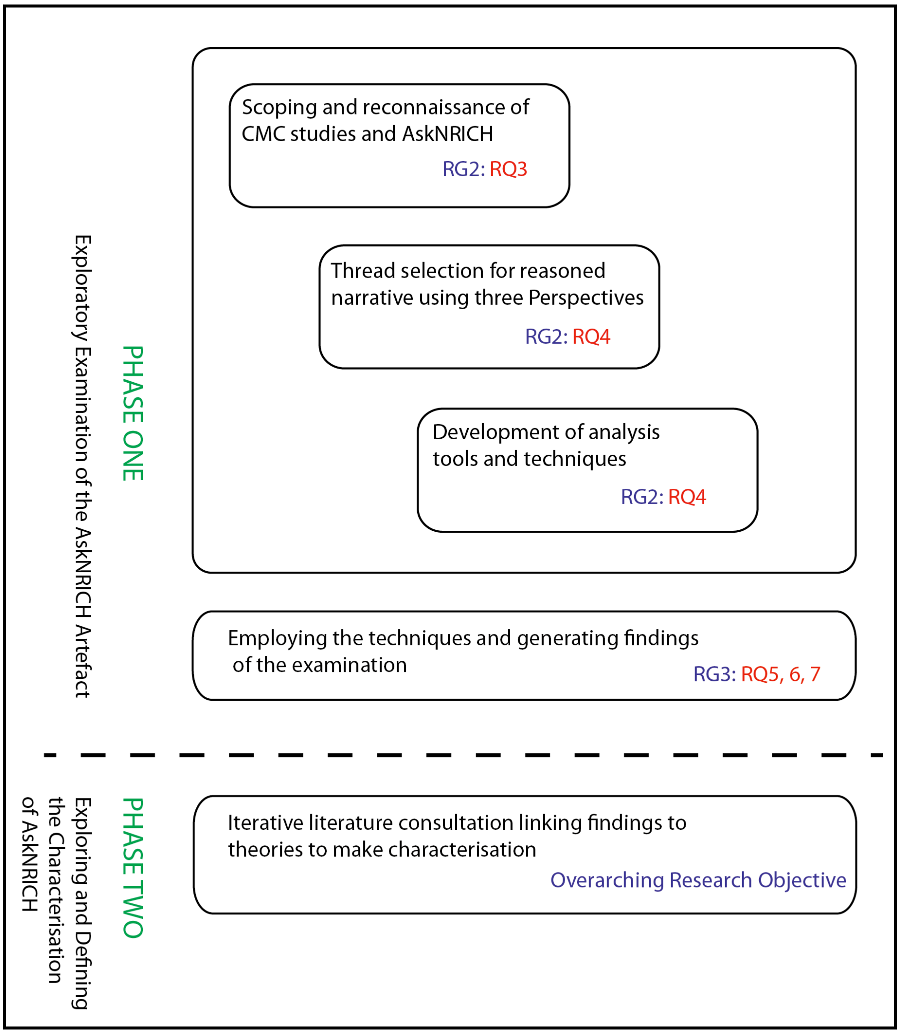

Figure 6.1 Skeletal Overview of Methodology

The literature review (LRII) of the previous chapter both addressed **RQ3** and *informed* the way in which the sub-questions of **RQ4** were addressed. The answers to **RQ3** and **RQ4** shape the approach to the Exploratory Examination of AskNRICH as an artefact, determining the analytical techniques employed in achieving **RG3** (Phase One). This chapter also describes the methodology of the subsequent steps, Exploring and Defining the Characterisation of AskNRICH, to reach the overarching objective (Phase Two).

The purpose of this chapter is to:

i. present the epistemological viewpoint and theoretical perspective that frame the study including the rationale for the emergent methodological design
ii. provide an account of how the Exploratory Examination of AskNRICH was organised to impose order on, and make sense of, the complexities of the vast data source available
iii. explain and justify the analytical approach, methods and techniques adopted to analyse threads and their posts
iv. explain and justify the choice of the three Perspectives[1] from which AskNRICH was investigated
v. detail and justify the range of representative threads chosen for the analysis of these Perspectives
vi. present the techniques employed for analysing threads
vii. present a reflection on the multi-dimensional exploration of AskNRICH through illustrating how the individual Perspectives were combined to present a wholistic picture
viii. describe the process of iteratively reviewing literature to produce a characterisation
ix. relate how the issues of reliability and validity are addressed in this study

The remaining part of this chapter is in four main sections that parallel the division used in setting out its purpose above. Section 6.2 frames the study, purpose (i) above. Purposes (ii) to (vi) are contained within Section 6.3, planning, structuring and executing the Exploratory Examination. Section 6.4 presents the processes through which the wholistic picture was formed and a characterisation generated, purposes (vii) and (viii). Section 6.5 focuses on purpose (ix).

6.2 Methodological Discussion

Within this section, the first sub-section presents the study's epistemology and theoretical perspective whilst the second describes the rationale for the methodological design.

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[1] As noted in Chapter One, not to be confused with a theoretical perspective.

6.2.1 Epistemology; Theoretical Perspective and Methodology

Crotty [1998: 2] presents epistemological viewpoint, theoretical perspective, methodology and methods as a chain of four distinct elements of any research project. Each element shapes or is shaped by the adjacent one(s) both top-down and bottom-up. For this study, the choice of what to adopt for each was not necessarily a straightforward task: the object (AskNRICH) and subjects (the AskNRICHers) are located within a virtual environment, thus in effect providing access to the subjects almost entirely through their written messages texts and not through human contact. Nevertheless, in exploring the phenomena that AskNRICH and its participants exhibit, the research would involve using those message texts to construct meaning of both people and artefact.

This naturally fits my epistemological world view of constructionism that “meanings are constructed by human beings as they engage with the world they are interpreting” [ibid: 43]. Although these restrictions to a virtual world and written text bring degrees of complexity to the research [returned to later] the resources’ access ensured a clear adoption of a theoretical framework residing within the interpretive paradigm [Crotty 1998]. Even though I came to the study with some prior knowledge of AskNRICH and with substantial pedagogical subject expertise[2] developed within the social, interactions world of classrooms, this was simply the starting point. I wanted to make sense of what was appearing to be going on, to the interpreter’s eye, as a result of the actions and activities of the AskNRICHers but in utilising my expertise, ‘a wise lens’, I would bring a mixture of objectivity and subjectivity to the process. Although I would be placing myself, or rather my professional experience at the centre of making my interpretations, I would also be focusing objectively on AskNRICH to determine the range of interpretations that could be made.

However I, as researcher, could neither socially interact with the AskNRICHers, nor construct meaning with anyone but myself. Nonetheless, and in keeping with my epistemological view of the world, I knew my descriptions of AskNRICH and my narrations of what I have perceived to be ‘going-on’ within it could only be meaningfully constructed

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[2] See my ‘confession’ at the beginning of Chapter One.

within the given boundaries and culture, in this case teaching and learning as I knew it from the classroom.

As mentioned above, the restrictions also bring a complexity that is repeated in adopting a methodology. Once more this is not a straightforward task due to the virtual environment. As shown in Chapter Five, researching within this particular virtual environment does not correspond wholly or easily to any examples reported in the CMC literature. That literature appears to contain little reference to the philosophical underpinning of methodological texts, focusing only on the learning theory selected to frame the analysis against. Similarly, at least at the time that this research was being undertaken, there appeared to be little adaption/consideration in research texts to the new technologies, including using a resource one-stage removed from actors in either methodological or philosophical terms. This point is illustrated by now considering two components of research within an interpretive paradigm: ethnography (being with actors) and hermeneutics (studying texts), that might in 'normal' circumstances have been considered possible for the artefact.

In terms of ethnography, conducting the in-depth study of AskNRICH included logging on to the site on a daily basis for a three-month period (late January to mid-April 2008) and periodically for a further six weeks until late May 2008. Within Jeffrey and Troman's [2004: 538] ethnographic time modes, this would fit the compressed time-mode. Furthermore, when logging on to the site on a daily basis, activities could be observed in real time. This 'coming to know' and 'living with' AskNRICH extended to feeling part of AskNRICH, an essential element of ethnography [Goldbart & Hustler 2005]. There was therefore, a developing sense of gaining familiarity with both the participants' working practices and the web-board protocols. Simultaneously, the in-depth study of the content of the artefacts (the product of the participants) within the web-board made the research more than simply observational per se. But ultimately I could never live my life with the AskNRICHers. Crucially I could only ever be an "observer participant" [Ely 1991: 42]; the web-board participants' being unaware of any outsider's presence ensured that I was not part of the culture [Crotty 1998: 76] nor would I be physically "just 'hanging around'" [Walford 2001: 8]. Thus it would be incorrect to ascribe the term ethnographic to this study without a re-interpretation of the term. If Walford's [2001: 7-11] listing of seven features indicative of an ethnographic focus were considered with respect to this study, the first five at least would need to be 're-visioned' to relate to this study [see Appendix 6.1 for a comparison].

In terms of considering this as a hermeneutical study, interpretation of texts (my intention) has been considered historically to reside within the field of hermeneutics [Schmidt 2006; Bleicher 1980]. However my study could never be a hermeneutical study in the sense that I would not be concerned to uncover authors' meanings and intentions hidden within the text for the authors themselves [Crotty 1998: 91], but to interpret the texts to and use the results of what I uncovered [i.e. interpreted] to explain the events I perceived to be occurring. Nevertheless I found Dilthey's differentiation[3] between understanding (needed in human and social science) and explanation (focused on causality and located within the natural sciences) [Schmidt 2006] useful. This differentiation is reported at some length in research method texts under the heading of hermeneutics [see e.g. Crotty 1998, Counsell 2009] and fits both with my epistemological view [see above] and with this study's aim. I wished to gain understanding in order to explain my interpretations that lead to the conclusions I have come to about AskNRICH, never to definitively explain AskNRICH. However, if this study were to be claimed as hermeneutical, it would have the potential to fall into the category where hermeneutics does not "seem much more than a synonym for interpretation in many contemporary [reading of texts] instances" [Crotty 1998: 110]. Nonetheless, as with the argument presented above in considering ethnography, it would be incorrect to dismiss hermeneutics entirely; in particular "the relating of part to whole and whole to part discernable in the interpretative practices of the ancient Greeks would become an enduring theme within hermeneutics" [p89], which provides a description of the hermeneutical circle [Schmidt 2007: 15]. As the interpreter I would "move from the text to the historical and social circumstances of the author, attempting to reconstruct the world in which the text came to be and to situate the text within it – and back again" [Crotty 1998: 95], a description afforded by Crotty to Dilthey's work of late 19th Century. A similar mutual inter-dependency is also present in exploring AskNRICH, where I would not be able to make sense of the individual posts or threads without having a sense of AskNRICH itself and vice-versa.

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[3] The exemplification given here for the distinction is often attributed to Weber [Crotty 1998: 94].

Thus, so far in evaluating the match to either ethnography or hermeneutical study, the virtual environment in which the actors 'live' and purpose of the study to interpret beyond articulating the authors' thoughts make this study, at best, one that might require re-interpretation of some elements from both ethnographical and hermeneutical studies. I remain in doubt as to whether using prefixes such as 'quasi' [i.e. 'being partly'] or 'pseudo' [i.e. 'seemingly but not really'] would be sensible, although they do imply a connection to the fields. It would perhaps be one step too far to take Crotty's [1998: 49-51] account of the term '*bricoleur*' and the required ability of such a person to " … 're-vision' … bits and pieces, casting aside the purposes which they once bore and for which there were once designed and divining very different purposes that they may now serve in new settings" [p51] and apply it to the meanings associated with ethnographical and hermeneutical, but there is a need to establish new 'bits' to this study to account for the 'newness' of the artefact. In essence there was a need to develop an emergent methodological design, the focus of the next section.

6.2.2 Emergent Methodological Design

The previous section presented reasons why the innovative nature of this study favoured an emergent methodological design. Moreover the Exploratory Examination of the *content* of AskNRICH would itself need to be open and evolving as distinct investigations into selected areas (referred to as Perspectives) would each impinge on, and inform 'dynamically', the others. This evolving process would continue until there was a sense that the Exploratory Examination had been completed in a sufficient and rigorous way to allow a valuable and knowledge claiming narrative. Thus both the development of an analytical approach and the Exploratory Examination of AskNRICH proceeded in parallel, although interacting with each other, each traversing a series of multiple, iterative loops combining both inductive and deductive steps [Cohen, Manion & Morrison 2007]. This method of working is portrayed within Figures 6.2 and 6.3 later.

The methodological advance for exploring AskNRICH began in a deductive mode by reviewing CMC literature reporting previous studies [LRII] by considering the frameworks, learning models and methods employed, searching for a methodology to be used for analysing AskNRICH. If that search had been successful then the work would have continued as theory testing [Bassey 1999], but an off-the-shelf solution proved elusive. In reaction to this outcome, an inductive stance was taken in the sense of adopting, adapting, developing and building an analytical approach for exploring AskNRICH systematically.

Likewise the process of Exploring and Defining the Characterisation [‘below-the-line’ in Figures 6.1 and 6.5] of AskNRICH proceeded in a similar evolving manner with an end point of going “beyond description of events to produce models that explain what is going on” [Taber 2009: 218]. The open nature of the study necessitated an iterative organisation with further returns to searching the literature [LRIII and LRIV] as new avenues emerged, precipitating either the addition of different topics or a change in emphasis. This is justifiable approach for exploratory studies [ibid: 222]; without first obtaining findings there were no theories to be tested. In testing out theories this part of the study could be classed as more confirmatory the opposite of exploratory [ibid: 218] but in reality the confirmatory facet here is embedded within an exploratory whole. As later chapters reveal, the eventual characterisation was established through a further development adopting and adapting respectively theories proposed by van Lier and Gee [as already indicated in Section 1.5 p27]. This partial adoption/adaptation mirrors Calissendorff’s “owing a debt” [2006: 92] to a published model to present a modified one [Taber 2009: 223].

It is equally important to consider the interplay of subjectivity and objectivity when justifying the methodological design for this study. I have already made a claim that this research will be valuable as it would be undertaken with a strong professional knowledge base and previous experience that could bring advantages. This could nevertheless be problematic, I was acutely aware that this could move the study further towards subjectivity. Indeed I did not wish to ‘bracket out’ in Husserl’s [Schmidt 2006] phenomenological sense this experience and knowledge, but on the contrary embrace it when undertaking the message interpretations. Thus I would not be “washing the mind clean” [Measor & Woods 1991: 70] as Measor was required to do when engaging in very different classroom observations for her research from those made as part of her everyday work. However I argue that the intended embracement does not in any way negate the need to avoid preconceptions, presupposition and assumptions [Counsell 2009: 270] or with reference to Habermas’ work [Schmidt 2006] of no prejudice, no pre-judgement. I considered that it would still be possible to interweave objectivity and subjectivity as illustrated in this chapter's opening quotation about the difference between an 'empty-head' and an 'open-mind'. Indeed, as Walford [2001] makes clear, being subjective is not a matter of "anything goes" [p9]. The intention was to realise Dilthey's "Objective Mind" [Crotty 1998: 93] or "human consciousness" [ibid: 95] as far as possible whilst simultaneously acknowledging that "the interrelationship between interpreter and interpretation is indissoluble" [Thomas & James 2006: 782]. In other words: it is as it is, but only as it is within the limits of the researcher and that which is researched.

Thus it will be shown that, within the limitation of any constraints, particular care was taken to ensure that the research would be as rigorous as possible. The remaining parts of this chapter demonstrate this in setting down the planning and implementation of the many faceted components involved.

6.3 Planning, Structuring and Executing the Exploratory Examination of the AskNRICH artefact

The previous chapter reported a review of literature on analysing CMCs (web-boards) undertaken during the iterative execution of the Preparation Tasks [see Figure 6.2 p117]. The review concluded that, although there were methodological issues that were clearly pertinent, there could be no direct appropriation of any one framework or method. Nonetheless, there were a number of elements that could be adapted or adopted in devising an analytical approach appropriate to the nature of AskNRICH. In stark contrast to most reported studies, due to the sheer amount of thread data available, the need to select and the method for selecting threads was a paramount consideration in ensuring that the analysis would provide a suitable basis from which to construct sufficient meaning in order to make a justifiable characterisation of AskNRICH. Naturally, it was also necessary to decide how an individual thread should be analysed, again in order to properly capture the sought-for essence of the AskNRICH world. These two decisions evolved in tandem in an iterative process during the whole exploratory examination. This process was initiated during the Preparation Tasks that also finalised the sub-questions of **RQs 5 to 7** [Table 6.1].

6.3.1 Contextual Information

Before discussion of these necessary formative decisions some contextual background relating to data collection is set out: the timeline of the exploratory examination, the means for providing a degree of triangulation of the data and a description of how the dynamic nature of the web-board was managed.

6.3.1.1 Timeline of Data Collection

Table 6.2 [an extract of Appendix 1.2] below presents the timeline for data collection in preparation for the characterisation of AskNRICH. Most of the work involved using the threads and posts contained therein [entries 13 and 14 of Table 6.2] by taking on the role of a non-participatory observer, studying the board's resources and activities, over a ten-month period. There was a period of three months of intensive work during this time that involved visiting and being constantly immersed in the site daily [see Table 6.3 later]. Observation of the site (lurking) also continued throughout subsequent analysis stages and indeed until the end of the study.

Data Collection for Exploratory Examination				
	Time	Data Item	Participants	Rationale for/Use of Data
11	August 2007	Tape recorded Individual Interviews	Director of NRICH Deputy Moderator of AskNRICH	Gain increased knowledge from two key personnel involved with NRICH
12	October 2007	Email correspondence	One female user of NRICH site One male regular participant in AskNRICH	As result of web-survey analysis, investigation into NRICH and AskNRICH users' activities
13	August to December 2007	Trawling AskNRICH [live and archived]	AskNRICHers Considering some 250 threads involving some 2000 posts	Scoping the Research and Reconnaissance of artefact, recognising the means of establishing order on the vast amount of material available, leading to finalising sub-questions of RQ3-7
14	January to May 2008 (and at times until study's end)	AskNRICH website (daily visiting late January to mid-April)	AskNRICHers Reading in excess of 500 threads involving 4000 posts	In-depth study of the workings of AskNRICH, consider and evaluate possible analytical approaches and techniques, selecting threads
15	March 2008	Email correspondence Tape recorded Individual Interviews	One male member of AskNRICH answering team One male regular participant in AskNRICH One female 'lurker'	To add to information being gained from AskNRICH explorations

Table 6.2 Timeline of Collection from AskNRICH Data Sources

6.3.1.2 Triangulation

The text-based content of the web-board was the primary, but not quite the sole data source at the Exploratory Examination stage. Some desirable triangulation [Evans 2009a: 120, Cohen et al. 2007: 141] during the qualitative analysis of data could be obtained in three ways. Firstly [as shown in entries 11, 12 and 15 of Table 6.2] it was possible to undertake some email correspondence and taped interviews with six people involved in varying levels with AskNRICH[4] [see Appendix 6.2 for sample interview schedules]; secondly, local involvement with both the Moderator and Deputy Moderator of AskNRICH was maintained throughout the study and personal contact with NRICH continues. Immersion and continuing lurking provided a third means of obtaining triangulation. Appendix 6.3 demonstrates how immersion facilitated the interpretation and analysis of the threads where two posts one year apart have been used to affirm an interpretation made of an individual post. Hence, the careful deliberations on one thread made with reference to other threads have gleaned greater knowledge than any reading of a single thread would provide. These three ways of triangulation relate to Denzin's [1970] designated types: methodological; combined level, and time, respectively.

6.3.1.3 Addressing the Dynamic Nature of the Web-board

There were two potential problems due to the dynamic nature of the web-board that needed to be addressed from the outset. Firstly, participants come and go according to different patterns and intensity of participation. In this study, however, contributions of all participants were considered without regard to any such differences. Secondly, new posts can be added to a thread at any stage before it is entombed in the archive. Thus threads are often left open for a six-month period or longer allowing for the possibility of re-opening the debate and taking the thread either further or in a different direction. In order to give consistency to the timing of when threads were active they were identified by the *date of their last post.* Even so, data within any tables presented in this thesis are accurate only *at the moment of* constructing the specific table and thus there may appear to be inconsistencies

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[4] Two school-aged participants of AskNRICH (one of whom is subject of the case study) engaged in email exchanges whilst two taped interviews were conducted with two undergraduates, one is an AskNRICH team member active on an almost daily basis. The second has continued undertaking NRICH problems having first encountered the website in school but only 'lurks' on AskNRICH – admitting to reading and working through the mathematics of many posts, but has never posted a message. The remaining two were staff from NRICH.

across tables when in reality no exact correspondence between tables constructed at different times could exist.

6.3.2 Volume of Threads and Posts Perused

Around five hundred and fifty threads, containing a total of approximately 4500 posts were perused in the scoping and reconnaissance stage. These were both live and archived threads taken from the two sections, aimed at school-aged pupils (up to 18 years old), Please Explain [**PE**] and Onwards and Upwards [**O&U**] [see Section 8.3.2 p169], but mainly from the latter due to the greater activity there.

Following this, to gain a more detailed but still manageable snapshot of the discussion board posts, the archive of twenty two threads from **PE** posted between September and December 2007 was systematically recorded in terms of (a) number of posts, (b) day, date and time of first and last post posted within the thread and (c) comments about the thread [see Appendix 6.4 for selection]. Furthermore, live threads were constantly being read and notes made of ‘incidences’ within the exchanges that appeared to offer insight into the working practices and conversations amongst the participants. A number of threads (45 from **PE** and 70 from **O&U**) representative of those being read, were also kept in anticipation of more detailed analysis.

A breakdown of the numbers of live threads and posts available from each section during the data collection period are shown in Table 6.3 below. Many of the posts were read more than once at different times and different stages during the research study. For example, once the case study subject had been selected all retrievable threads in which he had posted were perused (c.1500 posts in 134 threads). In addition to the threads listed in the table, further threads were read in a general perusal of postings in the post-school section, Higher Dimension [**HD**], as some of the posters in the ‘younger’ sections also post in this section for University Mathematics and in the Private Area of the web-board. In all, approximately 15% of available threads were perused at some point during the study.

Research Activity	Date of last post in thread	AskNRICH Section	Live Threads	Live Posts
13. Scoping and Reconnaissance	Aug 07 to Dec 07	Please Explain Onwards & Upwards	40 329*	285 2766*
14. Logging on daily	Jan 08 to mid-April 08	Please Explain Onwards & Upwards	47 171	494 1315
Casual visiting to maintain monitoring	mid-April 08 to late May 08	Please Explain Onwards & Upwards	28 83	209 617
		(Total)	(698)	(5686)

* only a proportion of these were read in full (approximately 200 threads and some 1600 posts)

Table 6.3 Number of live Threads and Posts available during Initial Explorations of AskNRICH

In the following sections the analytical approach devised is presented in a structure framed by the two formative decisions discussed above: the selection of threads to be analysed and how individual threads are analysed. The rationale for the choices made and the details of the methods and techniques employed are reported. The multiple inter-dependent pathways followed by the analysis of threads, selected from those collected in the preparatory forays described above, are shown in diagrammatic form in Figure 6.2.

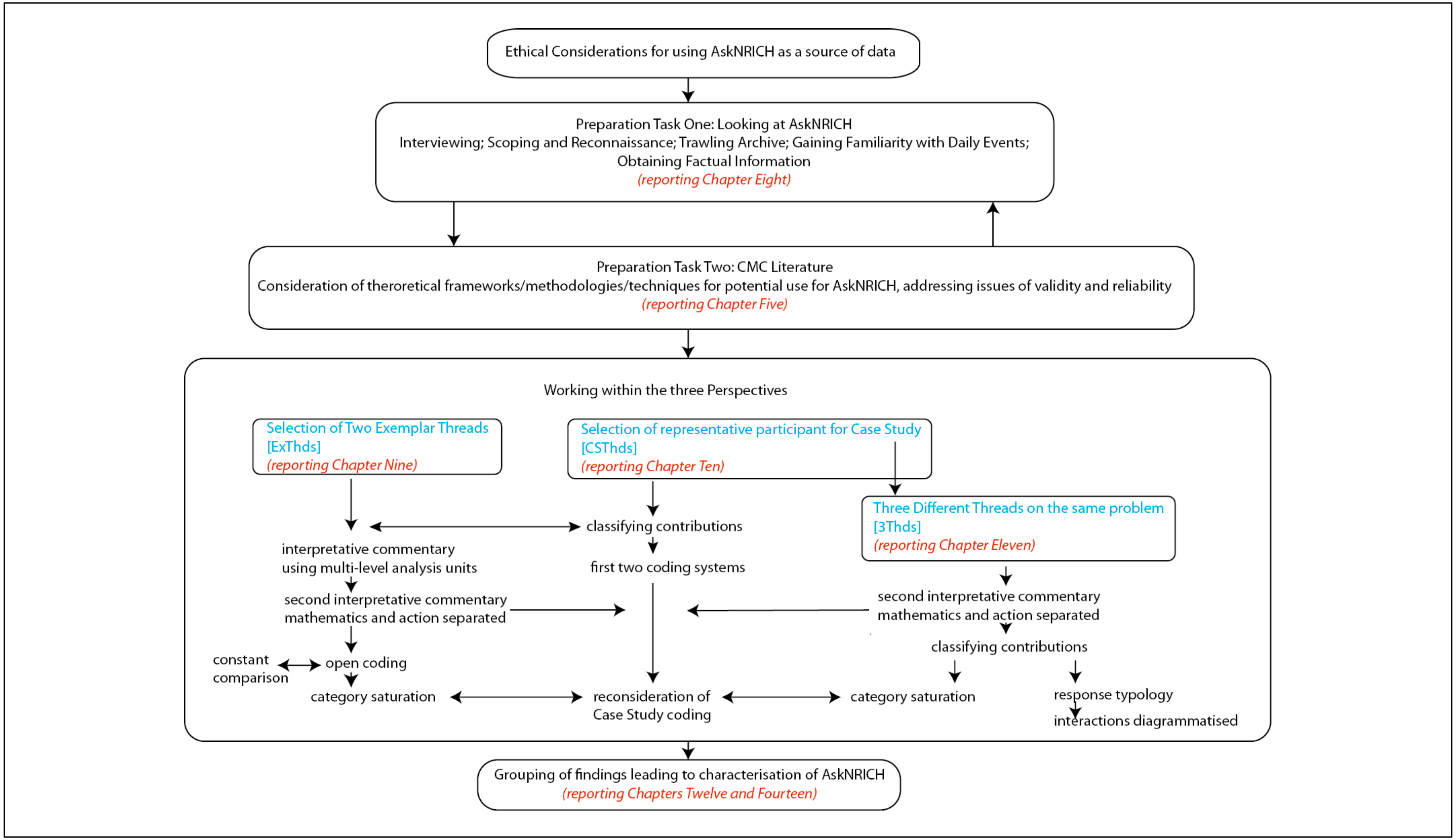

Figure 6.2 A Diagrammatic View of the Structuring, Development and Execution of the Exploration of AskNRICH

6.3.3 Selection of Threads for Detailed Analysis

As already argued, the size of AskNRICH data available dictated the need for a systematic, and rigorous mechanism for selecting a limited number of threads to be analysed from which sufficient meaning could be constructed for the characterisation of AskNRICH [see Claim 1 Section 15.3 p307]. This requirement was addressed by looking at AskNRICH from multiple Perspectives, whose selection was finalised by the end of the scoping and reconnaissance stage [See Figures 6.1 & 6.2]. Three Perspectives were chosen: firstly, a set of exemplar threads [**ExThds**], identified by studying a sufficient number of threads to establish the general/common practices and use made of the web-board; secondly, a case study that would focus on one representative AskNRICHer and include an examination of all threads [**CSThds**] in which they had posted. A third, harder to define, Perspective aimed to compare the AskNRICHers' interactions and ways of working on mathematics with those of 'professional mathematicians', a very different context from the in-school mathematics norm. The selection of threads for the first two of these Perspectives was relatively straightforward to make, but it was initially not clear how to undertake the selection for the third. Although working on the first two Perspectives would inherently provide data for the third, whilst undertaking the Case Study Perspective, three separate threads [**3Thds**], all on the same mathematical problem, posted over a period of time were discovered, that could be used to achieve a full exploration of the third Perspective. Taken together the three Perspectives enable a more comprehensive and rounded view through the integrated consideration of both the individual and the collective group, as argued in Chapter Five [Section 5.5 pp96-97]. The way in which the combination of findings from these three Perspectives was used to create a wholistic view is discussed later in this chapter [Section 6.4]. Further detail on how the threads for the first two Perspectives were selected is now set out.

6.3.3.1 Selection of Two Exemplar Threads from General Postings

Detailed consideration of the selection of threads to become exemplars evokes the paradox that 'all threads are the same and every thread is different'. All threads are the same in the sense that each begins with a participant posting a mathematical question that they wish to solve for a personal reason and should not conclude before a solution is reached to their satisfaction. Equally, however, every thread is different as they vary according to the question posed and the way that the participant can make sense of the help offered by those who decide to contribute.

Nevertheless as the number of threads read increased, it became clear that, whilst the mathematical problem posed was different, there was an observable consistency in the approach and characteristics displayed, [as the findings in Chapters Eight to Twelve will show]. As a result, two complementary, representative threads were chosen as exemplars for an in-depth analysis. One [**ExThd1**] contains posts primarily relating to the actual mathematics being undertaken. The other [**ExThd2**] begins with exchanges that discuss the process of doing mathematics, before settling into considering the specific mathematics. Thus the two threads taken together cover a range of types of exchanges and participants' actions that is widely replicated in other threads.

6.3.3.2 Selection of Representative Participant for a Case Study

The choice of an AskNRICH participant to be representative of the core contributors was initially made on four criteria: age to coincide with compulsory schooling; school attended to be a state comprehensive school; the opportunity to have email correspondence with the selected participant to probe more than could be managed with only a clinical reading of the posts, and the number of posts to be manageable to study in their entirety (in the event 501 posts). The first two criteria locate the case within the every day life experiences of the majority of children in the country and thus findings drawn from this study could both relate to, and be relevant to 'ordinary' education. This type of case study can be categorised as *Exploratory* [Yin 2009], *Theory Seeking* [Bassey 1999] and in terms of Stake's [2000: 437] definitions, that he states are not necessarily discrete, as *part-Intrinsic* and *part-Instrumental*. By the exploratory nature of the research the first of these three categorisations is self-explanatory and likewise emerging themes derived from the data would therefore make the case study theory seeking. In terms of Stake's categorisations, it is intrinsic because the particular choice of subject for the case study is of interest for the reasons stated above, but also instrumental as the case would also facilitate understanding of other generalities within AskNRICH since all threads inherently involve other AskNRICHers' posts.

6.3.4 Techniques Used for Analysis of Selected Threads

In order for the study of AskNRICH to be a wholistic exploration, in-depth, formal, written discourse or conversational analysis was avoided; an appropriate strategy for such circumstances explicitly advocated by Gee [2005a, 2008]. Furthermore, as literature reviewed in LRII [Section 5.3 p93] implies, both subject and pedagogical mathematical knowledge are indispensible in achieving a valuable, valid and replicable analysis of the AskNRICH web-board. It would be inadequate to describe characteristics of AskNRICHers without consideration of the mathematics, as some actions and accomplishments only appear through understanding of the mathematics and how it is being taught and learnt. Thus it is vital to work through the mathematics[5] (not always a straightforward task!) before starting the analysis. This has the additional, beneficial effect of transforming some of the latent content to become manifest and providing a deeper level of understanding of pedagogical interactions that can, in turn, be applied to the latent pattern and latent projective [Potter & Levine-Donnerstein 1999] content of other parts of posts [see Section 5.6.1 p99].

Nonetheless, the aim of this research is to understand the interactions, practices and outcomes of 'doing mathematics' that are present both explicitly and implicitly (manifestly and latently) in the threads, leading to a characterisation of AskNRICH, and not to study the mathematical content of the AskNRICH web-board per se. Such a position is conveyed by a second paradox that: 'the mathematics is simultaneously both at the centre and peripheral to each thread'. The mathematics within the topic of a thread is the central theme of the posts within it, but the actual topic itself is unimportant to the consideration of the actions and accomplishments of the AskNRICHers involved. In other words, *how* the mathematics is 'being done' through the exchanges is central to the analysis, but the *actual* mathematics (i.e. the topic) being studied is of little importance to the analysis.

As alluded to earlier [Section 6.2], the analysis of threads in the three Perspectives was conducted iteratively in distinct, interacting investigations [see Figure 6.2]. This meant, for example, that, having made some progress with the **CSThds** and the **3Thds**, more could be

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[5] Doing the mathematics though vital for the analyst, is not necessary for the reader.

brought to a second iteration of analysis of the **ExThds**. This second iteration would, in turn, enhance subsequent iterations of analyses in the other two Perspectives, and so on … .

This complicated overlapping and interweaving structure is by necessity presented below in a flat, linear order through four sub-sections. These sub-sections detail:

i. generation of multiple, parallel, separate commentaries tracking through all posts in a thread
ii. analysis of case study threads and codes generated
iii. analysis of exemplar threads and codes generated
iv. generation of a diagrammatic representation of the interactions and connections within/across threads

Within each sub-section reference is made as to whether the methods used have been devised specifically for this study, or adapted or adopted from previously reported studies. Where a technique has been devised or adapted the explanation is presented in sufficient detail to assist replication and examine validity.

6.3.4.1 Generating Interpretive Commentaries

In order to analyse and construct meaning from a thread it is first necessary to understand what is going on within the text. In this study the unpacking the meanings embedded within threads is achieved through writing down a clear, ‘understandable-by-others’, explanatory commentary on each post using multi-level, dynamic units of analysis [see Section 5.6.2 p100]. The first steps towards generating a commentary are to work through the mathematics and write notes on a printed copy of the thread [as later illustrated using **ExThd2** in Chapter Nine Appendix 9.6] that have taken into account the mathematics and information gleaned from the notes made during the data collection stage [see Section 6.3.2]. In creating commentaries against each post, contextual information, including factual data linked to posts, from any other part of the whole thread, i.e. both all previous and all succeeding posts, is taken into account. Constantly keeping in mind both the individual post and the whole thread in creating a commentary is an explicit example of the hermeneutical circle referred to above [Section 6.2.1].

The **CSThds**' commentaries used in reporting in Chapter Ten were created using the method described in the previous paragraph. For the **ExThds**, initial commentaries were made in the same way, but later reconsidered and improved in two further applications of the same method [see Chapter Nine Appendices 9.3&9.4 for the first two iterations of commentaries for **ExThd1**]. Finally, a refined method that helped to further unpack the thread was used on both the **ExThds** and the **3Thds**. In this refined method the mathematics and actions within the commentary were disentangled to produce two separate, parallel streams, one focused on the mathematics and the other on actions [see Figure 6.2]. Table 6.4 [next page] presents an example extract. This presents an even richer picture that, in turn, enables a further, deeper understanding of the thread content. Throughout the process of improving the commentaries the goal was to make the mathematics within any commentary as clear as possible to present non-expert readers of this thesis with a greater opportunity to access the process and outcome of the commentaries (and hence the overall methodology). Moreover, the disentangling of mathematics and actions into two parallel commentaries is critical in enabling the subsequent coding [see later] to properly reflect the 'doing mathematics' rather than the mathematical content per se. The commentary in the actions column emerges as a result of the mathematics being studied.

As will become evident through further work in this and later chapters, codes emerge either as a consequence of the actions or from explicit data on the web-board e.g. time, day, name. Hence the codes are predominantly non-subject specific and devoid of mathematical language even though subject and professional expertise was required in their formation. The implementation and use of this novel, parallel commentaries, technique [see Claim 1 Section 15.3 p307] was central to building a characterisation of AskNRICH.

Post Day/time	Post extract [précised]	Commentary Mathematics focused	Commentary Actions focused
P1 Friday 7.29pm	***Plea1:*** *We are solving simultaneous equations, one linear, one quadratic. I am stuck on two. I know the answers but I can't work out how to get them. Any help is greatly appreciated.* [Two questions and answers stated]. *I can usually solve them, but these two got me really muddled. Thanks in advance.*	Solutions values known and by substitution can be seen to be correct Plea wishes to understand how to work the solutions out	Posted at beginning of weekend, out of school time Content to show current inability Politeness
P2 Friday 8.13pm	***Help1*** [Provides a worked solution to an alternative question]: ... *See if you can do it for yours now. If you can't, post your working and we can see where you've gone wrong ...*	Method required explained and illustrated by a carefully considered (same structure) example	44 minutes before reply Relevant example especially devised Offers encouragement to try with reassurance of further help if required
P3 Friday 8.46pm & P4 8.54pm	***Plea1***: *I'm sure I've made a really silly mistake ... THESE AREN'T THE RIGHT ANSWERS. Thanks in advance.*	Knows error exists as for one of the problems it does not factorise though known that it should, and for the other problem the values derived are not the same as for the known solution	Has spent over half an hour (assumedly) trying to get correct solutions Posts mathematical workings Suggests own inabilities Apparently frustrated but is persevering Asking for further help with no explicit 'write down the solution for me' Perseverance
P5 Friday 9.58pm	***Help2:*** *For 1 you've just made a mistake in expanding the expression, can you see it?* *For 2 in a few steps you have divided/multiplied by x, which means that you have to check the extra case x=0.* *Additionally you've made a silly mistake in expanding ...* *Can you solve it now?*	Common error in algebraic manipulation Error made manipulation more complex and included a special case of division. This will also be true with correct expansion	Second helper [and first time poster] now involved Teaching locates and signals error but leaves ***Plea1*** to attempt to correct for self Teaching aware of special case, anticipates misconception, 'future-proofs' Supportive atmosphere, still asking if explanations are sufficient to complete solution plus 'silly mistake' is a repeat of ***Plea1***'s own turn of phrase

Table 6.4 Example of Commentary Created using the Refined Method on the first five (of twelve) Posts of ExThd1

6.3.4.2 Using the Interpretive Commentaries

The fundamental purpose of generating the interpretive commentaries was to provide a means of gaining the ‘valuable and knowledge claiming narrative’ referred to earlier. This was facilitated in three different ways. Firstly, the microscopic examination of threads required in creating a commentary would assist the application of any coding process or processes. Secondly, the parallel commentaries were the primary source for the diagrammatic representation of threads reported in Section 6.3.4.5. Thirdly, and more generically, the process of generating the interpretive commentaries also produced greater familiarity with and insights into AskNRICH retained ‘in my mind’ thus facilitating improved observation and interpretation when reading other threads. Consequently, the overall quality of analysis was enhanced even if, due to time limitations, these other threads could not be written up in the same amount of detail as a two-strand commentary.

As the interpretive commentaries developed so did the sophistication of coding. Thus the remaining sub-sections are presented in the order of the case study followed by the exemplar threads and, finally, the diagrammatic representation generated by the commentaries to reflect this sophistication.

Although all the threads are accessible electronically and can be easily exported into MSWord documents (with a mathematical text editor), the mathematical text could not, at that time, be imported easily into Nvivo. Hence open-coding on print-outs, threads and/or commentaries, reverted to manual means i.e. key phrases/events highlighted and a descriptive ‘work-in-progress’ code written alongside. Throughout, commentaries and threads were used in tandem, a constant movement back and forth between the two. Though aware that only intra-rater reliability was technically possible for this study, the desire that there would be inter-rater reliability in any further analysis remained nonetheless [see Section 5.6.3 p101].

6.3.4.3 Analysing Case Study Threads and Codes Generated

Every retrievable thread that contained posts made by the case study subject (***Peter***) were printed and scrutinised at least three times. To be systematic all ***Peter***’s posts were sorted by date of first post and then indexed by day and time of posting, web-board section, thread name (mathematical topic) and whether ***Peter*** had asked for help or was in a helping role [see Section 10.2.2 p218]. Threads where ***Peter*** was asking for help were further differentiated by separating out those that were started by ***Peter*** and those where ***Peter*** joined part-way through. This sorting also facilitated the task of investigating ***Peter***'s increasing proficiency either in a learning or in a teaching role.

The analysis of threads followed the pathway presented in Figure 6.2 above. Initially, threads started by ***Peter*** seeking help were examined, key phrases were highlighted and notes made to record the pertinent, relevant actions, both of ***Peter*** and of other contributors. A first trial of open coding [Corbin & Holt 2005: 51; Strauss & Corbin 1998: 101], applying descriptive labels to sections of text [Evans 2009b], was undertaken directly on these threads. Subsequently, interpretative commentaries were constructed for 15 threads and a second trial of open coding undertaken using the commentaries and thread-texts in tandem. Table 6.5 presents the themes, eight in the first, refined to five in the second, emerging from these trial attempts [see Chapter Ten Appendices 10.5&10.6 for examples from these trials].

	First Trial Coded Themes		Second Trial Coded Themes
1 2 3	Friendly Communication Etiquette Openly shows knowledge limitations	1	C – Community Characteristics
4	Mathematics	2	M – Mathematics*
5	Thinking	3	T – Thinking
6 7	Understanding Development in work, often through explanation	4	U – Understanding
8	Ways of working	5	W – Ways of Working (Mathematically)

* to represent: mathematical subject knowledge, or mathematical facts or the specific mathematics required for the problem

Table 6.5 Themes from first two Trials of Open Coding

The two themes Thinking [T] and Understanding [U], in Table 6.5, are problematic because they are inherently internal processes[6] whose presence can only be inferred by interpretation of the content of the posts, albeit sometimes aided by explicit personal comments and showing working. This difficulty was overcome by further development of the coding system during the re-examination of the **ExThds** that followed immediately after the two

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[6] The implicitness of these internal processes may go towards providing reasons for why others have been critical of Henri's model when their replicability rates have been lower for the cognitive dimension [Garrison & Anderson 2003: 138].

trial rounds of coding. In this re-examination of **ExThds**, which used the dual interpretive commentary process explained earlier [Section 6.3.4.1], instances attributable to Thinking and Understanding became differentiated by the type of event and were finally re-grouped under two main headings of 'Features relating to the Learning Role' and 'Features relating to the Teaching Role'. At the same time, in reconsidering the other case study codes, the M and W themes listed in Table 6.5 were subsumed within the appropriate features of these two main headings. Finally, to provide a more accurate portrayal of commentaries, the community characteristic theme used in the second trial was discarded and its three constituent parts from the first trial were re-allocated as follows. The theme 'Openly shows knowledge limitations' became part of the theme Features in a Learning Role and was renamed 'Openness of current difficulties'. 'Friendly Communication' and 'Etiquette' became part of the Social and Personal Theme, exemplifying some of the positive personal qualities resulting from the use of CMCs highlighted by earlier researchers [e.g. Mason 1994, Rourke et al. 1999]. Although the themes presented in Table 6.5 did not at the time involve examination of Peter in a teaching role, by the end of the exploratory examination, ***Peter***'s practice when in a teaching role had been included [see Figure 6.2]. Thus the themes and codes have been reconfigured and incorporated into the final allocation of 29 codes presented in Section 6.3.4.4 below.

6.3.4.4 Analysing Exemplar Threads and Codes Generated

Section 6.3.3.1 provided details of how two threads [**ExThds]** were selected to represent general activity. Developing an interpretative commentary on **ExThd1** was undertaken concurrently both with a continuing in-depth analysis of the **CSThds**, which by that stage included some trial coding, and examining the **3Thds**. All this activity preceded coding the **ExThds** and by this stage the interpretative commentary on the **3Thds** had evolved to being separated into two strands [see Section 6.3.4.1]. Thus the same type of two-strand commentary was used for the final coding the **ExThds** [see Figure 6.2] and this coding was only undertaken with the evidence that had already emerged in the work and analysis of the **CSThds** and **3Thds**. By this time there was also an increased awareness of AskNRICH, since threads within the other two Perspectives had already been studied. Nonetheless, perseverance with open coding allowed for improved capture of individual features/practices emerging from the interpretative commentaries at a sentence, post and thread level. Indeed, the substantial number of codes generated, as compared with the trial coding of the case study, implies a finer level of detail and a more informed, improved analysis.

Table 6.6 [next page] lists the codes and indicates the thread in which each was first generated. During coding of the threads and interpretive commentaries, procedures were adopted that are analogous to those commonly used on raw data by grounded theorists [Strauss & Corbin 1998] to in effect probe and ensure the robustness of the results of coding and to also ensure that the list in Table 6.6 is exhaustive. The (17) discrete codes that emerged from coding **ExThd1** were grouped and (4) overall categories/themes derived in a process analogous to the activity of *"axial coding"* [ibid: 229], where "conceptual categories … reflect commonalities among the codes" [Harry et al. 2005: 5]. Here the themes would correspond to conceptual categories and the connections between the features assigned individual codes reflect the commonalities. A second exemplar thread [**ExThd2**], specifically selected for its differences from the first thread, was then coded resulting in (10) further codes but no new categories. By this stage, the coding process was following the *"constant comparison"* [Strauss & Corbin 1998: 67] approach. Since it was thought unlikely that just two threads could produce a complete list, other threads were re-examined. The codes obtained from the **ExThds** were (re)-considered against the interpretative commentaries of the **3Thds** (generating no new codes) followed by the **CSThds** (generating two new codes). Although only two additional codes had emerged beyond the exemplar threads a final examination was made against thirty other threads, but this revealed no new additional codes, a point at which *"category saturation"* [Corbin & Holt 2005: 51] had been reached [see Figure 6.2].

As alluded to at the end of Section 6.3.3.1 above in discussing the 'same but different' paradox, the codes emerged either as a consequence of the actions, a combination of seeking direct clues and interpretation of what is bringing about that action, i.e. Potter and Levine-Donnerstein's [1999] pattern and projective latent content, or posting facts obtainable from the web-board's structure. The result is a set of codes that are predominantly non-subject specific and devoid of mathematical language. To elaborate further *'Here's an example …. See if you can do yours now'* [in **ExThd1**] is assigned the code TREG as the poster is in a teaching role (TR) providing a worked example (EG) different to the original question. The coding system is principally independent of the specific topic (Simultaneous Equations) and the mathematics (algebraic manipulation techniques required to solve the equations.)

Code Index	**Themes** and allocated features	First generated
LR – Features in a Learning Role		
LRA	**Seeking re-assurance** that solution/chosen method/idea is correct	**ExThd2**
LRB	Seeking whether there is a **better** (alternative) **solution** than own obtained	**CSThds**
LRC	Seeking aspects that **constitute a proof**	**ExThd2**
LRI	Feeling or **intuition** for solution or path taken being correct/wrong	**CSThds**
LRJ	**Joining in to find a solution** to the problem that someone else had initiated	**ExThd2**
LRO	**Openness of current difficulties**	**ExThd1**
LRP	**Perseverance**	**ExThd1**
LRU	**Developing signs of** (deep/relational) **understanding**	**ExThd1**
LRW	**Showing working**	**ExThd1**
TR – Features in a Teaching Role		
TRAD	**Anticipating difficulties**	**ExThd1**
TRAM	**Alternative methods offered**	**ExThd2**
TRDE	**Direct explanation/working through the problem**	**ExThd1**
TREG	**A worked solution to a different example**	**ExThd1**
TRMA	**Mathematical Advice**	**ExThd2**
TROD	**Open Discussion**	**ExThd2**
TROH	**Overlapping help**	**ExThd1**
TRRR	**Restricting Response**	**ExThd1**
TRSE	**Signalling error**	**ExThd1**
TRSM	Providing **specific method** to adopt	**ExThd1**
SP – Social and Personal		
SPB	**Banter**	**ExThd2**
SPC	**Care for others**	**ExThd1**
SPH	**Humour**	**ExThd2**
SPO	**Opinion**	**ExThd2**
SPP	**Politeness**	**ExThd1**
SPT	**Non-mathematics talk**	**ExThd2**
T – Temporal		
TA	**Significant influence of asynchronous communication**	**ExThd1**
TB	**Time between responses**	**ExThd1**
TE	**Working on a mathematical problem sustained over an extended length of time**	**ExThd1**
TM	**Mathematical teaching present beyond the confines of the school day**	**ExThd1**

Table 6.6 Coding System for AskNRICH Threads

Moreover this ‘same but different’ paradox [see Section 6.3.3.1] in part implies, each thread had its own distinguishing/different features and thus potentially could produce new code(s). This would make the analysis as unwieldy as the vast number of threads available as data.

However with hindsight, given the posting protocols, a purpose of people learning and teaching mathematics and the asynchronous nature of the board, the four categories obtained might have been those expected at the outset. Nevertheless the codes have evolved through a meticulous, systematic and reflective process involving commentary generation, interpretation and open coding. The fastidiousness with which the two threads in **ExThds** were selected to represent general activity appears justified as they account for 27 of the 29 codes generated. This might be in part due to these two exemplars being longer than the norm, thus increasing the potential for a larger number of different incidences that might produce new codes.

6.3.4.5 Forming a Diagrammatic Representation of Interactions and Connections

Threads capture every word that has, with deliberation, been written down and thus represent the totality of the AskNRICHers' talk and conversations. The web-board's linear sequencing of posts, determined solely by the time of the posting, hides the complex nature of the network of interactions. The interpretive commentary accompanying each post recognises the intricacies of AskNRICHers engaging with each other by considering a post in relation to the whole thread. Nevertheless, the analysis of the Three Threads Perspective highlighted the potential benefits of a complementary investigation using a simple visualisation of the interactions and connections between all participants and posts, both within and across the threads.

Although visual mappings are not a new concept, the purpose, rationale and implementation of the diagrammatic representation needed for this study are different to those of previously reported studies, as stated in Chapter Five [Section 5.5 p98]. The unique nature of AskNRICH fosters forms of free exchanges whose subtleties and complexities, not seen in other studies, need to be accommodated and clearly, explicitly, visible.

A type of visual mapping, a **connection diagram,** and an associated **typology of responses** were devised for this study. These diagrams portray interactions between participants in terms of the five response types explained and illustrated in Table 6.7 [next page]. The response typology was derived, initially using the three threads, through considering: the chronological order of the individual posts; the immediate interaction that resulted from a post, and the overall actions evident within the thread. As an example, the connection diagram portraying the first of the three threads is shown in Figure 6.3 [next page]. The labelled rectangular boxes indicate the individual participants, the lines represent responses and are colour and line-style coded to show their type. Alongside each line is a number indicating the post that formed the response [Section 11.4 p253 explains in detail the multi-mapping between post and response type]. See Appendices 6.5 and 6.6 for a series of connection diagrams tracking the progress of a thread, from two different viewpoints, as each post arrives.

Type of Response	Description	Example of Response [Taken from **3Thd1** used in Chapter Eleven]	Line Representation
[DR] Direct	the reply to a statement/question from one participant to another posting protocols determine that direct responses are formative in nature	***HelpB***: *If you look back over your proof, you used the fact that ALL primes are 6n-1 and 6n+1. However, is the converse of *this* true? Are all 6n-1 and 6n+1 prime? Using this, you can construct a counterexample*	
[FR] Follow-on	from a participant who makes a direct reference to a post, having picked up on a suggestion from a different poster	***HelpD***: *Not to be a spoil sport, but I don't think 24 quite cuts it as a counterexample* ☺	
[MR] My (Mine)	from a participant, other than the originator asking for help, offering their own solution having picked up on another's hint	***Help A***: *or 24* ☺	
[OR] Open	a statement/question offered to anyone (who may or may not respond) 'out in the virtual world' including the first post of any thread	***Peter***: *for anyone who's interested one counter example is 48*	
[PUR] Picked Up (an) Open Response	from a participant who has picked up on an OR (Open Response), either from the trigger or other post(s)	***HelpC***: *Or if you really want to do no work whatsoever when it comes to multiplication just use 720*	

Table 6.7 The Five Response Types with Examples and Line Representations

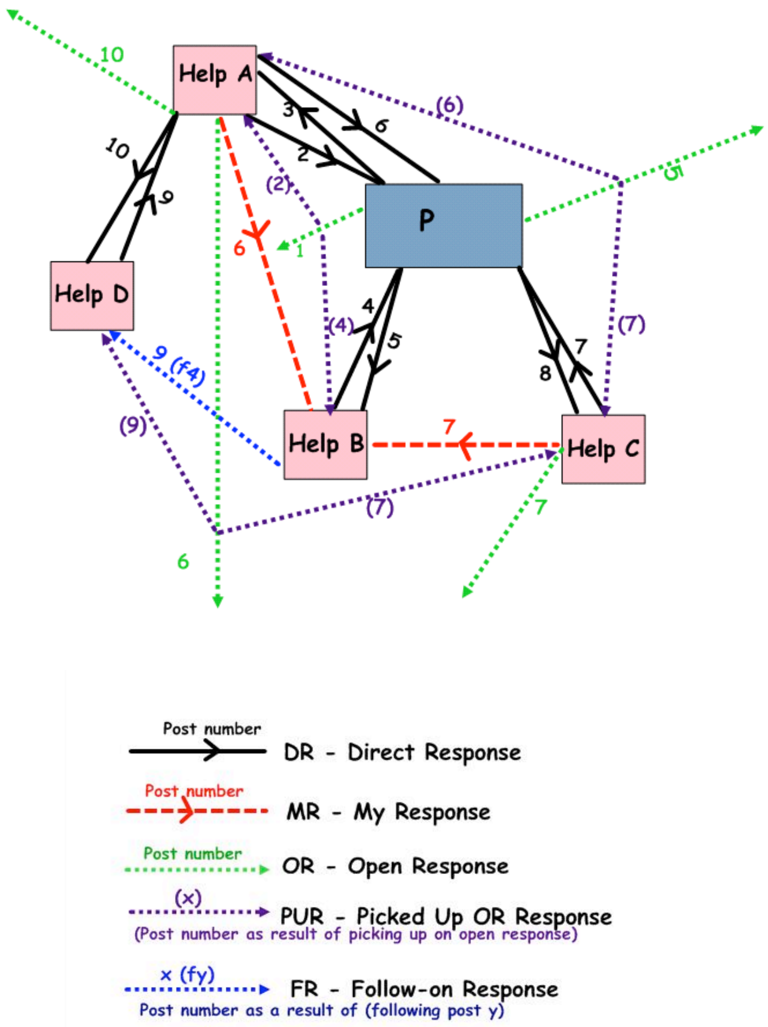

Figure 6.3 Connection Diagram for 3ThD1

Thus the form of connection diagram devised for this study differs from others found in the CMC literature. The derivation of different response types and the multi-mapping between participants, posts and response types directly addresses the deficiencies in previously reported visualisation methods discussed in Section 5.5 [p98] [see also Claim 1 Section 15.3 p307].

This section has focused on the analytical approach, techniques and methods adopted to analyse AskNRICH web-board content (Phase One). The next part of this chapter reflects on the process of Exploring and Defining the Characterisation of AskNRICH (Phase Two), the activities 'below-the-line' in Figures 6.1 & 6.5.

6.4 Exploring and Defining the Characterisation of AskNRICH

This section explains the methodology employed in the final stages of the study under two sub-sections. The first uses an illustrative framework to depict the synergy between the individual Perspectives that were combined for a wholistic view of AskNRICH. The second presents the method of iterative reviewing of the literature used to generate a characterisation of AskNRICH that was consistent with, and fully explained, the findings of the preceding Exploratory Examination.

6.4.1 Forming a Wholistic View of AskNRICH

As already stated above, it was decided that AskNRICH would be explored from multiple Perspectives and the Perspectives chosen so that the combined findings from each could be combined to form a comprehensive and coherent whole. This wholistic view would provide a basis for exploring and defining the Characterisation of AskNRICH (and the narrative to explain the 'goings-on' within it). The combination process was undertaken through an examination of the interconnections and relationships between the findings that led to their organisation into five groupings (Features Catalogues). The network of complex interrelationships between (a) the five Feature Catalogues, (b) the three different Perspectives, and (c) the Perspectives and Catalogues, is illustrated in Figure 6.4.

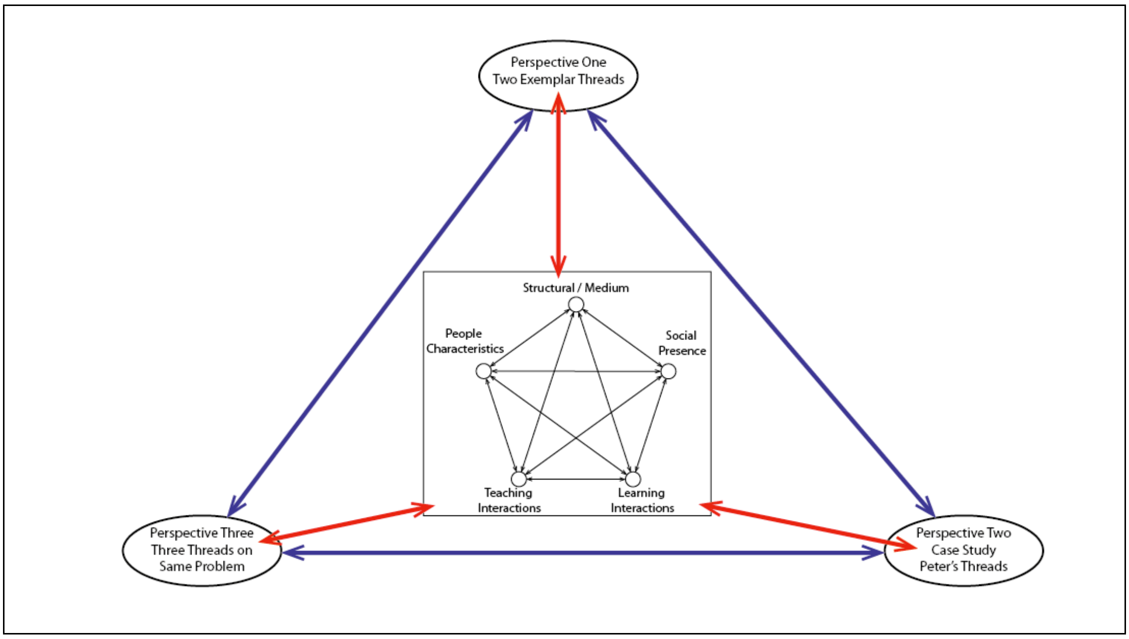

Figure 6.4 Diagrammatic Representation of Interconnections between Perspectives and Features Catalogues

In Figure 6.4 the three Perspectives are each placed at the vertices of the large triangle. The vertices of the five-sided figure of inter-connections[7] are labelled with the names of the five Features Catalogues constructed *at the end of the Exploratory Examination process* (Phase One, 'above-the-line' in Figure 6.5). In essence, both the Perspectives and Catalogues interweave, intricately bound to each other and hence blue and black double-headed arrows, respectively, are used to show this. Additionally, red doubled-headed arrows are used to indicate the inter-connections between Perspectives and Catalogues.

6.4.2 Iterative Reviewing of Literature

The need for an iterative reviewing of literature, appropriate to the open nature of this study has been argued in Section 6.2. above. The Introduction to this thesis framed the research [Section 1.5 p27] through discussing the three key areas that shaped the study, highlighting the relevant literature areas that came to be reviewed at various stages throughout the study. The Introduction also includes a framework summary diagram [see Figure 1.1 p36] that shows these literature areas within the portrayal of interconnections between research goals. However, the flow-chart in Figure 6.5 below is a general representation of how, having generated the findings from analyses during the Exploratory Examination stage of the study, the iterative reviewing of literature process was undertaken. It also indicates the inductive/deductive nature of the steps of the process and the paths followed in the case of van Lier's and Gee's theories which formed crucial elements of the final characterisation.

Thus in particular, the theoretical underpinning themes incorporated in reporting the three Perspectives [Chapters Nine to Eleven] resulted from theory seeking reviews of literature [Chapter Seven LRIII] made in passes through the first (upper) iterative Loop A in Figure 6.5. Further iterations followed the lower (left) Loop B[8] and resulted in adaption and refinement of theory [Chapter Thirteen LRIV] and a successful termination with the final characterisation [Chapter Fourteen].

This chapter concludes with a section considering issues of reliability and validity building on the discussion in Section 5.6 of the previous chapter.

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[7] Mathematically this is the complete graph K_5.

[8] Successful completion occurred in this study without following the right hand lower loop.

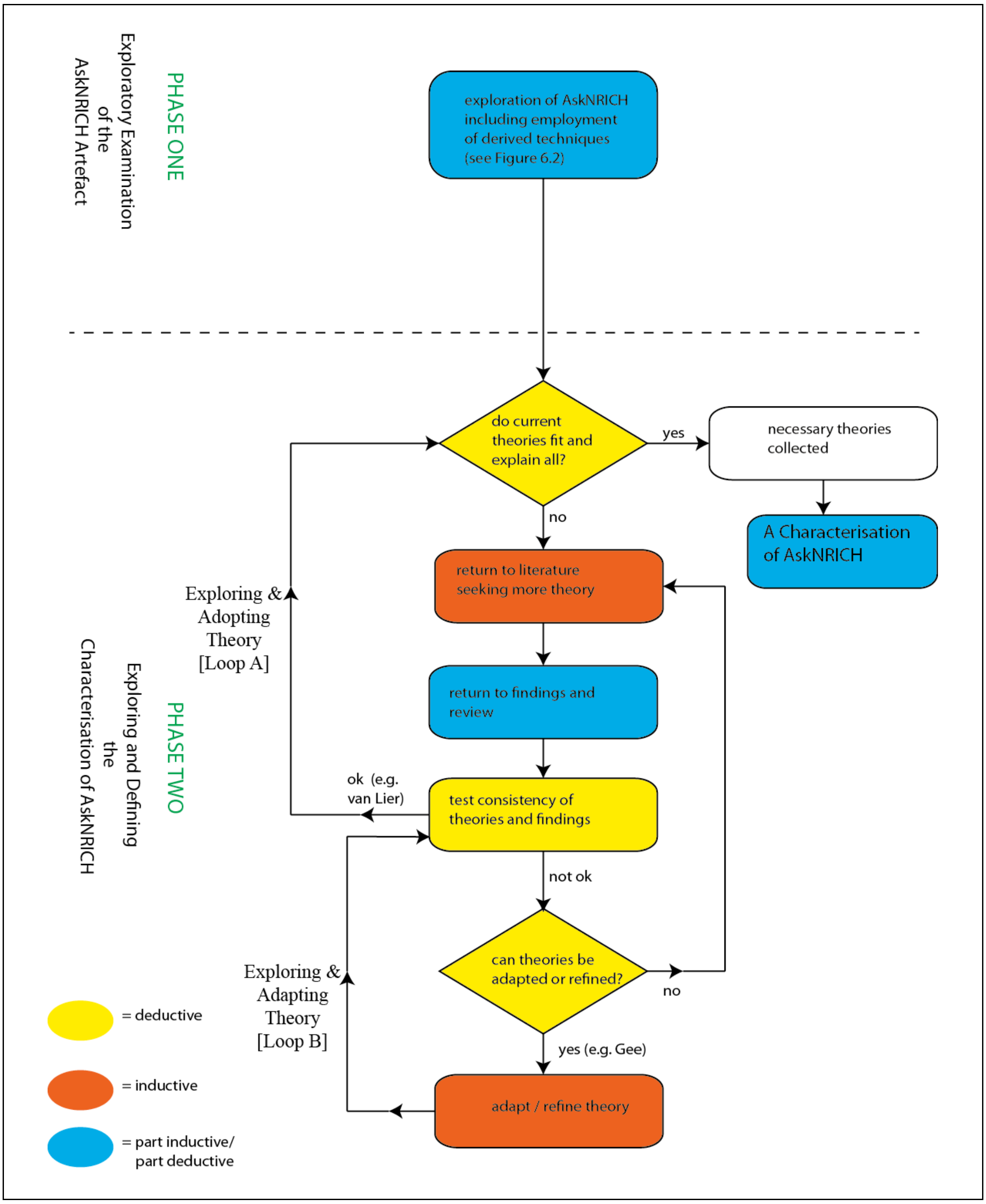

Figure 6.5 Iterative Reviewing of Literature following Exploratory Examination of the AskNRICH Artefact

6.5 Reliability and Validity

Although issues surrounding reliability and validity are always important to consider in any research, they are of increased importance here as this study has an emergent methodology rather than a frequently used methodology where the arguments have already been well rehearsed in previous qualitative studies. Section 5.6 of the previous chapter [pp99-101] briefly considered these concepts in the narrow context of the analysis of CMCs. The discussion below brings in more general research methodology literature to look at the issues related to this study more generically.

Evans [2009a] states that “reliability refers to the rigour, consistency and above all the trustworthiness of the research” [p116] and explains why it is “a precondition for validity” [Lincoln & Guba 1985: 292]. Indeed similar phrases were the starting point in Section 5.6.3 [p101] and Potter and Levine-Donnerstein [1999] exemplify the explanation in consideration of the combinations of high/low reliability/validity in relation to the different forms of content.

For qualitative work, both external reliability (a replication study in same or similar circumstances reaching the same conclusions) and internal reliability (another researcher analysing the same original data reaching the same conclusion), are operationally more problematic than when applied to quantitative work [Evans 2009a]. Evans points out that external reliabilty is “totally problematic” [p116] as it is impossible to replicate any study in the exact context as the passage of time automatically brings changes to the situation and participants. Replication in a different context or with different participants is extremely likely to introduce new factors. Internal reliability may be a ‘little less impossible’ but at best is limited to the self-confirmation of original judgements made. Thus replicability as the measure of reliability [see also Section 5.6.3 p101] in these terms is infeasible and this results in attention turning to conveying the operationalisation of the research for an ‘imagined’ replication [Evans 2009a].

However in relation to my own study, I would argue that there is the potential for a degree of external reliability. As a web-board, AskNRICH has more inherent stability, or perhaps fewer factors potentially producing instability, than the classroom situations that are the subject of the type of qualitative studies that Evans uses to illustrate his arguments. Thus there are circumstances in which another researcher could replicate this study. Similarly I would argue that in response to the need to make my emergent methodology explicit, this study also has the potential for a degree of internal reliability as someone else could re-interrogate my data as it remains unchanged on the web-board. Even if this were not true, I have stressed repeatedly that there no off-the-shelf methods and techniques I could take. I have used this chapter to explain the strategies and processes I have employed in order to be as open about them ensuring that the "process and product are intricately linked … an assessment of the latter cannot be made without also taking into account the former" [Evans 2009a: 116].

As far as the issues for validity are concerned, the general tenet of qualitative studies is that they are for example context-bound/people-specific and thus external validity, concerned with generalisability/transferability in quantitative studies is also inappropriate. Indeed qualitative studies by their nature, would epistemologically and methodologically oppose generalisability as implied by quantitative studies [ibid: 118]. To re-imagine the concept of external validity for qualitative studies, Lincoln and Guba [1985] appropriated[9] the term "thick description" [p125] arguing that by providing one it would at least allow others to see if they could take a study into another context. However unlike internal reliability that has only a modicum of feasibility, internal validity is highly applicable and indeed extremely important to qualitative studies [Evans 2009a: 118]. Taking Evans' references as a basis, internal validity concerns the extent that a study "actually investigates what is purports to investigate" [Nunan 1992: 14]; the quality of the "authenticity" or "credibility" of the findings [Miles & Huberman 1994: 278] echoing the 'soundness of study' [Section 5.6.3 p101] referred to in the previous chapter. For Evans [2009a] a key element of checking internal validity is "evidence of a persuasive connection between the conclusions made in the outcome of the study and the procedures and methodology used in collecting and analysing the data" [p118]. Although I have earlier stressed that my subject and pedagogical knowledge is of great advantage to undertaking this research, this does not by itself bring authenticity or credibility to the findings that must stand scrutiny in their own right. It is only through presenting the amount of detail that I have within this methodological chapter, i.e. a 'thick description', that I will have provided the means in which to judge the soundness of my findings and the extent to which it can be transferred to another context. Explaining issues of reliability and validity in as transparent a way as possible has been central to the reporting of the methodology of this study.

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[9] according to Evans [2009a] the term was appropriated from Geertz who had popularised it, borrowing in turn from Ryle.

6.6 Concluding Remarks

This chapter has reported on the methodological approach and methods to research and characterise the virtual world inhabited by contributors and unknown lurkers of AskNRICH, an open-access asynchronous web-board. In addition to general mindfulness of Fahy et al.'s [2001] plea to avoid methods that could be judged impressionistic and/or anecdotal [see Section 5.6 p99], there were three major methodological challenges:

- the study would be predominantly confined to only using the text of the posts in the threads
- a review of the literature revealed that few other CMC studies had reported research on web-boards of such a large size and none had reported web-boards used in a similar context; this implied AskNRICH had an unique nature
- how could the vast amount of data potentially available (six thousand threads contained within fifty thousand posts) be used[10] whilst imposing some degree of order to manage the exploration and the resulting analysis

In response to these three challenges an emergent methodological design, situated within an interpretive paradigm, was developed. In response specifically to the second and third challenges, a large proportion of the methodological approach and methods were newly devised, supplemented where possible, by adaptation or straightforward adoption of others reported in the literature. The final methodology that evolved is composed of two phases: Exploratory Examination of the Artefact and Exploration and Defining a Characterisation of AskNRICH. Both phases were constituted of multiple, interacting, iterative processes that incorporated both deductive and inductive steps.

The unique nature of AskNRICH both dictated and enabled a novel approach to the Exploratory Examination of the Artefact in which three distinct but intertwined Perspectives were employed, each using different sets of selective threads, to both span the content and analyse their content in a systematic and rigorous process. The three Perspectives were: general activities finally represented by two exemplar threads; a case study based on 151 threads involving one participant; and, thirdly, three different threads on the same

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[10] 'Fighting one's way out' seems an appropriate metaphor here.

mathematical problem posted over a four-month period. In the event the study involved using over four thousand (of the fifty thousand) posts from AskNRICH and tracking activity over a ten-month period to 'get-to-know' and 'to make sense' of the complexities. In-depth examination of threads involved studying each post to separate subject knowledge and people actions and making interpretative commentaries of both that were subsequently open coded. Analysis in the third Perspective was further enhanced by the use of a novel diagrammatic representation derived specifically for the purpose of tracking the posts, the types of responses and interactions between participants within and across threads.

The Exploration and Defining of a Characterisation of AskNRICH included the devising of a pictorial representation to capture the synergy and depict the complex interconnections in the multi-dimensional exploration of AskNRICH and illustrate how the findings from the individual Perspectives could be combined to present a wholistic picture of the activity within.

In following through a constructionist epistemological stance and interpretivist theoretical perspective into the emergent, exploratory methodology, the consequences of my long professional experience as they affect the subjectivity/objectivity of the study are discussed. Similarly, it is argued that the nature and context of AskNRICH as an artefact make this study different from qualitative studies in general in relation to the issues of its reliability and validity. The underlying principles adopted throughout this research can be usefully encapsulated by the following:

> [Postpositivists] are united in believing that human knowledge is not based on unchallengeable, rock-solid foundations – it is conjectural. We have grounds, or warrants, for asserting the beliefs, or conjectures, that we hold as scientists, often very good grounds, but these grounds are not indubitable. Our warrants for accepting these things can be withdrawn in the light of further investigation. [Phillips & Burbules 2000: 26]

The work reported in this chapter addresses in full **RQ4** of **RG2**: organising the exploration of AskNRICH. The methodological innovations reported are incorporated within Claim 1 of this thesis [see Section 15.3 p307]. The following chapter, LRIII, starts the third part of the thesis by covering a selection of education literature that anchors the reporting and discussion of the findings from the Exploratory Examination of the AskNRICH Artefact.

Part Three

Chapter Seven

Literature Review III
Interactions whilst Doing Mathematics

> *... what is said is considered independent of the individual who has said it, and treated as something which may need to be modified or augmented. Establishing an atmosphere in which people are expected to and are supported in expressing half-formed thoughts make a vital contribution to mathematical development.* *[Rowland 1995: 44-45]*

7.1 Introduction

This chapter reviews the bodies of literature listed in the next paragraph that are used to support the narrative of the reporting in the following four chapters. The selection of literature emerged during part of the iterative process of Exploring and Defining the Characterisation of AskNRICH [Exploring and Adopting Theory [Loop A] in Figure 6.5 p134]. Section 1.5.3 [p30] explained the rationale for using existing literature on classroom practice and learning to consider the learning and teaching practices within AskNRICH, despite the differences between the physical and virtual environments. Literature in each area selected is reviewed with its important concepts explained, and the significance for AskNRICH discussed.

This chapter begins with a discussion of the characterisation of, primarily teacher-led, classroom talk. It moves on to a consideration of more collaborative situations that produce conversations that are more symmetrical. This leads to a review of van Lier's [1996] work on conversation-for-education and his categorisation of four types of pedagogical interactions, each with decreasing teacher direction, and his definition of the concepts of transformative pedagogical interactions and contingency. This is followed by a consideration of the influence of learning theories within mathematics education. A Vygotskian perspective is adopted in reviewing the concepts of Socratic Dialogue, scaffolding, self-regulated learning and metacognition. The chapter concludes by examining a two-stage conceptualisation of the Zone of Proximal Development [ZPD] as single and multi-zoned.

7.2 Characterising Classroom Talk

On the basis of classroom research Sinclair and Coulthard [1975] formed the concept of the Initiation-Response-Feedback[1] [IRF] Exchange and concluded that it was predominant in interactions between teacher and pupil. There is a general consensus that it remains the dominant exchange [Kyriacou & Issitt 2008; Littleton & Howe 2010]. It has been so prevalent that Wood [1994], in criticism of it being present in the mathematics classroom, was able to simply refer to it as "the well known tripartite exchange" [p150]. Although such an IRF exchange is little more than the transmission of information [van Lier 1993: 149], Littleton and Howe [2010] argue that when the exchange is well managed it can lead to more meaningful work. Indeed, van Lier calls such exchanges IRF Questioning [van Lier 1993: 149] in order to distinguish it from Transmission, i.e. narrow IRF.

Kyriacou and Issitt's [2008] conclusion of the dominance of IRF exchanges resulted from a government-funded review of literature seeking out strategies in teacher-initiated teacher-pupil dialogue that would promote conceptual understanding in mathematics lessons in Key Stages 2 to 4. Although Smith, Hardman, Wall and Mroz's [2004] study found higher levels of pupil participation and engagement with open-questions, as Myhill's [2006] study points out, a teacher-pupil dialogue is not necessarily promoted just by the teacher asking questions. Moreover, Smith and Higgins [2006] argue that "emphasis should be less on the questions teachers ask, and more on the manner with which teachers react to pupils' responses to questions" [p485].

Wood [1994] reports on two different primary based teaching episodes that she proposes are inquiry led. The teacher is able to keep the whole class 'on-board' by maintaining the dialogue by the type of questioning. The first episode involves a joint activity between the teacher and the class, but it leads to a pre-determined solution procedure dictated by the teacher. Wood identifies this as having a *funnel* pattern of interaction [Bauersfeld 1988], described by [Tanner & Jones 2000b] as a teacher with expert knowledge who "selects the thinking strategies and controls the decision process to lead the discourse to a pre-determined conclusion" [p21]. In Wood's example [1994: 153] the questions become

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[1] Feedback has at times been replaced by Evaluation (IRE exchange) but it is suggested that as feedback encompasses evaluation, to remain with IRF is preferred [Littleton & Howe 2010: 3].

increasingly refined (simplified) so that, by the end, the pupils can correctly answer the questions without necessarily needing to be aware of the concept that they have been designed to highlight. A pupil's incorrect answer to 9+7 of 14 is 'picked up' by the teacher saying that 7+7 =14 and then, in determining what 8+7 would be, says that 8+7 is just adding one more to 14. In response to the pupil's answer of 15, the teacher continues, then 9 is one more than eight so 15 plus one more is? As Wood comments, the teacher has used the incorrect answer as a starting point to *guide* the pupil to the correct answer by using the strategy of starting from a known fact (double of 7+7) and increasing by one each time to arrive at the correct answer. However, as Wood later points out, the crucial mental activity involved belongs to the teacher, thus curtailing the possibility for the pupil and the class to engage in their own meaningful thinking [ibid: 157].

The second, contrasting episode, is described as having a *focus* pattern, where the teacher's questions "indicate to the pupil the critical features of the problem that are not yet understood" [Wood 1994: 160], "leaving the pupil to resolve perturbations which have thus been created" [Tanner & Jones 2000b: 21]. Wood's example uses a lesson where there is a whole-class discussion on the pupils' own methods of solving two-digit subtraction problems such as 66-28, where the unit digit of the second number is larger than the unit digit of the first. One pupil's correct but convoluted response, a type of response that often accompanies personal mental strategies [NCC 1989, DfEE2001; Tanner, Jones & Davies 2002] is given with some 'halting explanation' from the front of the class with the teacher moving to the back of the room. The pupil has dealt with the tens first (60-20) and then deals with putting 6 back to 40 (46), subtracting six of the eight (40) and then subtracting two more. The teacher decides to link this response with a common addition strategy that 'misses' the importance of dealing with 6-8, as addition is commutative, subtraction is not. When others realise that they cannot use the same strategy as for addition, they are "confronted with a conflict that they now need to resolve" [Wood 1994: 157]. The lesson continues with the teacher's comments and clarifying questioning *focussed on* the critical points of the pupil's solution. This interaction allows the pupil to perfect their explanation to the class, in doing so gaining a better understanding of their own solution, and, in addition, provides the opportunity for the remainder of the class to gain an understanding of the processes.

The mathematics is at a level far below AskNRICH, but the detailed account of these two episodes has been included to provide background against which to later view the way that the AskNRICHers offer support that might at times *funnel*/*focus* when in a teaching role.

In analysing the funnelling pattern, Wood makes reference to the work of Mercer who, with colleagues, has continued to research classroom discourse [Mercer & Howe 2012; Mercer & Littleton 2007], chiefly in Science and the Primary sector, in two contexts of interactions: teacher-led and peer group. Incorporating their own work of '*Exploratory Talk*' [see Mercer, Dawes, Wegerif & Sams 2004: 362], Mercer and his team provide guidance for teachers to engage in and facilitate their pupils engaging in such interactions through their '*Thinking Together Programme*' [Thinking Together nd]. The programme has two strands: one to increase pupil knowledge in the subject domain, the second to increase pupils' critical thinking skills through collaborative peer questioning. In the first, interaction is associated with the guiding role of more knowledgeable 'persons' who help others to gain greater familiarity with the particular discourse associated with the specific knowledge domain [Mercer et al. 2004: 361]. This had been precisely the goal the founders of AskNRICH intended (for the mathematics domain) and thus this strand is of particular interest when investigating the AskNRICHers' exchanges. The objective for the second strand, establishing peer dialogue is, in essence, concerned with a critical thinking type of collaboration that, as already established in Section 5.3 [pp92-93], does not match with AskNRICH. However, the key consequence of any peer dialogue is the shift that it makes towards a more equal form of discourse, which is important to AskNRICH. Such discourse presents different opportunities for participating in reasoned arguments and discussion than teacher-pupil exchanges where the teacher explicitly remains 'in charge'. Nonetheless, in focused collaborative work, the teacher remains fundamentally in charge as they have control of the agenda (curriculum) [van Lier 1996: 180].

7.3 Conversation

Contemporaneously with Mercer, research by Alexander and colleagues led to '*Dialogic Teaching*' [Alexander 2008] which "harnesses the power of talk to stimulate and extend pupils' thinking and advance their learning and understanding … attend[ing] as closely to the nature and quality of teachers' talk as to the pupils" [pp37-38]. Three talk repertoires for:

everyday life, teaching, and learning are defined and for each, different types of talk are set out. The types listed under teaching and learning talk are those Alexander suggests should be present in classroom practices. In particular, Alexander positions the discussion and dialogue types of teaching talk and all types of learning talk, for example narrating; explaining; asking different questions; exploring and evaluating ideas, as key so that "children may be empowered both in their learning now and later as adult members of society" [ibid: 39], enabled thus to participate in Alexander's third repertoire of 'everyday talk'.

The types of talk that Alexander [2008] defines are comparable to those used in Jenlink and Carr's widely cited (1996) work in what they termed *educational conversations* and which they classified into four differentiated elements: dialectic, discussion, dialogue and design. These elements can also be viewed as being implicit in the intentions of Mercer et al.'s [2004] '*Exploratory Talk*', a form listed by Alexander [2008: 39] in his repertoire of everyday talk. Schrire's [2004] CMC research, reviewed previously in Chapter Five, references Jenlink and Carr's classification in introducing the term "Learning as Conversation" [p480]. This view of learning formed the basis of the research questions addressed in her categorisation of patterns of interaction within CMCs with a similar knowledge-building collaborative purpose to that of the classroom-based work of Alexander and Mercer. However, it is van Lier's concept developed in the classroom, given the similar sounding name of "conversation-for-education" [1996: 167], that was chosen as the model for a consideration of the free-flowing exchanges within AskNRICH due to its focus on contingency of pedagogical interactions [see later].

As Mercer et al. [2004: 361] point out, talk within any peer group is generally more 'symmetrical' than talk between teacher-pupil, a premise that van Lier [1996: 180] had similarly used earlier in defining his perspective on conversation. van Lier, a specialist in English as a second language, argues that within dialogue there are varying degrees of 'un-equalness' between participants, that general conversation is a symmetrical form of talk-sharing and, consequently, that conversation-for-education encompassing such symmetry will improve the learning experience.

In an earlier publication van Lier had proposed five characteristics that form the basis of conversation: face-to-face interactions (considering computer forums a derived form); local assembly (not overly planned in advance); unpredictability of sequence and outcome; potentially equal distribution of rights and duties, and reactive and mutual contingency which he has now reduced to two [see van Lier 1996: 169]. The first of the five in essence remained, re-invented as the *means of access to the conversation*. The remaining four are regrouped to form just one characteristic that he calls *contingency*. Knowing the original five not only makes it easier to understand how van Lier uses the word contingency and by inference conversation-for-education, but is subsequently valuable in relating these concepts to conversations within AskNRICH. van Lier relates his choice of the word contingency to its two common, though contradictory, situation-dependent meanings: "*dependency*" and "*uncertainty*" [ibid: 170]. Conversation is dependent upon the utterances of others, but as these utterances are in essence free from constraints, 'anything goes' and there is thus no [real] predictability as to how the conversation will 'flow'. This "jointly managed talk" [ibid: 180] is even freer than versions of classroom talk described above, as no one person is in charge of the agenda, rather it is "shaped by all participants … contributions are self-determined or produced in response to others' requests" [ibid]. van Lier defines such exchanges as the transformation type of pedagogical interaction which has the potential to change the agenda in the 'here and now'. van Lier describes the collaborative work set by the teacher as a transaction mode of teaching with less freedom than the transformation mode but nevertheless substantially more than either type of IRF exchange.

Thus van Lier has defined four types of pedagogical interactions: Transmission, IRF Questioning, Transaction and Transformation. Each is less restricting than the previous in terms of less teacher direction and by implication involvement of peers increases. This categorisation, depicted in the next section, was found to be key to enabling the analysis of the AskNRICHers' exchanges which possess the freedom of 'doing' the mathematics that they have chosen 'to do' together.

7.4 Characterising and Categorising Pedagogical Interactions

van Lier [ibid: 179] uses a diagram (reproduced as Figure 7.1 below) to show his four distinguishing types of pedagogical interaction as four concentric circles, moving from the inner circle of Transmission outwards to Transformation. He draws six radii across the circles, without any obvious ordering, each representing a different form of interaction along a continuum line, referred to as an axis of polarity, from minimum/no interaction at the centre to maximum interaction on the outer circle's circumference. These are:

- talk that is from monologic, to dialogic, to conversational
- talk that is from asymmetrical to symmetrical
- specification from that which is product-oriented [e.g. examinations] to process-oriented [engage and grow academically]
- an 'understanding gap' that is from elliptic through to proleptic i.e. from assuming missing information can filled in by an individual through to inviting mixed competencies to share/bridge the gap [of understanding]
- teacher actions from being authoritarian, to authoritative, to exploratory
- students' actions from being externally controlled through to self-determined

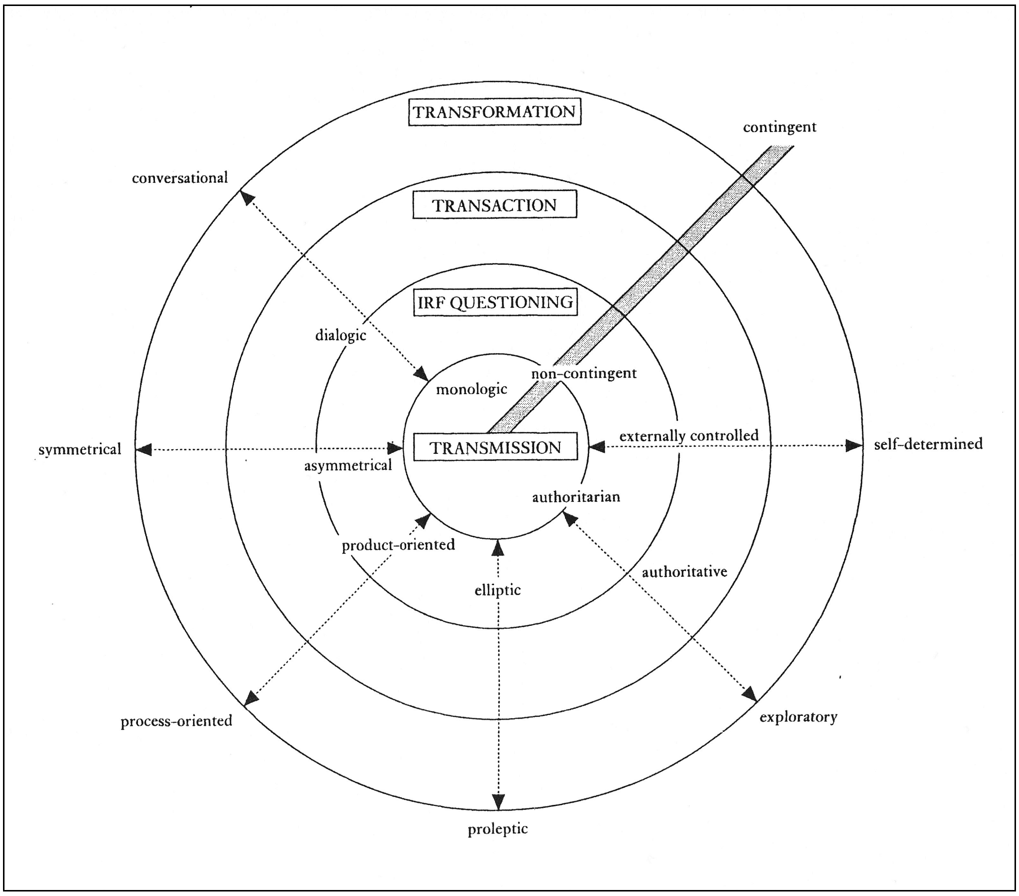

Figure 7.1 Types of Pedagogical Interaction [reproduced from van Lier 1996: 179]

A seventh axis of polarity, drawn as a shaded radial shaft that highlights its criticality, represents the phenomenon of contingency: moving from non-contingent out to contingent interaction, which van Lier describes as follows:

> As interaction moves towards the outer realms, sharedness of perspectives increases, and expectancies are effectively created and exploited. When that happens we can speak of contingent interaction, and this will mean that a number of the other polarities will be 'pulled out' outwards, opening up and enriching the discourse. [p183]

Evidence that the AskNRICHers' exchanges are substantially towards the outer circle of Figure 7.1 and thus their interactions contingent is considered through the analysis in the Three Threads Perspective of the Exploration [Chapter Eleven].

The discussions above focused on the differing degrees of 'conversational' freedom between teacher-pupil and pupil-pupil. In so doing, it moved away from the actual teaching and learning that is taking place. This review now returns to consider theories in relation to the teaching and learning process.

7.5 Frameworks for Learning

The first part of this section presents an overview of the debate during the late 1990's between proponents of the Piagetian and Vygotskian perspectives. This is followed by a brief justification of the adoption of a Vygotskian perspective for this study. In the subsequent sub-sections three key concepts for the analysis within the exploration of AskNRICH are presented and their relation to it discussed. However, this section begins with a return to further examination of Wood's [1994] paper to provide illustrative examples giving context to the presentation of the debate.

In reporting and considering two classroom episodes [see Section 7.2] Wood was opening up and contributing to the increasing debate on socio-cultural aspects of mathematics education. Learning was beginning to be perceived as a much more social activity than had hitherto been the case [Greeno et al. 1996]. Wood [1994: 160] assigns the *funnel* and *focus* patterns of interaction to Vygotskian and Piagetian theory respectively. This assignment is made from consideration of the teachers' *intentions* for the lessons. However, it is difficult to relate the example narratives of how the lessons *actually unfold* to the respective theories, a point returned to in the presentation of Howe's arguments later. Thus, although Wood states that the teacher of the *'focus'* lesson had engineered the opportunity for pupils to experience cognitive conflict, one key element of Piagetian Theory [Swan 2001], the help provided also appears to resemble *scaffolding*, a strategy [also discussed later] developed from and typically associated with Vygotsky's theories [van de Pol, Volman & Beishuizen 2010]. So, for example, the responsive guided help visible in the narrative of the *'focus'* lesson appears to better exemplify scaffolding than that in the '*funnel'* one intended for that purpose. Indeed, considering how the teacher actually managed the questioning, the *'focus'* episode appears little different from the secondary mathematics classroom lessons, described by Goos, Galbraith and Renshaw [1999], also discussed later, that were based explicitly on Vygotsky's theories.

The consideration of these examples illustrates Orton's comment that it is (potentially) difficult to ascribe one universally accepted learning theory to mathematics [Orton 2004: p175]. Greeno et al. [1996] contains an extensive review from a psychological viewpoint of theories extant in Europe and the United States in the early 1990s, separated into the three perspectives of behaviourist, cognitive (more ascribed to Piaget) and situative (more ascribed to followers of Vygotsky). In their conclusions they propose two ways in which some sort of a synthesis of the three might be achieved. Element of the continuing sharp debate between proponents of the cognitive and situative perspectives in a series of papers is captured here by Anderson, Greeno, Reder and Simon [2000] responding to Cobb and Bowers [1999]. The former, from a cognitive/psychological stance, puts forward an identification of common ground and calls for further evaluative research towards synthesis. The latter from a classroom-based/situative viewpoint continues to emphasise differences in its conclusions.

Concurrently, Sfard [1998] had ascribed two metaphors, acquisition and participation, to the cognitive and situative perspectives respectively. She details the relative worth of each one and brings her arguments to a powerful conclusion by pointing out the dangers of choosing just one rather than an amalgam in scenarios where one of the two might predominate but nevertheless the other has some influence. It is clearly not possible to encapsulate such a major debate in a few lines. However, I have intentionally made a passing reference to it

since I consider it important to mention Sfard's two metaphors in context because of their relevance to the AskNRICHers' *participation* on the web-board as a means of *acquiring* individual mathematical knowledge. On this basis, the way that the mathematics is undertaken within AskNRICH would fit a social constructivist framework
[see Ernest [1999] for a clear exposition of such a framework in mathematics education and Burton [1999] for examples of working within this framework].

More recently, Littleton and Howe [2010] describe how learning has been theorised by Piaget and Vygotsky and in the further developments by disciples of each:

> From a Piagetian perspective, the germs of intellectual progress are to be found in the socially motivated resolution of conflicting perspectives arising through discussion between peers. In contrast, it is the *guided* construction of knowledge that is underscored in Vygotskian and neo-Vygotskian accounts. [ibid: 5]

The difference, succinctly explained here, is clearly evident in Wood's [1994] assignment of focussing and funnelling to the teachers' intentions for the two lessons. Indeed, Littleton and Howe [2010: 9] go on to make the crucially important observation that a classroom of some thirty pupils is not the setting in which these theories of development and learning (in individuals) were originally established. Nevertheless as Greeno et al.'s [1996] review indicates elements of either theorists' work can be used as a framework for classroom peer collaboration and the assignment of one perspective to each of the two examples in Wood's paper illustrates this. AskNRICH in effect straddles the individual and the classroom situations and thus it is even less straightforward in this context to prioritise one perspective over the other.

In her own chapter Howe [2010] argues that the emphasis Piaget placed on child development through engaging in dialogue between (same aged) peers, contrasts with Vygotsky's proposal of more capable peers (including adults). For Howe, the differences lead to either a focus on the *dialogue* of the exchanges (Piaget) or the *outcome* of the exchanges (Vygotsky) [p33]. Although Howe theoretically makes the distinction clear, in practice, as with Wood's *'focus'* lesson, deciding who has guided/helped to initiate the peer dialogue is less clear. Furthermore, once more capable peers' help has been provided, opportunities can then exist for further work developed through equal peer dialogue.

Indeed within the exploration of AskNRICH, both the message text (dialogue) and also what results from the text (outcomes) are examined. Moreover evidence concerning cognitive conflict within AskNRICH is far from clear-cut. A query might arrive because an individual has already [somehow] experienced a cognitive conflict that they wish to resolve. Alternatively a cognitive conflict might be set up but it would be only one of many means through which an AskNRICHer offers (guided) help. Ultimately the situation of AskNRICH is that (the virtual) participation is in a cultural and social setting, and the posting protocols ensure that an individual is guided to understand (their) mathematics through constructing their own meaning with the help of their more capable peers. As such, this is more in keeping with the Vygotskian view.

The first type of interaction in Mercer et al.'s [2004] work on [collaborative] '*Exploratory Talk*' described earlier is similarly predicated on the guiding role of more knowledgeable others, effectively teachers [p361]. A range of concepts is ascribed to this situation: *scaffolding*, a metaphor for the support given during the learning process [see Wood[2], Bruner & Ross 1976 and Wood & Wood 1996]; *guided construction of knowledge* the process by which [one] person helps another to gain knowledge and understanding [Mercer 1995]; and in more collaborative situations *dialogic teaching*, using talk to deliberately stimulate and extend pupil's thinking [Alexander 2004] and *guided participation*, process by which children actively acquire skills through participating in meaningful activities with adults [Rogoff 1990].

Brown and Palincsar's [1989: 393] concept of *guided, cooperative learning* is similarly predicated on the guiding role of more knowledgeable others, but is described as specifically located in a cooperative learning environment and thus, it may be inferred, encompasses both teacher-led and peer group interactions. The concept is embodied in a procedure they label '*Reciprocal Teaching*', giving an example of its use in introducing group discussion techniques aimed at understanding and remembering text content [ibid: 413]: "the teacher guides the process, sometimes models effective interpretation strategies and systematically hands over cognitive management of the activity to the students." [Resnick 1989: 22]. The approach in this description seems to closely resemble scaffolding [Wood et al. 1976].

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[2] Not to be confused with T. Wood whose paper has just been used in Section 7.2 and earlier here.

Guided, cooperative learning is also apparently similar to Mercer's collaborative '*Exploratory Talk*' in the promotion of discussion techniques, although Mercer's work relates to knowledge-building in science problem-solving. Brown and Palincsar [1989] explicitly state that they have adopted a Vygotskian perspective and that their work is underpinned by "three theories of guided learning that share a family resemblance: ZPD … , expert scaffolding … and Socratic dialogue" [p409].

Chapter One signalled that the Posting Protocols were designed to engender a particular type of discursive talk rooted in *Inquiry/Socratic dialogue* [Collins & Stevens 1982]. Furthermore, the Posting Protocols' entreaty to avoid giving the solution in offering help and in sharing current progress and thoughts when seeking help will promote a scaffolding approach and foster the development of metacognitive skills. Thus the following sub-sections explain and discuss, in relation to AskNRICH, the topics of Socratic dialogue, scaffolding, self-regulated learning and metacognition and finally, the concept behind all of these: the ZPD.

7.5.1 Socratic Dialogue

In writing about Socratic Dialogue, Brown and Palincsar [1989] draw directly on Collins and Stevens' [1982] paper on analysing the problem-solving principles and strategies of teachers with "diverse teaching goals and strategies … [but using] some version of the case, inquiry, Socratic or discovery method of teaching" [p65]. Brown and Palincsar do not explicitly define 'Socratic dialogue' but state that it is a "classic example of expert-guided group discussion" [Brown & Palincsar 1989: 411] and describe what a Socratic Teacher would do: "A Socratic Teacher employs a range of standard questioning activities to force the students to elaborate, justify and provide warrants and backing for their statements" [ibid], all requirements of critical inquiry. In so doing the teachers would "routinely use five main discussion ploys" [ibid], mimicking the way that Socrates is reputed to have questioned his subjects. In other words the teachers are having a dialogue with the students using questioning based on the Socratic method. Although in their work Brown and Palinscar use the term Socratic Dialogue, Collins and Stevens [1982: 94] originally used the term '*inquiry dialogue*' within their theory generated from six source dialogues, five from teacher observation and only one from Plato's writings on "Socrates with slave boy Meno dialogue” [p70]. Collins and Stevens suggest intelligent computer-assisted instruction systems as potential application of their theory [p97]. Self [1990] in writing on the construction of Intelligent Tutoring Systems and having worked through one of their six dialogues comments that “The process of Socratic tutoring is formalised in terms of the drawing out of a student's implicit beliefs (or "thoughts which need to be awakened into knowledge", as Socrates put it)” [p6].

Thus the views of Socratic Dialogue above portrayed it as questioning that forces students to make their own current position explicit and enables the students to question themselves. What is relevant for AskNRICH from these descriptions is the requirement that the teacher (or the AskNRICHer in the helping role) ascertains where the learner is and builds from there. However, in contrast to Brown and Palincsar’s [1989] Socratic teachers, the AskNRICHers are unlikely to have a *conscious agenda* of employing discussion ploys such as for example “in taking up errors before omissions [and] easy misconceptions before fundamentally wrong thinking … model[ling] modes of scientific thought, thereby teaching students how to think …” [ibid: 412]. Nonetheless, these are key strategies that the AskNRICHer might engage in. In other words the AskNRICHer in the teaching role does not have an agenda as a classroom teacher would have, the point also made by van Lier [see end of Section 7.2 above]. Also relevant to AskNRICH is the onus on the learner to fully participate/engage in replying/responding to the help offered in a way that allows “feedback that is tailored to the [learner’s] existing levels” [ibid: 417]. In AskNRICH it is the equal two-way dialogue between the helper and learner to gain common/shared understanding of the learning situation and move forward together that makes it acceptable to label the exchanges a Socratic-*Style* of Dialogue.

As will become clear in following section, it is also an inextricable part of a scaffolding approach for the teacher to ascertain where a learner is and build from there, possibly the viewpoint underlying Tanner and Jones [2000b] statement “one person’s scaffolding being another’s Socratic questioning” [p20].

7.5.2 Scaffolding

The concept of scaffolding was introduced by Wood et al. [1976], derived from Vygotskian theories [van de Pol et al. 2010] and the name was deliberately selected to indicate a "temporary support for a completion of a task that learners otherwise might be able to complete" at that time [ibid p272]. In the intervening years, the true meaning of the term has become lost and it is often erroneously used, without care, for any support given [Puntambeckar & Hübscher 2005] rather than when support is offered, then faded. True scaffolding is thus support adjusted/reduced as learning takes place until it is finally removed at the stage when the learner can stand alone [Wood et al. 1976].

Through their extensive review, van de Pol et al. [2010] have developed a framework, part of which derived from Tharp and Gallimore's [1988] earlier classification of means of 'assisting performance', to analyse the scaffolding process. van de Pol et al. [2010] list these components as: Feeding Back; Hints; Instructing; Explaining; Modelling and Questioning. Anghileri [2006], using examples from her previous mathematics education research, and starting with the seminal sources of Wood et al. and Tharp and Gallimore, attempted "to identify a hierarchy of [scaffolding] interactions which relate to teaching practices that can enhance mathematics learning" [p33]. Just as van de Pol et al. [2010] view the interactive process between teacher and student as crucial, so Anghileri picks up on Tharp and Gallimore's [1988: 42] "questioning – calling for linguistic response [and] cognitive structuring – providing explanations and belief structures that organise and justify", as the two strategies that are more suggestive of the interactions that typify good classroom exchanges [Anghileri 2006: 35]. To exemplify such interactions Anghileri draws heavily on Wood's [1994] paper, used extensively above [in Section 7.2], detailing the *funnel* and *focus* patterns of interaction. Wood [1994] is also used by Tanner and Jones [2000b] in reporting their classification of teachers by assigning scaffolding modes according to teaching styles observed. In addition to reaffirming the erosion of rigour in the use of the term scaffolding "an ill defined construct in the literature" [ibid: 20], the authors add the comment on the difficulty of differentiating it from Socratic questioning cited earlier. Moreover, Bliss, Askew and Macrae's [1996] study in three primary schools across three subjects including mathematics derived five categories of scaffolds: actual (actions maintaining a scaffolding presence); prop (offering suggestion to help); localised (specific help to an individual);

step-by-step or footholds and hints & slots. The authors suggest that the last two are "really like cueing" [p47], relating this to 'funnelling': *step-by-step* a series of questions and *slots* a narrowing down to only one answer. This carries the implication that the hinting strategies might not be considered 'true scaffolding'. However Anghileri's [2006: 39] three level hierarchy of scaffolding has both 'funnel' and 'focus' at Level 2, the former represented by "showing, telling, or explaining", i.e. *prompting questions*, and the latter by "reviewing and restructuring", considering prior experience to consider pertinent aspects and adapting help to closer understanding, i.e. *probing questions*.

Kyriacou and Issitt [2008], cited earlier [Section 7.2], discuss the use of "slow-down strategies" [p10] in engaging pupils in dialogue beyond IRF. Furthermore, Tanner and Jones [2000b] refer to their earlier work in which a "Start-Stop-Go" [p22] strategy was observed. Both these strategies are providing learners with time for reflection. The two highest (of four) classifications of teachers used by Tanner and Jones [ibid], both employing the start-stop-go strategy, were labelled dynamic and reflective scaffolders. The distinction was that the reflective scaffolders afforded their pupils the opportunities to reflect in a manner that could develop their metacognitive skills [see below]; "reflection-*on*-action" [Schön 1991: 1], rather than remaining at the "reflection-*in*-action" [Schön 1983: 49] level.

7.5.3 Self-Regulated Learning and Metacognition

The discussion on scaffolding above implies that if there is a scaffolding need then the learner is unable to move beyond the point where they can do something on their own and are at a stage where they can only access a further range of knowledge and skills through someone's assistance. van Lier [1996] refers to actions that a person can do confidently on their own as *self-regulated* [see also Section 7.6 below]. Implicit in the can/cannot description above of a learner is their awareness of their current state. That state is viewed as composed of a series of components: "cognitive, affective, motivational and behaviourial" [Zeidner, Boekaerts & Pintrich 2000: 751]. Each of these components in turn has their own influence on the learning process. Indeed, Zimmerman [2000] has described the self-regulation of learning as a cycle involving forethought, performance and self-reflection.

Metacognition, “the capacity to reflect upon one’s own thinking and thereby to monitor and manage it” [Greeno et al. 1996: p19] is, through its definition, a constituent part of self-regulated learning, directly relating to the first and last of Zeidner et al.’s [2000] four components above. In early work on metacognition Flavell [1971] made the distinction between metacognitive knowledge and metacognitive experience. More recently, Tanner and Jones [2000b] present a ‘loose’ summary of the twin aspects of metacognition: passive, metacognitive knowledge (knowing what you know) and actively working with metacognitive skills (knowing what to improve). This clearly and succinctly captures present meanings that have been derived from Hacker’s [1998] two central components: knowledge about one’s cognitive processes, and the monitoring and regulation of these processes.

In the preface to a special issue ‘*Self-regulated learning in a digital world*’ Steffans and Underwood [2008] point out that, in the new digital age, the term self-regulated learning is now being used to mean that students are regulating their learning if they choose what, when and where to learn [p169] which is indeed true of the AskNRICHers. Simply participating in AskNRICH implies an inclination to self-regulation as just defined, and is likely to indicate a predisposition to metacognition.

In summary, Socratic-style Dialogue as defined in Section 7.5.1 above provides one opportunity for scaffolding to occur. For this study scaffolding should be seen in terms of helpers scaffolding the learner’s learning, with the aim that, once the problem has been completed, the learner is in a position, the next time such a problem arises, to undertake the work with less or no help. The level of success in the engagement with these two processes and the scaffolded achievement is in part due to the metacognitive capabilities of those involved.

Vygotsky’s theory of the Zone of Proximal Development [ZPD] led directly to Wood et al.’s [1976] development of scaffolding. In addition, each AskNRICHer is developing their mathematical knowledge through participating with their peers but is left to make sense of it by internalising it individually. Zuckerman [2003] captures this situation precisely with her descriptive phrase: “Independent, Yet Not Alone” [p186] before quoting[3] Vygotsky’s [1978] proposition:

> Every function in the child’s cultural development appears twice: first, on the social level, and later, on the individual level: first, *between* people (*interpsychological*), and then *inside* the child (*intrapsychological*). [Vygotsky 1978: 57]

In going beyond his ‘basic theory’ to examine its ‘educational implications’, Vygotsky proposed ‘a new approach’ to the interaction between learning and development and in so doing created the key concept of a ZPD. The concepts of Socratic Dialogue/Questioning and Scaffolding have been developed out of this conception and thus the ZPD is the subject of the next section.

7.6 The Zone of Proximal Development

Vygotsky, in translation, defined the ZPD as

> … the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers. [Vygotsky 1978: 86]

Although this concept, as just defined, is widely cited and the ideas associated with it well-known, it has been conceptualised in a variety of ways, some of which contain assumptions that were never intended by Vygotsky’s original proposal [Chaiklin 2003: 41]. My own conceptualisation of Vygotsky’s ZPD has been shaped by considering the consequences of the AskNRICHers conversational exchanges that support mathematical development. This section begins with Chaiklin’s [ibid] listing of three misconceptions about the ZPD found in the literature as reference will be made to these where appropriate in the later discussion. I then return to the work of van Lier [1996] who developed his arguments further on contingency by making his own interpretation of the ZPD. This interpretation has been instrumental in my own conceptualisation of the ZPD as it relates to AskNRICH. The section continues with a further brief reference to the three year long mathematical study of Goos et al., [2002], which is similar to van Lier’s work in that it also explores the notion of a peer *collaborative* ZPD. This view of interactions within the ZPD has more in common with the situation in AskNRICH than the more traditional situation of

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[3] there is a slight discrepancy between Zuckerman’s full quotation and Vygotsky’s translated text.

an adult helper working with pupil(s). However, there is, as with van Lier's work, still a teacher present. It is difficult to be precise about the effect of this in comparisons with AskNRICH, but the work of Goos et al. and van Lier provided a pragmatic starting point. A brief discussion follows, using primarily mathematics sources, on a variety of appropriations of the ZPD that are renamed as Zone of X. The section concludes by using van Lier's dynamism within a ZPD to finally encapsulate my conceptualisation of a ZPD.

7.6.1 Misconceptions and Assumptions

Chaiklin notes that although the term ZPD had first appeared in a translated publication in 1962 it was a chapter in the 1978 publication quoted above that made it commonplace. van Lier [1996] references both publications whilst Goos et al.'s papers [1999, 2002] rely heavily on secondary sources and the 1978 publication. Chaiklin's 2003 publication not only post-dates these but also references Vygotsky from 32 different sources, in at least six languages, including seven in the original Russian, all of which implies considerable scholarly effort on the interpretation of Vygotsky's work. Nevertheless it is my view that whilst recognising that the misconceptions that Chaiklin lists may well be prevalent in the literature and are important to be aware of, I consider that they do not figure detrimentally in either van Lier's or Goos et al.'s work, as is argued in the discussions below.

The three misconceptions (or *common interpretations*) listed by Chaiklin [2003: 41-43] are:

- the *generality assumption* – a focus on tasks and skills, not the development of the individual
- the *assistance assumption* – an implication that the learning is dependent on adult helpers rather than [see Howe 2010 earlier [Section 7.5 p149]] understanding the meaning of the interaction that is taking place
- the *potential assumption* – an implication that the potential for a person's development is the property of the individual rather than the presence of certain maturing functions

Chaiklin [2003: 49] also explains that Vygotsky defined the ZPD to be used for two different purposes: one *objective*, i.e. not referring to any specific individual, and the other *subjective*. The *objective* purpose is to identify the kinds of maturing psychological functions and social interactions associated with them (in general). The *subjective* purpose is to identify the individual's current state in relation to developing these functions needed for the transition from one stage of development to the next.

7.6.2 Conceptualising the Zone of Proximal Development: Stage One

The work that van Lier presents on contingent interactions and educational conversations is followed by his detailed perspective on Vygotsky's ZPD [van Lier 1996: 190-196]. He begins this by presenting two simple diagrams, reproduced within this section.

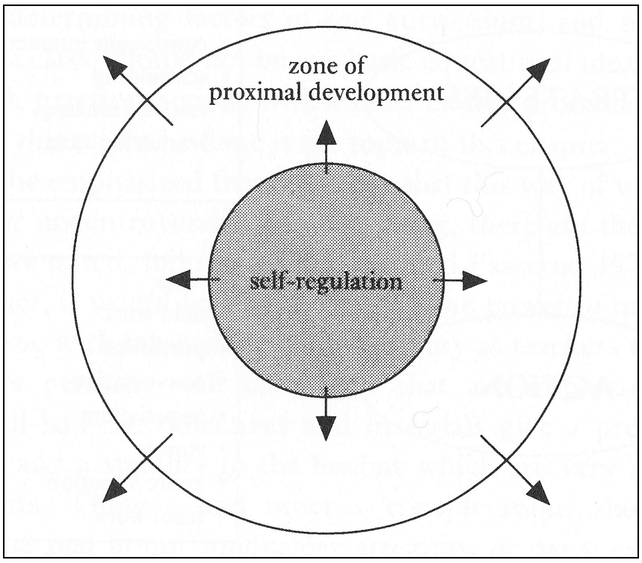

Figure 7.2 Zone of Proximal Development [reproduced from van Lier 1996: 190]

Figure 7.2 [reproduced from p190] consists of two concentric circles. The inner one is labelled self-regulation, indicating the work that an individual can confidently do unaided [see Section 7.5.3]. The second, outer circle, the ZPD, indicates the 'extra space' containing a range of knowledge and skills, the material that is within that same individual's grasp, but that they can only currently access with the help of someone else. There is further space beyond both circles that is beyond current reach and not (yet) available for learning [pp191-192].

Although van Lier has mentioned knowledge and skills, a potential trap into the *generality assumption*, he avoids the misconception by going on to stress Vygotsky's assertion that "any learning is, of necessity, in advance of development" [p191]. The label of self-regulation within the first circle and the second of van Lier's three curriculum principles, Autonomy, [van Lier 1996: 13] might be, in part, connected to the *potential assumption*. However, an emphasis on maturing developmental processes that have both an *objective* and *subjective* state is implicit in his explicit use of the label self-regulation.

Moreover, van Lier goes on to present a more detailed analysis. The additional detail leads to a second pictorial representation of the ZPD. Figure 7.3 [reproduced from p194] is a redrawing of the two concentric circle diagram with the outer ring (the ZPD space) neatly quartered, for presentational purposes, to illustrate four different resources available within what then in essence becomes multiple zones of the ZPD.

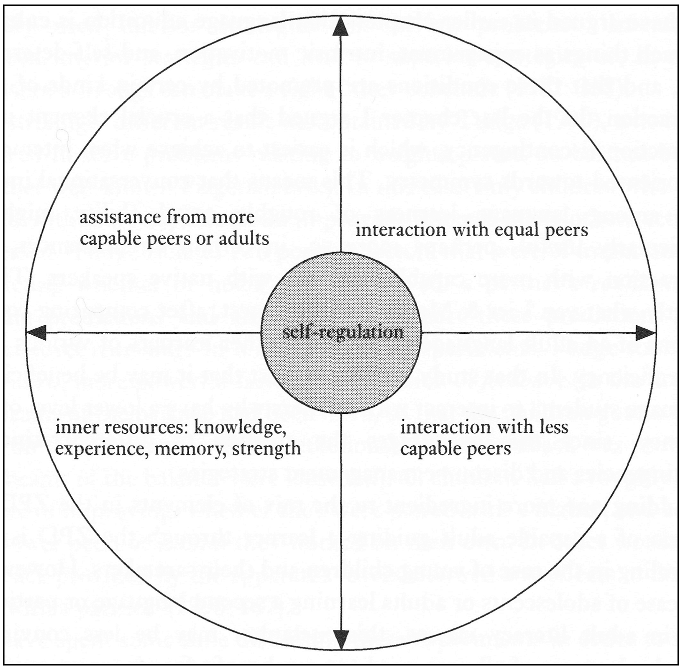

Figure 7.3 Multiple Zones of Proximal Development [reproduced from van Lier 1996: 194]

The first three concern different means where all involved (helpers and learners) can develop: assistance from more capable peers and adults; interaction with equal peers; and interaction with less capable peers (in the sense of learning more through teaching others). The detailing of this is extremely important to AskNRICH where all three are naturally present amongst the mixed-experienced AskNRICHers. The fourth zone represents inner resources of knowledge, experience, memory and strength and is shown distinct from self-regulation that remains at the core as maturing functions. Again, these inner resources could be seen in the exploration of AskNRICH along with a situation that enables increasing maturity, both objective and subjective development as will be reported in later chapters. In these later chapters van Lier's model of multiple zones will be referred to as a four-part ZPD model.

In discussing these multiple zones of proximal development van Lier firstly emphasises the importance of considering the individuality of the subject and the task and thus avoids the *generality assumption*. Secondly, he stresses the *ability* of a more knowledgeable person (teacher or peer) *interacting* to succeed in providing the support and in doing so van Lier avoids the *assistance assumption*.

Moreover, taken together with his work on contingency, the labels van Lier gives to at least three of the divisions of the ZPD seem to imply a *collaborative* model. However, as Chaiklin [2003: 54] points out, Vygotsky reserved use of the term collaboration to refer to assessing development through joint working together on a problem rather than working with a more knowledgeable person providing help by, for example, what would now be called scaffolding. Indeed, in his conclusion, Chaiklin advises that the term ZPD should be used to refer to the "phenomenon that Vygotsky was writing about and find other terms e.g. assisted instruction and scaffolding to refer to specific (subject) practices and skills" [ibid: 59]. As will be seen in the two sections that follow, whilst Goos et al. [1999, 2002] use the term 'ZPD *as* Scaffolding', and van Lier [1996] refers to 'ZPD *and* Scaffolding', it is difficult to determine the degree to which either is adhering strictly to Vygotsky's definitions. The discussion of all of these terms is necessary since in this thesis, as the previous section explained, the scaffolding that the AskNRICHers offer each other plays as important a role as considering the individuals' development within their ZPD.

7.6.3 Variations on the Zone of Proximal Development Theme

Goos et al. [1999, 2002] had been involved a three-year study in senior secondary school mathematics classrooms. One part of this project had been to explore the notion of a collaborative ZPD, "identifying mechanisms of peer interactions that mediate collaborative metacognitive activity in problem solving tasks" [Goos et al. 2002: 198]. Thus Goos and her team, studying the acquisition of metacognitive skills, though peer-pair interactions and analysing conversational moves, had chosen to do so as: "teaching and learning in the ZPD entails moving students past their present capabilities towards new forms of reasoning and action" [ibid 218], therefore avoiding the *potential assumption* by focussing on maturing processes. In an earlier article Goos et al. [1999] set their introductory discussion [pp39-43] within four contexts, two that are relevant here: 'ZPD as Scaffolding', see discussion above and 'ZPD in Egalitarian Partnerships' both avoid the *assistance assumption* by focussing on interactions.

Others have also explored variations on the three words of the ZPD acronym to deliberately maintain an association with Vygotsky's ideas. The Realm of Developmental Possibilities was the construct derived by Cobb [1995: 29], a mathematics educator, in his desire to shift the focus away from the adult's role of helper to concentrate on the individual's interpretations and interactive contributions. This redefinition with its focus on interactions clearly avoids the *assistance assumption*. Newman, Griffin and Cole [1989] refer to the ZPD as a Zone of Construction, similar to experiencing 'cognitive conflict' in a Piagetian sense, though within a strong emphasis on the socially mediated interactions that would bring it about.

Mason, Drury and Bills [2007] make reference to eight different 'zones of …' in a paper where they share their deliberations as to why teachers and students are resistant to expressing generalities in [school] algebra. With reference to Vygotsky's notion that "teaching converts ability to do something into ability to do something knowingly" [p43] they propose a Zone of Proximal Generalisation as a particular case of a Zone of Proximal Awareness.

> The idea was to use the term to describe awarenesses which are imminent or available to learners, but which might not come to their attention or consciousness without specific interactions with mathematical tasks, cultural tools, colleagues, teacher or some combination of these.
>
> [Mason et al. 2007: 53]

Mason et al. again argue that this conception places the emphasis back on potential *developmen*t, rather than the misconceived attribution to "behavioural aspects of the human pysche" [ibid].

The authors discuss potential barriers that can inhibit pupils recognising, or remembering, generalities in the classroom. In part, they refer to Rowland 's mathematics educational research on linguistic hedges [1995] observing how learners were very tentative or 'hedging' in putting forward their thoughts in case of being wrong or thought silly. This had led Rowland to recommend the creation of a Zone of Conjectural Neutrality, which is described in the quotation at the start of this chapter. The description could be considered as a near-perfect match to the intentions of the Posting Protocols of AskNRICH [set out in Section 8.3.3 p170].

This section on the ZPD now concludes with the second stage of the conceptualisation that formed my view of the AskNRICHers' ZPD.

7.6.4 Conceptualising the Zone of Proximal Development: Stage Two

As mentioned earlier my own conceptualisation for AskNRICH evolved from van Lier's characterising "the dynamism of working within the ZPD" [1996: 195]. He does this under a sub-heading: '*The ZPD and scaffolding*' [see above]. van Lier provides detail in describing Bruner's notion of scaffolding [ibid: 94] and (re)-interpreting the idea in terms of his own view of the ZPD given above. He lists in general terms, six features or principles that enable and characterise the dynamic nature of working:

- continuity – repeated occurrences
- contextual support – a safe but somewhat challenging environment
- intersubjectivity – mutual engagement and attention
- contingency – activities subject to change dependent on participants' actions and reactions
- handover – as soon as the learner shows signs of readiness
- flow – actions jointly orchestrated or synchronised for subsequent natural actions

These six dynamic features together with van de Pol et al.'s [2010] six means detailed earlier [p153] may be seen as presenting an entire picture of scaffolding by intentions and by type of help. However van Lier is careful to stress that Bruner based his ideas on the work he had undertaken working with mothers and young children [Bruner 1983]. As a consequence he [van Lier] suggests that without further study it cannot be assumed that items in the list describing the dynamism of working within the ZPD will be present with older age-groups. Nonetheless, given the nature of AskNRICH, van Lier's listing presents a sensible framework within which to consider the actions and activities undertaken: the AskNRICHers' work together doing mathematics and the outcomes of that work are dependent on each individual and the role they take. I have constantly stressed that AskNRICH is not a classroom, teacher-led environment, though Vygotsky neither specified study situations [Littleton & Howe 2010: 9] nor, I assume, could he have imagined the current virtual worlds that have become commonplace.

In conclusion, my conceptualisation of the ZPD relates the knowledge and skills associated with the (mathematical) study to an individual's (mathematical) development. This development is demonstrated through teaching, learning and social aspects of free and contingent interactions between mixed-experience contributors within the ZPD. The contributions of an individual that are observed, during dynamic engagement with the subject, are analysed to assess developing maturity both in the subject and personally.

7.7 Concluding Remarks

In this chapter, as part of the inductive/deductive methodology of Exploring and Defining the Characterisation of AskNRICH [see Figure 6.5 p134], the literature has been reviewed and critiqued to establish positions, embedded within a Vygotskian standpoint, on: talk, leading to conversation-for-education; transformative pedagogical interactions evoking the maximum level of contingency; Socratic Dialogue, the nature of scaffolding, self-regulation and metacognition, and a four part ZPD. All of these provide a theoretical underpinning for the reporting of the analysis of findings obtained from the Exploratory Examination of the AskNRICH Artefact and later for Exploring and Adopting Theory within the Exploring and Defining the Characterisation of AskNRICH. However, for simplicity and clarity in reporting, each of the three chapters focuses on one Perspective and selected underpinning(s) as illustrated in Figure 7.4 below.

<table>
<tr><td>Chapter</td><td>Nine</td><td>Ten</td><td>Eleven</td></tr>
<tr><td>Perspective</td><td>One</td><td>Two</td><td>Three</td></tr>
<tr><td>Title/Content</td><td>Exemplar Threads</td><td>Case Study on Peter</td><td>Three threads
on same problem</td></tr>
<tr><td rowspan="3"></td><td colspan="3">Theoretical Underpinning</td></tr>
<tr><td>Socratic Dialogue
Scaffolding</td><td>Four-Part ZPD</td><td>Conversation-for-Education</td></tr>
<tr><td colspan="3">Pedagogical Interactions:
Transmission IRF Transition Transformation</td></tr>
</table>

Figure 7.4 Reporting in Chapters Nine to Eleven

The next chapter provides essential background information about the AskNRICH web-board and its participants. Each Perspective is then reported separately in Chapters Nine to Eleven. Chapter Twelve then presents a summary of the Perspectives' findings before a further theory seeking review of the literature in Chapter Thirteen [LRIV] in preparation for the final Characterisation of AskNRICH.

Chapter Eight

Introduction to the AskNRICH Web-Board

> *.... and then one day* [I] *ventured into askNRICH. ... whenever I am stuck with a problem I always know that I can go onto NRICH and ask somebody to help me.* *[Peter aged 16, email communication]*

8.1 Introduction

This and the following three chapters report on the analysis of AskNRICH based on three research questions (**RQ**) given in Table 8.1. Although each of the four chapters contributes to all of the research questions, Table 8.1 also shows which chapter is the major contributor to each question.

RG3: to undertake the exploration of the AskNRICH artefact		
RQ	**Research Question** and *sub-questions*	Thesis Chapter
RQ5	**What does AskNRICH offer to participants to enable them to pursue their mathematical practices?** *Necessary background information about AskNRICH:* *What is it? What are the different sections of the web-board? What are the posting protocols on AskNRICH? Who are the participants? Why do they participate? How is AskNRICH typically used?*	Eight
RQ6	**What are participants' common practices when using the AskNRICH web-board?** *What characteristics do participants of AskNRICH exhibit as they pursue their interest in mathematics?* *What mathematics teaching and mathematics learning roles are manifested within AskNRICH?*	Nine & Ten
RQ7	**What results from participants' practices when using the AskNRICH web-board?** *What types of interactions are shown between the participants as they engage with mathematics?* *In what ways does the behaviour of AskNRICH participants emulate the working practices of professional mathematicians?*	Eleven

Table 8.1 Research Questions for the Exploration of the AskNRICH Artefact

The purpose of this chapter is to set out essential background information about the AskNRICH web-board and its participants to provide a context for subsequent chapters. Thus this chapter explicitly addresses the first of the three research questions (**RQ5**) at an initial, basic level and implicitly contributes to addressing the other two.

This chapter has four further main sections. The first introduces AskNRICH through the participants' voices. In the second section the AskNRICH environment is described by: defining its nature; detailing the components that make up the web-board, and discussing the posting guidelines. The third section describes AskNRICH participants and reasons for their participation. A section providing statistical data on postings to the web-board completes the introduction to AskNRICH, and the chapter concludes with two summarising sections.

This chapter is primarily informed by a ten-month intensive study, including trawling the Archive, and three months of 'living' on AskNRICH by visiting the web-board on a daily basis [p108, p116]. The detailed information presented in this chapter is further warranted by my insider knowledge as a co-founder of NRICH who has subsequently maintained a 'parental' watching brief.

8.2 AskNRICH Portrayed through Participants' Voices

AskNRICH would not exist without its users; it thus seems appropriate to introduce it through the voices of four of them taken from three different sources. The first source is three comments made by respondents to the web-survey:

> *AskNRICH is a good place to ask and answer questions, as well as discuss maths.* *Female KS4 oh*[1]

> *It's interesting to read comments from other people who are interested in maths so much, as these people are uncommon. Also, when I have the time to try some harder maths, it is a fun change to ordinary school maths.* *Female KS4 oh*

> *I don't use the main NRICH website much; rather the forum section (AskNRICH). I feel the atmosphere is very good, and it's great to be able to talk and discuss with other talented mathematicians - an opportunity which I don't really have at school.* *[Nick] Male Year13 hs*

The person, anonymised here as Nick[2], making the third comment above subsequently elaborated on his use of AskNRICH in email exchanges with me. His intensity of using

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[1] Notation used previously in Chapter Four Web-Survey.

[2] co-incidentally ***HelpA*** in **ExThd2** discussed in the next chapter.

AskNRICH, albeit simultaneously with other sites, is illustrated by this response:

> *I first came across NRICH in the lower end of high-school; as I said, a teacher introduced me to it. I'm not sure exactly how old I would've been ... Hmm, the search function seems to indicate I first used AskNRICH in 2004*[3] *or 2005. I was most active in 2007 probably… I was perhaps spending 10 hours per week on AskNRICH when I used it the most, though I would've been on other websites/forums, and MSN etc. at the same time.* *[Nick email exchange]*

Finally, during a face-to-face interview, John[4], now an AskNRICH undergraduate team member, recalled that during the second half of his last year at school he found AskNRICH and has become 'addicted' to a daily logging on:

> *I think I had heard about it a couple of times but hadn't then sort of had forgotten as soon as I heard. But ...* [Cambridge Mathematics Lecturer] *was giving a talk and mentioned it and I thought it would be a good idea to check it out to see what it was like and it's kind of kind of addictive after awhile I mean ... well you go on it and it's not just sort of asking questions it is obviously partly that it is a very good resource outside of school ... and I think that it quite a fun thing to be able to sort of check up on you know a couple of times a day just to see what sort of discussion is going on.* *[John Interview]*

These comments speak for themselves and provide a succinct introduction for AskNRICH.

8.3 The Nature of the AskNRICH Environment

This section provides an overview of the components and philosophy of AskNRICH and its basic level of operation. This overview is given by addressing three sub-questions of **RQ5**:

What is AskNRICH?
What are the different sections of the web-board?
What posting protocols are used on AskNRICH?

This section expands on the relevant parts of the 'About AskNRICH' information on the web-board. All quotations italicised below were taken verbatim from the web-board at the time of writing.

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[3] Nick would have been aged 14 at about this time.

[4] co-incidentally HelpC in **3Thds** of Chapter Eleven.

8.3.1 What is AskNRICH?

AskNRICH is the universal abbreviation for the Ask-a-Mathematician service created by the NRICH founders to provide a space where anyone of any age and from any place could ask 'an expert' for assistance with any mathematical problem. The primary source of expertise is currently a team, overseen by a member of the NRICH staff, comprised of mathematics undergraduates and postgraduates studying at Cambridge University. However, there is no exclusivity as to who can take the role of the expert, as this extract makes clear: *'You can join in existing discussions or start a new conversation of your own. To post anything, you need to be registered, but this is free; just click on the link ... '*. Thus simply by registering anyone can voluntarily become a poster either seeking or offering help or indeed, as can occur, participating in both ways as they see fit. Anyone is free to come and go as they please.

AskNRICH is not a commercial organisation. The AskNRICH team members are volunteers living in the UK, thus the board makes it clear that: *'We're not one of those help services which guarantees an answer within so many hours'* and *'at some times of day there might be quite a delay before anyone sees the thread'*. However, the Moderator ensures that one member of the NRICH staff will voluntarily look at the board at least once on all 365 days of the year. Nonetheless, as AskNRICH exchanges examined later will show, responses often come remarkably promptly – at times within minutes.

As the web-board language is exclusively English there can occasionally be some difficulties with nomenclature, usually overcome by asking for clarification. Additionally, some newcomers can have problems with posting mathematical notation. However, there is a specific section containing instructions on mark-up that enables the mathematics to be more easily read [threads in Chapters Nine and Eleven exemplify this].

The design of the web-board has so far remained unchanged from its inception. The significant design decisions involved the division of the board into separate sections addressing differing purposes and the creation of Posting Protocols for participation in the activities on the web-board. These aspects of the design are discussed within the two following sub-sections.

8.3.2 Sections of the Web-Board

There are three sections of the web-board devoted to different levels of mathematical problem. Although the three levels correspond in UK terms to mathematics studied by students of compulsory school age (5-to-16 year-olds), sixth form study (17-18) and university level, all sections are open to all ages. The first, **Please Explain** (**PE**), is described as the section to *'ask about mathematics that you meet in school, or that your school maths lessons never explain!'*. The section, **Onwards and Upwards** (**O&U**) is for those '*who have started learning things like calculus*' and **Higher Dimension** (**HD**) for *'those studying maths at university level to discuss it with their peers'*. During every academic year since 2002, the Moderator has archived message threads which then become read-only but remain accessible partitioned according to the three section names.

All three sections listed above are on public (open) access, thus although it is necessary to register (which is free) in order to post to the board, anyone can visit AskNRICH and read any of the postings. However, only registered users have access to four additional private discussion sections: **NRICH talk** *'a place for discussions which are not actually about maths*'; **Who's Who** *'to find out a bit more about the people you are talking to'* – although this is dependent upon participants actually contributing a little about themselves in the first place and certainly not all do; **Reviews** *'of books, websites and other mathematical resources you think might be of interest to other NRICH members'* and **Follow it up** *'for discussions linked to maths events, whether local or national'.*

There is an implied guarantee that all questions in the **PE** and **O&U** sections will be responded to[5], but: '*Don't expect us to do your homework for you - we'll give you a hand in the right direction, but we won't provide a list of answers'*. Thus the AskNRICH team is vigilant in detecting such posts and tries to ensure that, where help given relates to coursework, the poster prints out the thread to give to their teacher. Questions in **HD** are not guaranteed a reply as they may well be in a highly specialised field, although there appears to always be someone who can and will respond and thus the content is almost entirely constituted of peer-to-peer discussions.

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[5] This is further commented on in Section 8.5, Posting Statistics.

8.3.3 Posting Protocols

The Posting Protocols offer advice in the form of ‘ground rules’ and ‘user instructions’ for those participating in the Ask-a-Mathematician web-board. The ultimate intention was to create a stimulating, enjoyable, supportive and successful learning environment. The Protocols were formulated by people who had been classroom practitioners but were then involved in teacher training within the mathematics education department of the University of Cambridge. Thus the creation of the Protocols was embedded within what the designers of AskNRICH considered to be best practice for good classroom mathematics teaching and learning. In other words the Protocols were the result of the application of tacit knowledge and beliefs, based on personal experience, of what would produce the intended good learning experience. Furthermore, the design of the Protocols also addressed the new and different challenges and opportunities presented by using the medium of the web-board. Thus some of the Protocols include practical advice/instructions for the participants on how to work within the medium. The Protocols fall under three headings: “Asking questions”, “Answering other people’s questions” and “Writing an entry for Who’s Who”, Appendix 8.1 presents all the posting guidelines verbatim. However, the Protocols for ‘Answering’, presented in Table 8.2, provide further insight into the philosophy of AskNRICH and its designers’ implicit assumptions about what constitutes good teaching.

Answering other people's questions

1. *Don't just tell them the answer (tempting when you've just worked it out yourself).*
2. *Give hints and explanations to help someone understand for themselves.*
3. *If you're not sure whether what you are saying is correct, say so, so that others can check.*
4. *Remember that the team will probably answer, so if you don't know, leave it to them!*
5. *If someone has already started to help someone with their question, think carefully before joining in. It's often best to let the original poster respond before giving them more to think about.*
6. *If several people are trying to solve the same problem, and you want to avoid giving things away to those still working on it, you can post your answer in white by typing "In white: \white{your answer}". Those who want to can select the text to be able to read it.*
7. *Be tactful if someone is getting things wrong.*
8. *Be careful about humour; a light-hearted comment about a silly mistake will not always come across how you meant it when it's in print.*

Table 8.2 Posting Protocols when Offering Help

Protocols 1 and 2 set the 'good teaching' tone of an adopted pedagogy, analysed in detail in the next chapter, that scaffolds learning [see Section 7.5.2 p153]. Protocols 3 to 5 are there to reduce confusion, although the analysis of threads in later chapters shows that overlapping help [Protocol 5] generally appears to be advantageous. Protocols 7 and 8 give advice on basic social etiquette, probably even more important within an e-environment where there can be no "visual cues" [Garrison & Anderson 2003: 48] or "social cues" [Chen & Chiu 2008: 681]. The part of Protocol 6 stating that clues/answers can be temporarily concealed until the reader wishes to look at them, plays an important role in addressing different needs and experiences; a mixed-ability strategy.

As will become evident in later chapters the crucial guideline for 'Asking Questions' [see Appendix 8.1] is *'Tell us what you've tried so far'.* This, together with the additional request: *'To help our team of experts to answer at the right level, do tell us about why you are asking the question, and what you already know about the mathematics involved.'* facilitates the initiation of a dialogue based on contingent responses [see LRIII p145]. The open access/public face of posting combined with the potentially young age of posters obliges AskNRICH to have an explicit posting etiquette. Thus the etiquette includes instructions on 'Being Polite' with the clear message: *'We are proud that AskNRICH is a place where people are polite to each other. Let's keep it that way'.*

The next section starts to discuss the range of potential participants in AskNRICH; the in-depth analysis presented in later chapters develops a much fuller picture.

8.4 Who uses AskNRICH?

This section provides initial, basic answers to two further sub-questions of **RQ5**:

> *Who are the participants – the AskNRICHers?*
> *Why do they participate?*

The response to both questions starts from one, now adult, participant's description of who he *felt* he was, when he first found AskNRICH:

> *When I first came here I was a toddler (even though I was 15/16 years old). I knew nothing when it comes to mathematics, I barely knew some*

> *basic algebra and geometry. NRICH practically opened the gateway to a whole new mathematics for me. It showed there are things beyond numbers involved in mathematics.*
>
> *[International Participant, now aged 24*[6]*]*

The web-board's open access ensures that there is variety in participation. There is the passive silent participation of 'lurkers' like Julia who for several years had accessed AskNRICH regularly but never posted, instead looking interestedly at the discussions pertaining to her level and interest of study [Julia interview see Table 6.2 p113]. Active participants range from people who only post a few times[7] all the way to those who post on a regular, sometimes daily or even hourly, basis.

The threads studied in the Perspectives of the next three chapters were all initiated by questions posed by school-pupils asking for help. However it must be borne in mind that some posters offering help will be volunteer team members (including NRICH staff) who must be also be considered as AskNRICHers allowing the serious pursuit of study. Furthermore, participation within AskNRICH is dynamic and fluctuating. Thus, for the purposes of this study, an AskNRICHer is defined as anyone who has contributed in some way to the exchanges by posting on the web-board. More precise definition of an AskNRICHer is not straightforward[8], but some information about them can be obtained directly.

8.4.1 Explicit Information Obtainable about Participants

Posts by NRICH staff and AskNRICH team volunteers are easily recognisable as they appear in green and cyan respectively and show their full name. However, posts by ordinary participants are essentially anonymous. The administrators are the only people who have access to real names and email addresses. Everyone is invited to choose a screen name when registering that becomes their sole identifier. Real names as posting names are also acceptable and some participants will choose first name and maybe initial of surname. Such

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[6] Comments from the web-board are accompanied with participant description based on web-board information (which can be limited).

[7] Investigation of the reasons for both 'dropping out' and/or 'lurking' is beyond the scope of this study.

[8] This problem parallels Gee's [2005a] arguments on the difficulty of defining membership within a Community of Practice [see Chapter Thirteen p279].

a format probably indicates gender but even what sounds real need not necessarily be so, thus screen names may reveal nothing. Age information is optional at registration (year of birth stopping at 'before 1990') and is used only for statistical data i.e. this is not available to other members or the outside world.

The number of posts[9] by a participant is recorded and displayed in their posts on the web-board with their screen name and a classification of their status: new (up to 5), poster (6-25), regular (26-74), frequent (75-199), prolific (200-399) and veteran (400+). If a poster becomes at least 'regular' then it seems reasonable, given the voluntary nature of posting, to infer that they have a natural enjoyment of being involved, in various ways, with mathematics.

In addition, some information surfaces within a post e.g. *'I am in year 9'*, *'I go to the local comprehensive'*. Even more may be revealed within NRICHtalk, which is not accessible to non-registered users, where participants are invited to contribute a self-portrait to Who's Who and thus other registered users (not the outside world) can learn a little more about them. However, there are many more active participants than entries in Who's Who and a superficial scan of threads will show a diverse range of participants, across all continents and all ages.

Having described how each post provides some limited information automatically about the poster, the focus now turns to consideration of why posters might participate.

8.4.2 Reasons for Participating

Evidence of reasons for participating can mostly only be inferred from the content of posts or comments within them, although the interviews [see Table 6.2 p113] provided some explicit information.

AskNRICH provides an opportunity for any member of the general public who wishes to make use of the facilities on offer. Venturing into AskNRICH (as Peter phrased it in the

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[9] As AskNRICHers are usually interested in numbers, significant numbers in the eye of the beholder often do not go unnoticed!

opening quotation of this chapter) can be something of major importance and/or revelation for those who do enjoy mathematics; it can be the first time that some have found the chance to discuss mathematics. It can offer a safe, supportive and caring place in which to ask questions, something that later chapters confirm and is exemplified here:

> *I love the way everybody is so friendly, and not obnoxious when you post the most obvious of questions, which you see the answer to the minute you post it.* *[Female Upper school]*

> *I can't think of a more natural way to have learned math though. I would just pursue the topics and questions I was interested in, and I'd get guidance along the way, and maybe suggestions of other topics I ought to read about.* *[International participant now aged 24]*

Although there is no compulsion or expectation to do so, AskNRICH team volunteers often state a desire to be able to pay back as their reason for participating:

> *I joined the team because this website [had] encouraged me to get more interested in maths and I wouldn't be where I am without it.*
> *[Final Year Cambridge Undergraduate]*

and by Nick (quoted in the introduction) anxiously waiting in the wings:

> *I've been on NRICH for perhaps 3 or 4 years, though most intensively for the past 12-18 months. If I do get into Cambridge[10], I think I would like to become a team member and give something back for all the generous help I have received here!* *[Nick final year at school]*

The longest serving participant[11] who also has posted the largest number of posts (3100+) is now an AskNRICH team volunteer, aged 24 and completing a PhD. He started posting when he was 15 and is one of a growing number who have made the transition:

> *[from] being one of the people who asks the questions to one of the ones answering them. I learnt an awful lot through this board; I'm really glad to have the opportunity to repay the debt by helping other people.*
> *[Longest serving poster]*

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[10] Nick succeeded and became a Cambridge undergraduate. He made personal communications over the summer vacation to enquire how he could join the AskNRICH team and became a Cyan poster.

[11] excluding the moderator who, at the time of writing, has 3172 posts.

John in his interview whilst mentioning being able to put something back, also sees it as a way of testing his own knowledge:

> *A very good test of how well you know material is whether you are able to explain it to someone else so in a way it is the only test* [that you know the material] *because it shows you fully understand it and you can answer questions on it.* *[John Interview]*

Providing a place to ask questions captures students who are naturally enthusiastic for the subject and enjoy challenging mathematics problems. Many come to AskNRICH because they wanted to learn some mathematics that was beyond the curriculum work in school, in effect they are teaching themselves. This will be further explored particularly in Chapter Ten that follows Peter through his experiences. In the meantime, this comment below is typical:

> *I have* [been with] *NRICH since I was 14 after I saw a poster of this discussion Forum in my school. I love to come here and ask a lot questions on maths topics that I teach myself.* *[School Student Aged 18]*

Amongst these self-teaching and self-learning young people, there are, as evidenced in their postings, those, likely to be the most able at the subject in their year, who aspire to compete nationally and internationally in mathematical competitions. Many schools within the UK offer pupils the opportunity to take part in the Annual Mathematics Challenges and from these the highest scorers are invited to participate in Mathematics Competitions, culminating with the British Mathematical Olympiad (BMO). Thus one reason for participation is involvement in these competitions or through a Gifted and Talented (G&T) initiatives. Indeed, the United Kingdom Mathematics Trust (UKMT) book mentioned in **ExThd2** [Chapter Nine] is most likely to have been purchased as preparation for taking part in such competitions. As Nick's first comment in Section 8.2 reveals, AskNRICH does provide a means by which these young people, generally isolated as being the only one in a school, can communicate with others in a similar situation.

AskNRICH also has some older participants who are avid recreational contributors rekindling childhood passions. One AskNRICHer gives an age of 65+, lives on a cattle ranch and is making up for lost opportunities to increase his understanding of higher mathematics generally through contributing to discussion, more than asking specific questions. Another contributor is a father of three secondary aged pupils. He visualises helping others to make connections and take on new challenges just as he remembers as a primary aged pupil begging his maths teacher to show him how to do long division and surprising them that he quickly understood the concepts.

This section finishes with an instance where the web-board, and the AskNRICHers, are put to the test. A new poster in **PE** received replies that assumed they had posted in the wrong section and thus were at a higher level than that expected for **PE**. When the Moderator stepped in stating '*given that you've posted in Please Explain, I guess you may not know most of the terms being used in the replies*' and continued to offer sympathetic simpler explanations, the original poster revealed all. '*Actually, I am a math teacher who is researching websites that can be used to explain things that my students might have trouble on. I picked a question that had given me some trouble when I first encountered it. You guys were the quickest to respond and definitely gave me more than I needed to understand the answer to my question. I will admit that the discussion went a lot further than I needed. I was able to follow along for the most part. Thanks for everything!*' (July 2006). This exchange illustrates the quality of response and the consideration and encouragement offered to new posters, often with an implicit invitation to continue to participate. Moreover it serves as an illustration of being unable to know anything about a poster unless they themselves reveal it.

Thus, as already stated, reasons for participation are wide-ranging and varied. To complete the introduction to AskNRICH and to initially address the final sub-question of **RQ5**, *How is AskNRICH typically used*, the following section presents statistical data relating to the sections, threads and messages (posts) that are retrievable from the board.

8.5 Posting Statistics

This purpose of this section is to convey a snapshot of the traffic over one year on the three open sections of the web-board in terms of the numbers of threads and posts and the distribution of thread lengths. It starts by detailing the size of all data accessible for study.

In order to contain the number of active threads on the web-board, the Moderator periodically selects threads, judged to be inactive, to be moved to the archive or (occasionally) deleted. Given the resulting dynamic nature of the web-board, the data presented here can only be that present on the day it was viewed. Although the more recent threads are likely to be a closer match to actual posting activities, the numbers presented within the archive are those for threads that remain, i.e. the number of archived threads is unlikely to equal the number that appeared on the board during that period.

Table 8.3 represents the number of threads and posts for each of the sections on the web-board as retrieved on May 22nd 2008. The grand totals of 49285 posts and 5966 threads highlight the vast amount of mathematical discourse that is available (for anyone) to read and to study. Based on this grand total the mean number of posts per thread is 8.2, a value that has changed little across individual sections and years [see breakdown in Appendix 8.2].

Period	Please Explain		Onwards and Upwards		Higher Dimension	
	Threads	Posts	Threads	Posts	Threads	Posts
Active	43	370	141	1208	173	1085
Archive 07-08	69	570	468	4188	430	2385
Archive 06-07	236	1908	952	8546	667	4364
Archive 05-06	228	2518	1066	10658	781	6265
Archive 04-05	29	141	22	132	5	23
Archive 03-04	137	1083	207	1359	20	144
Archive 02-03	23	85	269	2253	0	0
Total	**765**	**6675**	**3125**	**28344**	**2076**	**14266**

Table 8.3 Number of Threads and Posts (retrievable on May 22nd 2008) for each Section on Web-Board

Table 8.4 below provides data for **PE** and **O&U** relating to the total number of *retrievable* threads and posts each month[12] for August 2007 to July 2008. These are the sections and the academic year relating to the Main Study. Again there is no guarantee that this represents all the threads posted during this time.

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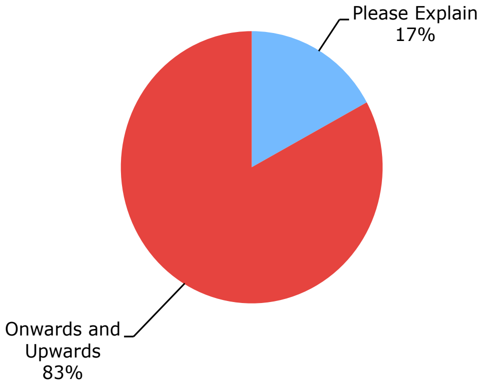
[12] The month attributed to the thread is the month on which the last post was made.

	PE Threads	O&U Threads	PE Posts	O&U Posts	Figure 8.1 Distribution of Threads between PE and O&U August 2007 to July 2008 
August 07	9	36	88	225	
September	5	68	34	668	
October	13	78	80	592	
November	12	109	100	930	
December	7	63	44	485	
January 08	11	48	126	790	
February	19	58	178	482	
March	17	69	188	645	
April	8	36	42	240	
May	20	46	169	369	
June	11	44	171	296	
July	10	34	59	355	
Total	**142**	**689**	**1279**	***6077**	
Monthly Mean	12	57	107	506	Mean number of posts for PE Threads: 9
Median	11	53	96	484	Mean number of posts for O&U Threads: 8.8

** Two threads of 401 (December) and 201 posts (March) were not included in these figures. The content of these two outliers would be more appropriate placed in NRICHtalk as general discussion.*

Table 8.4 Number of Threads and Posts in Please Explain and Onwards & Upwards Sections of the Web-Board for the twelve months August 07 to July 08

Figure 8.1 indicates that during this period the number of **O&U** threads (689) is about five times that of **PE** (142), indicating that, for the two school-aged sections, work beyond GCSE level predominates. However, this statistic cannot be used to make any inference about the age of the posters as, for example, some who are of pre-GCSE age are doing mathematics beyond GCSE and therefore contributing to **O&U**.

For each of the threads (except for those in **PE** for June and July where data had accidentally been deleted by the Moderator) represented in Table 8.4, the number of posts were grouped into five different sizes: 1 to 3 posts (corresponding to problem posed, help offered and in an ideal world thanks given); 4 to 8, 9 to 15, 15 to 25 and over 25 [see Appendix 8.3 Tables 1&2]. Figure 8.2 below represents the frequencies for each size category for each section's total for the year (ten months for **PE**). The ratio between the heights of the **O&U** and **PE** bars for the first four categories is approximately the same at around 6:1 and for the longest posts category it is around 4:1. Thus during this period there was little difference between the sections in the distribution of lengths.

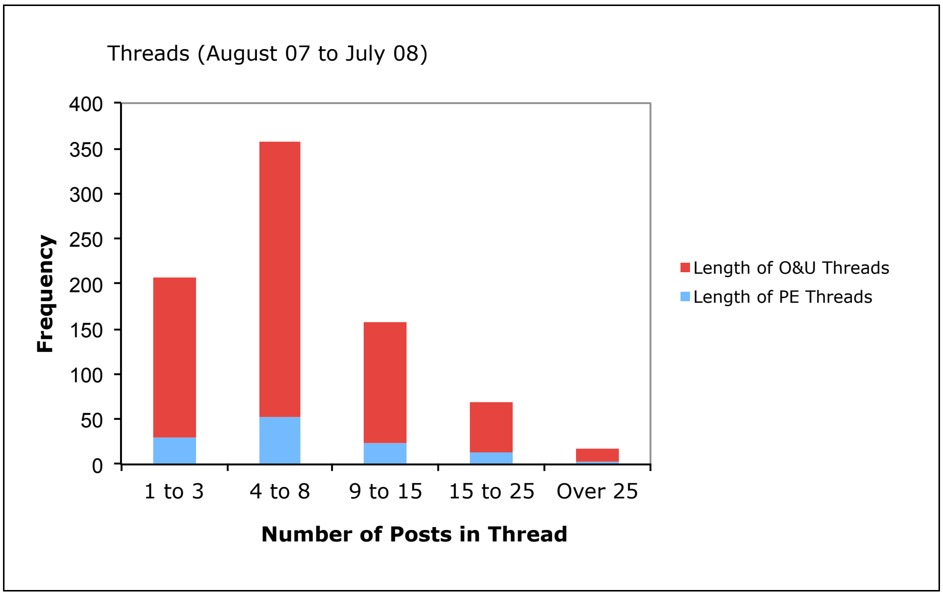
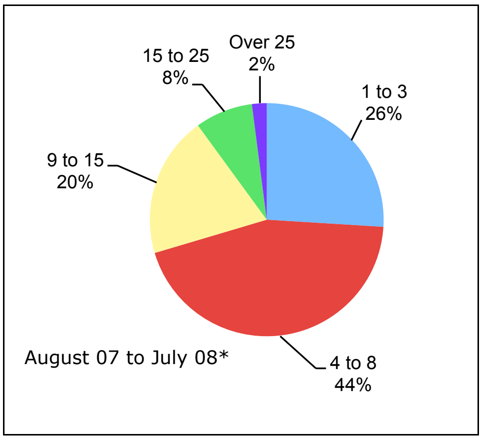


Figure 8.2 Frequency in each Length Group for PE and O&U threads

The thread length data from each of the two sections is combined and re-presented in Figure 8.3 to indicate the percentage of all threads that fall into the designated sized category. The most common size, accounting for nearly half (44%), is 4 to 8, indicating that at least some 'conversation' has taken place beyond the minimum that would results from: ask, receive help, give thanks. Adding in those threads of length 1 to 3 then 70% of all threads used in this data are completed within 8 messages.



** Please Explain threads available up to May 08 only*

Figure 8.3 Percentage of Threads in each Length Group

Although, as stated earlier, there is an implied guarantee that requests in **PE** and **O&U** will be responded to, 18 (2%) of the total number of threads were of length 1, all in **O&U**. However, inspecting the contents of these threads shows that no request has been ignored.

These posts are, for example, information giving, or multiple error postings as a new user gains familiarity with the mechanics of posting, or as one thread's title makes clear self-answering: *'Slope fields and integral curves: sorry have solved problem myself'*. Furthermore, no judgement may be made relating the length of thread to the quality of the content within it, as it may take as few as two posts to have a rich exchange.

The final two sections summarise this chapter.

8.6 Features Summary 1

In order to provide a means of managing the presentation of the large quantity of findings from this study, at the end of this and each of the next three chapters, a selected set of Features are catalogued. These Catalogues (five in all) form parts of a diagram illustrating all Features around a five-sided figure showing their interwoven interrelationships portrayed in Figure 8.4 [next page]. This diagram is the basis of the summarising of findings contained in the 'interlude chapter' [Chapter Twelve]. The Features selected for each chapter's Catalogue are always present in that chapter, although not all Features present in the chapter are necessarily included in the Catalogue since they may have a stronger relation to another chapter's catalogue. Similarly, for the same reasons, later chapters may contain further or better examples than the one used to underpin inclusion in the earlier Catalogue.

Thus by the end of the four chapters all Features within the complete diagram will have been thoroughly considered. The Features Catalogue for this chapter, relating to Structural/Medium, is presented in Figure 8.4. The Catalogues of the other chapters are also shown greyed-out.

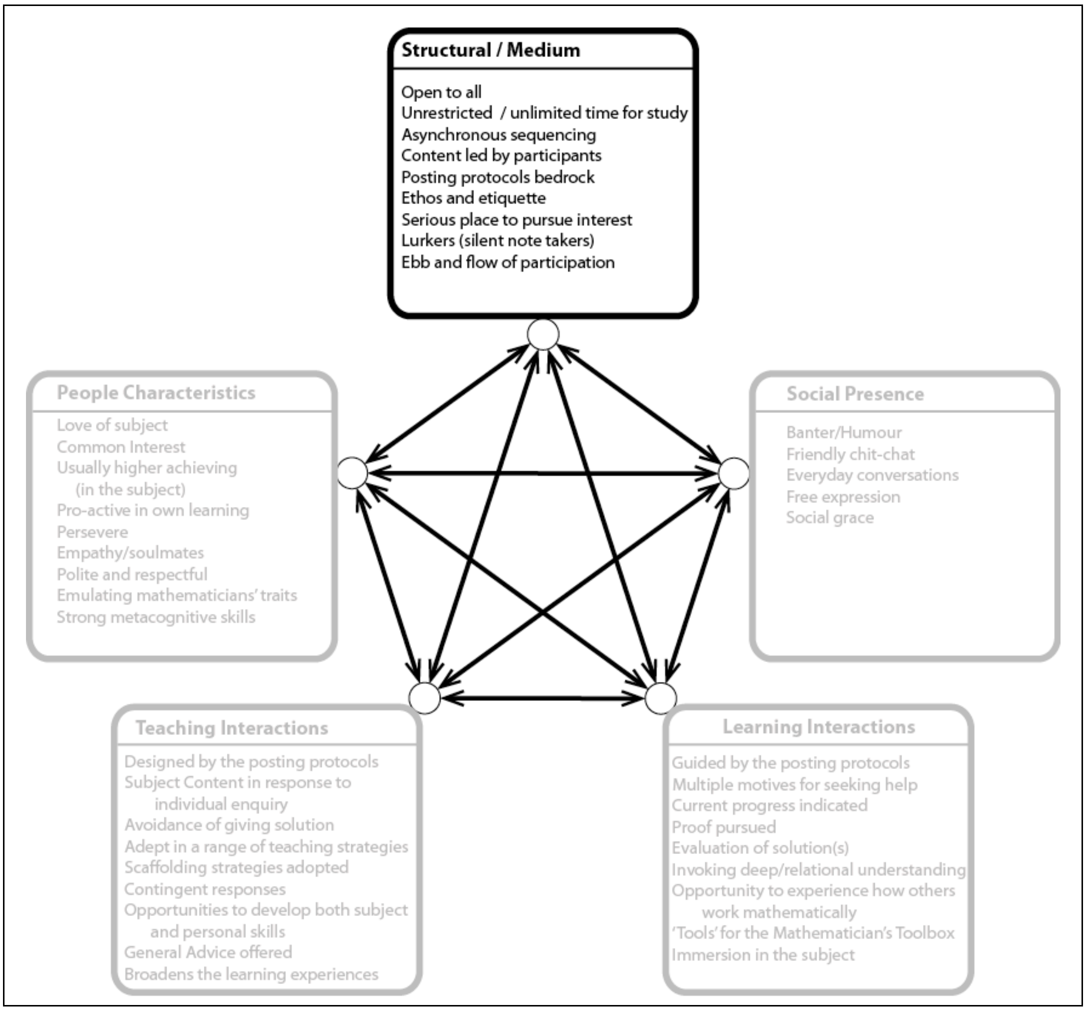


Figure 8.4 Features Catalogue: Structural / Medium

8.7 Conclusions

This chapter has set the scene in order to frame the further discussion in the remaining chapters. In essence it provide essential background information on AskNRICH, introduced through the voices of some of its participants, the AskNRICHers. The chapter has included a description of the different sections of the web-board. It has presented the Posting Protocols that, as later chapters will demonstrate, form the bedrock for the teaching and learning activities that take place within AskNRICH and the AskNRICHers' well-mannered conduct. The statistical data presented has conveyed the vast amount of mathematical discourse and other focused activities that can be accessed from the 5966 threads and 49285 posts available for study.

Although, intriguingly, the true number and motivations of people who might read and study AskNRICH material is unquantifiable due to the presence of passive lurkers, some data on active participants who actually post can be obtained. This comprises information about who the AskNRICHers might be and some of the variety of reasons for their participation. Overall, AskNRICH is shown to be an environment in which young people, often high-attainers working beyond the school curriculum, can be enthusiastic about mathematics, find help (that may not be available elsewhere) that enables them to develop their mathematical skills, and build their confidence among like-minded peers and mentors. The AskNRICHers' complimentary comments make clear how much they feel they gain by participating and being in-touch with such like-minded others, often returning later to voluntarily 'pay back'. As the AskNRICHers imply, and the next three chapters will continue to demonstrate, there is an obvious quality to the discourse.

The following three chapters, building on the content of this chapter, present analysis from different Perspectives using selected threads to typify the activities and behaviours evident in AskNRICH. The next chapter will use analysis of two exemplar threads in the portrayal of the common practices evident amongst AskNRICHers.

Postscript

When returning to the early evaluations of NRICH recently, I noticed that the following quotation had been included [Jared 1998: 17].

> *Hey why so little space [on the questionnaire] for AskNRICH. It's brilliant. I can ask all the questions [I want]. The answers are fascinating because they take you further than you were asking plus you can have a decent conversation with someone who knows much more maths then you, about maths.* *[Female student aged 17]*

With respect to the current work, it would appear that little has changed, the sentiments expressed by this student, who, at the time of writing, will be approaching her thirty-first birthday, have been echoed by many others over a decade later.

Chapter Nine

Common Practices of AskNRICH Perspective One: Two Exemplar Threads

> *Just keep practising, and only look at the hints when you're really, really stuck - you'll gain more if you struggle with the question a bit before looking at the hint. They will become easier if you keep hammering away at problems ☺.*
>
> *[Advice from a school-aged peer in* ***ExThd2****]*

9.1 Introduction

The previous chapter provided supporting background information on AskNRICH and the AskNRICHers. This chapter is the first of three, each focusing on the interpretive analyses of a selection of message threads from a different Perspective.

The work reported in this chapter, and the two following, contributes to addressing the last sub-question of **RQ5**, *How is AskNRICH typically used?* and the whole of **RQ6**, *What are participants' common practices when using the dynamic web-board?*. These chapters also inform **RQ7**, *What results from participants' practices when using the dynamic web-board?* [see Table 8.1 p165].

The Perspective for this chapter uses examination of threads that show the general nature, common practices and use made of the web-board by the AskNRICHers, reported here primarily through the analysis of two exemplar threads [**ExThds**].

Thus the purpose of this chapter is to:

i. briefly describe the two exemplar threads
ii. provide examples, using annotated extracts, of the outcomes of applying the data analysis processes to the exemplar threads
iii. convey the general nature, common practices and use of AskNRICH by presenting the themes derived from the features found by the coding process
iv. discuss the general practices revealed by these features and themes

The intention of this chapter is to report on general practices found to be common amongst the AskNRICHers from actions and activities evident within the **ExThds**. The theoretical underpinnings used in the discussion section within this chapter are the concepts of Socratic dialogue and scaffolding [see Figure 7.4 p164].

The remaining sections of this chapter start with a brief introduction to the **ExThds** each accompanied by their tabulated extracts of posts, commentaries and indices. The chapter continues by explaining the 29 features, grouped under four themes, created from the open coding process. The final section is a discussion on the general practices revealed by these features and themes.

9.2 Exemplar Threads

In the two threads, the participants asking the original question are ***Plea1*** and ***Plea2*** respectively. Another participant, who joins in the second thread also seeking help, is labelled ***Plea3***. Any information on participants such as, for example, age, is tied to the time of the analysis, May 2008. Appendices 9.1&9.2 present the two threads in html format.

9.2.1 Exemplar Thread One – Attempting to Solve Simultaneous Equations

The first thread [**ExThd1**] was selected for analysis as the topic, simultaneous equations, is taught in school as part of the National Curriculum [DfEE/QCA 1999: 62], and the problem is a classic routine exercise question. The thread, which is typical of all threads posted for the purpose of finding a solution to a ‘straightforward’ question, shows the nature and the type of help and the process by which it is offered. The question involves solving two simultaneous equations, one linear and one quadratic. This topic appears at GCSE higher-level mathematics (ages 15-16) though is often more accessible and ‘mastered’ in the early parts of a Year 12 (ages 16-17) A-level syllabus. Circumstantial evidence suggests that ***Plea1*** was working on two GCSE homework questions.

Plea1, still a fairly new poster in early 2008, was making their 20th post. Since then they have continued sporadically to post new problems that they need help with. ***Help1*** has veteran poster status and is at school (aged between 16 and 18). ***Help2***, making his first post ever, is also at school and continues to use AskNRICH.

Appendices 9.3&9.4 present the full text of the posts, with complete interpretative commentaries resulting from the first two stages of the iterative analysis process described in Chapter Six [Section 6.3.4.1 pp121-123]. Table 9.1 [pp187-189] shows the posts in précised form together with the final, précised, third-iteration, interpretive commentary, produced using the refined method in which that related to the mathematics undertaken and that related to the actions of the posters are separated into two columns. The allocated code, explained in Section 9.3 below, appears in the final column.

9.2.2 Exemplar Thread Two – Number Theory Topic

In this thread [**ExThd2**] ***Plea2***, *'by the way I am in year 10'* thus assumed aged 15 to 16, is preparing for National Competitions by attempting a series of number theory questions based on self-directed reading of the UKMT Number Theory Book. The topic is, at the level it is being worked on, not common in school mathematics and beyond that expected in GCSE examinations. However it is a topic that young, capable, aspiring mathematicians need to study [Houston 2009]. Part-way through, ***Plea2*** is joined by another school student ***Plea3***, one year younger. In this case the collaboration is akin to the type sought in critical thinking CMC studies. Both participants attempt to make sense of the more abstract and, at times incorrect, helping posts with only spasmodic interjections by more able participants acting as 'sages/experts' collaborating in the sense of hints or nudges to lead towards the solution (and being in a teaching role).

In contrast to ***Plea1*** seeking help with two homework questions, ***Plea2*** is seeking to increase his own knowledge both beyond his chronological age and on a topic outside of 'normal' school lessons. Thus ***Plea2*** appears to be pursuing mathematical study 'at leisure', as also evidenced by the thread starting five days before Christmas, i.e. in the school holiday, and the seven exchanges on that day and two further on Christmas Eve.

This thread also shows the nature of generic mathematical advice that more-experienced peers give to someone encountering such challenging problems for the first time, a significant, additional reason for its selection. Several posts, especially at the beginning, aim to support the participant seeking help to have confidence in pursuing the type of challenging mathematics problems that are not just routine practice and, as such, more likely to be studied by an individual at home. The thread only starts to focus on the specific mathematical problem[1] after some eight posts discussing problem-solving practices. Appendix 9.5 presents an initial interpretative commentary, at a pre-coding stage, on the first nine posts. My handwritten comments on the whole thread, developed over a period of time and including working through the mathematics, are documented in Appendix 9.6. Appendix 9.7 presents the thread with codes added. Table 9.2 [pp190-191] presents post extracts interspersed with an interpretive commentary and relevant codes on the first eight posts of the thread. Table 9.3 [pp192-195] presents the remaining specifically mathematical posts in the thread.

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[1] ***Plea2*** has subsequently and periodically returned to the thread to ask about other number theory questions. For the purposes of the exemplar thread only messages up until the first problem is resolved have been included here.

Table 9.1 Exemplar Thread One: Posts, Dual Commentaries and Codes

Post Number Day/time	Post précis	Commentary Mathematics focused	Commentary Actions focused	Code
P1 Friday 7.29pm	***Plea1**: We are solving simultaneous equations, one linear, one quadratic. I am stuck on two. I know the answers but I can't work out how to get them. Any help is greatly appreciated.* [Two questions and answers stated]. *I can usually solve them, but these two got me really muddled. Thanks in advance.*	Linear equation of form y=ax+b, a quadratic is a polynomial of degree two of form $y=ax^2+bx+c$ Solutions values known and by substitution can be seen to be correct. Does not indicate the method being used	Posted at beginning of weekend, out of school time Content to show current inability Politeness	TM LRO SPP
P2 Friday 8.13pm	***Help1**: The method you want to use here is substitution* [Provides a worked solution to an alternative question] ... *See if you can do it for yours now. If you can't, post your working and we can see where you've gone wrong ...*	Suggests the method to use, and explains it through working through an alternative example	44 minutes before reply Relevant example especially devised Offers encouragement to try with reassurance of further help if required	TB TREG TRSM SPC
P3 Friday 8.46pm & P4 8.54pm	***Plea1**: I'm sure I've made a really silly mistake* [includes workings] ... *doesn't factorise ...I'm not sure what happened ... THESE AREN'T THE RIGHT ANSWERS. Thanks in advance*	Used method given to find correctly $x = \frac{2}{3}y$, substituting into the quadratic but expands $(y+\frac{2}{3})^2$ not $(\frac{2}{3}y)^2$. Knows error exists as quadratic does not factorise as it must For second problem the values derived are incorrect	Has spent over half an hour (assumedly) trying to get correct solutions Posts mathematical workings Suggests own inabilities Apparently frustrated but is persevering Asking for further help with no explicit 'write down the solution for me' Perseverance	TB LRW LRO LRP LRU LRP

Post Number Day/time	Post précis	Commentary Mathematics focused	Commentary Actions focused	Code
P5 Friday 9.58pm	***Help2****: For 1 you've just made a mistake in expanding the expression, can you see it?* *For 2 in a few steps you have divided/multiplied by x, which means that you have to check the extra case x=0. Additionally you've made a silly mistake in expanding ….* *Can you solve it now?*	See above Error made in multiplication manipulation: multiplying a bracket by 5x would not make the denominator 5x times larger, a typical 'silly mistake' that mathematicians can be prone to even if here it was made through inexperience. The error made the equation more complex and included a special case of division though this will also be true with correct expansion	Second helper [and first time poster] now involved Teaching locates and signals error but leaves ***Plea1*** to attempt to correct for self Teaching aware of special case, anticipates misconception, 'future-proofs' Supportive atmosphere, still asking if explanations are sufficient to complete solution plus 'silly mistake' is a repeat of ***Plea1***'s own turn of phrase	TA TRSE TRAD SPC
P6 Friday 10.07pm	***Plea1****:* [obtains expression that] *cancels down to* $x^2 = 5x$ *is that correct and if so, what is the algebraic way then, to solve it to make 5, and 0. i can see how the numbers go in but not how to solve it algebraically*	$x^2 = 5x$ often causes more problems than quadratics which have all three terms. Solution involves the case where x=0 alluded to in P5	***Plea1*** has continued to work on the problems (2.5 hours since first posting) and is explicit as to where he has reached and what he would like to know	TE LRO
P7 Friday 10.12pm	***Moderator****: For 1, I suspect it's the sort of blunder you become blind to when going back, because you're too busy checking the steps you did do. So try this: ….*	'blind to blunders' is also something a mathematician can be prone to Required to expand $(y+\frac{2}{3})^2$ and $(\frac{2}{3}y)^2$ to highlight **Plea1**'s original error [see P4]	Third helper now involved. Postings beginning to overlap and offering correct advice but involving a range of perspectives as to what should be done Expansion will make ***Plea1***'s original error clear 'Comfort' offered with 'blunder'	TA TROH TRSE SPC
P8 Friday 10.15pm	***Help2****: For 2 we consider the following cases:*	Provides the solution for q2 involving the case of x(x-5)=0	Gives direct instruction (and completes the solution for q2), restricting **Plea1** in working it through personally (possible to infer advantages and disadvantages to doing so)	TRDE TRRR

Post Number Day/time	Post précis	Commentary Mathematics focused	Commentary Actions focused	Code
P9 Friday 11.33pm	***Plea1***: *Thank you. I really understand 2 now. Number 1 is coming to me too … taking a little more time*		Four hours since ***Plea1*** started. Assumed to have worked through ***Help2***'s solution to understand the mathematics involved Perseverance – still content to continue working on it	TE LRU LRP
P10 Saturday 2.34pm	***Plea1***: *Thak [sic] you so much. I understand it all now*		Completed before the end of the weekend, over a period of 19 hours (including sleeping!) Lets people know that all is well and offers thanks for the help received ***Plea1*** perceives that work is understood	TM TE SPP LRU

Table 9.2 Exemplar Thread Two: Posts 1 to 8, Actions focused Commentary and Codes

Post Number Day/time	Post précis	Commentary Actions focused	Code
P1 Thursday 12.03am	***Plea2****: I just wanted to ask a few questions about the ukmt number theory book. How much prior knowledge does it assume? Are the exercises meant to be challenging? I put particular emphasis on the last question as I find the exercises quite tricky*	Posting just past midnight Open to stressing current difficulties	TM LRO
P2 Thurs 8.11am	***Deputy Moderator [DM]*** *Well exercises aren't that much fun if they're easy!*	Brief reply advising gain pleasure in 'hard' mathematics	**TRMA**
P3 Thursday 8.22am	***HelpA****: found the number theory problems to be fairly easy in comparison.* **…** *Just keep practising, and only look at the hints when you're really, really stuck - you'll gain more if you struggle with the question a bit before looking at the hint. They will become easier if you keep hammering away at problems ☺.* **…** *Out of interest, how do you find the inequality problems, and if you have the geometry book, how do you find those problems?*	11 minutes after brief reply ***HelpA*** – more experienced peer, still at school, offering sincere, genuine advice and encouragement Inviting ***Plea2*** to engage in further the discussion	TB TROD TRMA SPC TROD SPC
P4 Thursday 1.18pm	***Plea2****: I haven't started the inequalities or geometry yet. I am doing them one by one. I asked … about the difficulty of the exercises … spent 30 minutes on one part of the* [primes] *exercise … (week2). After … struggling with the question I looked at the commentary and was extremely put off to know that I had not even been thinking along the lines of the solution …*[Includes scanned image of commentary]. *By the way I am in year 10*	First part of response immediate to ***HelpA***'s final comment Continues to stress difficulties Giving school year aids helpers to judge level, but also offered here in 'talk'	SPP LRO **SPT**

Post Number Day/time	Post précis	Commentary Actions focused	Code
P5 Thursday 1.27pm	***HelpA***: *30 minutes is not long in the grand scale of things. Often you can spend 3-4 hours or more on a difficult problem if you're really getting into it. I know what you mean though about not even thinking along the right lines. Often it's tempting when faced with a solution to think "Wow, I never would've thought of that", but it's best not to think in that way. Instead, make the solution your own! Use the technique in other problems now that you've encountered it ☺. Always look to improve your problem solving 'toolkit' and to add more tools to it.* *If it's any concellation [sic], I just spent 20 minutes on a question, approaching it in completely the wrong direction, and at the end I arrived back at the initial problem. Annoying, but it happens ☺. I didn't have the required knowledge to solve the problem in fact it turned out.* *Persistence is key, though once you've bashed away at a problem for a reasonable amount of time, it's not shameful at all to look for hints/solutions ☺ The more problems you have a good go at, the better you will become, I promise! If you keep at it, in six months time I'm sure a lot of problems you struggle with now will be very easy to you.*	Just 9 minutes later (during which this long reply has been written) ***HelpA*** provides reassurance that ***Plea2***'s state is normal. Offers advice Continues discussion, again reassures Offers more advice and further reassurance	TB SPC TRMA TROD SPC TRMA SPC
P6 Thursday 1.43pm	***Plea2***: *Thanks for the motivation. I was even contemplating giving up working through the books because I thought the exercises were too hard.*	Lets ***HelpA*** know they have been of great help [continued, authentic encouragement enabled ***Plea2*** to stay on board]	SPP
P7 Thursday 2.33pm	***HelpA***: *You're welcome ☺, never give up!*	….. who replies kindly and with a final word of advice!	SPC TRMA
These exchanges have taken place within the space of a morning and early afternoon (a break for lunch?) A little under an hour later ***HelpB***, a second more experienced peer, joins the discussion supporting ***Plea2*** and reiterating some of the ***HelpA***'s advice.			
P8 Thursday 3.21pm	***HelpB***: *30 minutes definitely isn't a long time when attacking a problem. No doubt your [sic] used to destroying gcse/alevel problems but i actually think it's more fun tackling a longer question. I remember being disheartened when attempting [] question because i couldn't instantly see the answer which is common in Alevel questions, but now i quite like the fact that i have to rack my brains in order to spot the path. It feels more rewarding when you do actually solve it. I've not done either of these books but if they are stretching you that's always a good thing because unfortunately i doubt Alevel will or does. Maybe parts of further Maths possibly*	Reiterating ***HelpA***'s advice Reiterating ***DM***'s advice Reassurance as ***HelpA***'s Reiterating ***DM***'s advice Criticism of lack of challenge in school mathematics	TRMA SPC throughout **SPO**

Table 9.3 Exemplar Thread Two: Posts 9 onwards, Dual Commentaries and Codes

Post Number Day/time	Post précis	Commentary Mathematics focused	Commentary Actions focused	Code
P9 Monday 1.12am	***Plea2****: In the commentary that I provided in post 4, are you just meant to 'spot' that 2^m- 1 is of the form 4t+3? I ask this because I would never have thought of doing that....*	Prove when n is a power of 2, the function (1/3)*(4^n-1) has a prime factor of the form 4k+3 Scanned image sent by ***Plea1*** includes the 'spot'.	Early hours of Christmas Eve. Late in the night posting for 15-16 year old Four days later, ***Plea2*** returns with same problem – indicating inadequacy	TM TA TE LRP LRO
P10 Monday 8.32am	***HelpA****: Well, it's not *too* hard to spot if you notice that 2^m is always going to be a multiple of 4 for m > 1. With more experience a lot of things like that will jump out at you quite quickly* ☺	Senses where ***Plea2*** is stuck as clue to 2^m-1 is in discussion of 2^m that would lead ***Plea2*** to 4t+3 ***Plea2*** does not respond directly to this (thought the 'spot' here is illustrated later by ***Plea3*** in P20 with slight error and corrected in P30).	***HelpA*** [continuity] replies around breakfast time of the same day ***HelpA*** suggests what to use to move forward Advice also intended to reassure	TA TRSM TRMA SPC
There were no pleasantries of wishing each other 'Happy Christmas' and no more posts until ***Plea2*** returns on 9th January when there are five exchanges between ***Plea2*** and ***DM***				
P11 Wed 8pm to P15 Wed 9.02pm	***Plea2****: seeking justification why 2^m-1 is of the form 4t+3 … has an odd number of primes of the form 4k+3 in its prime factorisation. Thanks* ***DM*** suggests: *multiplying two numbers of form 4k+3 say (4k+3 and 4l+3)* ***Plea2*** does so, shows working and responds: *I think I've got it* ***DM*** enquires about familiarity with modular arithmetic ***Plea2*** offers thanks.	Multiplication leads to [36(k+l)+24)] +3 hence [term] is divisible by 4 and thus of required form Both inputs from ***DM*** provide potential for ***Plea2*** to increase their mathematical knowledge	Just over two weeks later and ***Plea2*** is still working on the problem ***Plea2*** is seeking a proof for the justification and in doing so implicitly shows a wish to gain understanding of the fact. 2 minutes between ***Plea2*** asking for and ***DM*** giving hint Advice and suggestion of method within ***DM*** posts ***Plea2*** includes working implying has seen solution and adds thanks	TE LRP LRO **LRC** LRU TB TRMA TRSM LRW LRU SPP

Post Number Day/time	Post précis	Commentary Mathematics focused	Commentary Actions focused	Code
P16 Wed 6.02pm to P19 Thurs 4.55pm	***Plea2*** returns with another statement on a related problem based on $(2^n- 1)$: *Can someone show me … book just gives an example and no justification. I know that it does, but someone show me how to show it generally (like the one above).*	See Appendix 9.6 for mathematics involved.	One week later. Wanting to understand why the statement is true, not just accepting of it	TE LRP LRC LRU
	HelpC states a formula triggers two further messages*:* ***DM*** stressing that is not just for powers of 2 and the formula worth remembering; ***HelpD*** suggests generalising a^n-b^n	None of the suggestions are trivial; all three suggestions shift focus to memory in order to apply a useful fact to achieve a solution	***HelpC*** veteran poster still at school. Problematic to judge whether necessary for further study and/or balancing instrumental/relational understanding. But all help given has the potential to aid ***Plea2***'s'mathematical knowledge	TRRR TRMA TRSM TRAM
Plea3 making only their 9th post joins the thread				
P20 Sunday 4.52pm	***Plea3****: I'm in year 9* ☺ offers their own solution/method to ***Plea2*** first justification request (P11 above).	Solution holds a misconception (or misunderstanding) and a numerical coincidence [see Appendix 9.6]	Sunday afternoon ☺ – humour but with possible one-upmanship, 'cleverness' as one year younger ***Plea2***	TM **LRJ** SPT **SPB**
	…. However, after all this, I still don't understand how a number N… [gives own thoughts] … *if I am not mistaken …or then again I may be wrong. Can you please justify … .*	Problem in understanding ***Plea3***'s mathematical text (even though there is an error) as normal typing cannot produce the precise mathematical notation	Alternative given but clearly states not understanding, shows own thoughts and wants justification ('*Not mistaken*', '*may be wrong*' although open with current thoughts possible suggestion of actually being correct)	TRAM LRW LRO LRU LRC

Post Number Day/time	Post précis	Commentary Mathematics focused	Commentary Actions focused	Code
P21 Sunday 5.24pm to P24 Sunday 8.46pm	***HelpD*** provides example of correct notation ***DM*** Gives advice on how to mark-up mathematical text to appear ‘normal’ on the board Also gives very full explanation with numerical example to misunderstanding includes adding and answers own question: *why does m have to be more than 1? Does that help? Do post back if you’re still confused* Another Veteran Poster points out ***DM*** numerical slip: *24=2^3x3☺ ...* … to which ***DM*** replies: *Yes, all right, fair enough. Hopefully [**Plea3**] will understand what I was trying to say despite that. (Curiously, I thought that something was a bit odd when I wrote it, but still didn't spot it!)*	Example contains numerical slip/ howler /‘silly mistake’ writing 24=2^2x3 [NB This episode is revisited in Chapters Eleven]	Sunday Implicitly signal error ***DM*** Gives advice on how mark-up mathematical text to appear ‘normal’ on the board Extending the explanation but making error Invitation to ask for more help A light hearted exchange with an impression of ‘frisson’.	TM TRSE TA TRMA TRRR SPC SPB SPB

<table>
<tr><td>Post Number Day/ time</td><td>Post précis / commentary</td><td>Code</td></tr>
<tr><td colspan="3">The remaining messages settle down to a discussion started by <i>Plea2</i> expanding an expression previously suggested by <i>HelpD</i>. Here the exchanges become ‘messy’ and difficult to report here in order. The annotations on the thread [Appendix 9.6] provide a commentary of the connections, the mathematics involved and a judgement on the quality of help being offered. The main points are listed below with appropriate codes.</td></tr>
<tr><td>P25
to
P32</td><td>Seven days after last posting and two days after the <i>DM</i>’s previous post, <i>Plea2</i> returns and asks: <i>Does $x^{ab}-y^{ab}$ factorise to…</i> [gives suggestion with error of sign in first bracket that they correct], <i>HelpD</i> sends three messages (P26-28), first: <i>on right lines though</i> two minutes apart correcting twice, <i>whoops,</i> a factorisation that is related to <i>Plea2’s</i> attempt. 15 minutes later <i>Plea2</i> corrects their own post and one minute after <i>Plea3</i> suggests to <i>HelpD</i> the possible correct factorisation that <i>HelpD</i> had given and in doing so <i>Plea3</i> corrects own error (made in P20). Another minute later, two minutes since <i>Plea2</i> last posted, they start: <i>nah… forget that</i> (P32) suggesting second thoughts on what they were about to write down.</td><td>LRA
LRP
TE
SPC
TRRR
LRW
SPP
SPH</td></tr>
</table>

Post Number Day/ time	Post précis / commentary	Code
P33	It is not clear as to which of ***Plea2/3*** or both ***HelpD*** is responding to with: *nope nothing quite there yet, try doing some simple examples and spot a pattern. May I suggest powers of 3 and higher before trying to generalise .* 'Pattern spotting' needs to be treated with caution The difference here is that by seeing how the powers in terms appear by trying a few examples, understanding will be built up to write the factorising by detailing the first few terms and then … to last term or two.	TRRR TRMA
P34 to P37	***Plea3****'s* attempt is not correct even though they comment: *I have tried this for a few values of a and b and they work fine.* ***HelpA*** *,* returning to the thread after one month and some 25 messages later, spots one minor error of sign in first short bracket (***Plea3*** corrects this) but does not comment on the main second incorrect bracket. ***HelpA****'s 'nearly'* offers encouragement. ***HelpE*** arrives with own (correct) suggestion mirroring ***Plea2****'s* (P25) efforts earlier: *shouldn't it be more like this?*	LRA LRP TRRR SPC TRDE
P38 & P39	***DM*** suggests a useful technique to simplify notation, which ***Plea3*** uses correctly but fails to correct errors in second bracket.	TRMA LRA
P40	***Plea2*** returns four days after completing the problem to politely suggest to ***HelpD*** that: *I believe your correction in post no 82 is partly incorrect* and explains why they think this. In the same message ***Plea2*** comments directly to ***Plea3****: I'm sure your factorisation is wrong. It should be* [gives correct form]. *I'm with [**HelpE**] on this one.*	TE SPP TRSE SPC
P41	***Plea2*** then sends another post 8 minutes later (just before midnight) thanking ***DM****: yours was a very helpful hint which made the problem break down much more quickly in this factorisation mess*…. Factorisation mess seems an appropriate (and humane) description for the intricacies and difficulties of getting both the algebraic manipulation and notation correct.	SPP SPH
P42	Finally two days later ***Plea3*** sends a final message which suggests that they now agree: *Thanks I have realised that my equation only works for when a=1, hence my misunderstanding.* ☺ Interpreting ☺ is problematical: it may for example indicate laughing at own 'stupidity' or relief that the problem is now finally sorted. Nonetheless it conveys a 'happy' banter type exchange	SPP LRO LRP LRU SPB

9.3 Explanation of Features

This section explains the features accompanied by their respective code index grouped according to the four themes resulting from the interpretative analysis of threads. Two of the four themes concern specific educational aspects related to teaching and learning interactions whose presence is inferred through the interpretation of message content. Although it can, for example, be inferred that something has been learnt or understood, no conclusion can be drawn as to the *degree* to which it has been learnt or understood. Neither can it necessarily be assumed that a teaching strategy adopted was a known pedagogical intention of the helper. For these reasons, these two themes have been labelled as *Features in a Learning Role* and *Features in a Teaching Role*, rather than simply 'Learning' and 'Teaching'. A third theme *Social and Personal* is again the result of interactions but these could be broadly termed non-educational and non-subject specific. The fourth theme, *Temporal*, relates to the medium / web-board structure in which the interactions and the subject study can take place.

9.3.1 Theme One: Features in a Learning Role [Prefix LR]

Table 9.4 [next page] presents the nine features assigned in this theme. The Posting Protocols expect that the person seeking help (the learner) will share their thoughts and include current work on the problem with those offering help. Thus the presence of '**openness of current difficulties**' [LRO] and '**showing working**' [LRW] should permeate throughout any thread. In continuing with a problem as far as seeking help in the first place, some degree of persevering must already be present, but '**perseverance**' [LRP] is further exhibited within the thread by staying in the thread and continuing with the problem until sufficient help had been given. Two features, **'seeking re-assurance'** *that a solution/selected method or presented idea is correct* [LRA], and *seeking whether there is a* **'better (alternative) solution'** *than own obtained* [LRB] are two consequences of knowing that help is at hand. The open access of the board automatically presents opportunities for **'joining in to find a solution'** *to the problem that someone else had initiated* [LRJ]. Participation in AskNRICH provides the opportunities to see others engaging in mathematics. Two features commonplace in such engagement are: *following a hunch or*

'intuition' f*or a solution or path taken being correct/wrong* [LRI] and the importance of having a rigorous proof, thus *seeking aspects that* **'constitute a proof'** [LRC].

LR – Features in a Learning Role		Explanation and/or Examples from the data
LRA	Seeking **assurance** whether a solution/chosen method/idea is correct	Posters can have partially-formed or tentative solution *'Does* x^{ab} *-* y^{ab} *factorise to: ....'* **ExThd2**
LRB	Seeking whether there is a **better** (alternative) **solution** than own obtained	Similar to but as an alternative to LRA, a poster can wonder whether there other solutions available *'i have got that 3,5,2 works but how do i prove that this is the only answer or find other answers? have i done this correctly? is there a nicer way of solving it?'* [also LRA,LRC] **CS-P363**
LRC	Seeking aspects that **constitute a proof**	Due to a common usage of AskNRICH as a means to discuss mathematics competitions some posts will be querying aspects of proof, either whether their own attempt is a proof or parts (present or missing) of a written proof *'Can someone show me why* $(2^n- 1)$ *contains a factor* $2^m- 1$ *where* $n= 2^s*m$ *and m is an odd integer. The book just gives an example and no justification. I know that it does, but someone show me how to show it generally'.* [also LRU] **ExThd2**
LRI	Feeling or **intuition** for solution or path taken being correct/wrong	The web-board provides the opportunity for tentatively expressing thoughts or feelings as to what the solution will be, rather than providing a solution straightaway *'.... Do you think the converse is true?'* Response to **CS-P125** and **3Thd1**
LRJ	**Joining in to find a solution** to the problem that someone else had initiated	The open nature allows anyone to join the search for a solution, working together *'I am doing the same .... I still don't understand how ...'* **ExThd2**
LRO	**Openness of current difficulties**	As with LRW, sharing thoughts is required by the posting protocols *'I know the answers but I can't work out how to get them. Any help is greatly appreciated'.* **ExThd1**
LRP	**Perseverance**	Continued engagement with a problem *'I'm not sure what happened ... THESE AREN'T THE RIGHT ANSWERS. Thanks in advance'.* [also LRO] **ExThd1**
LRU	**Developing signs of** (deep/relational) **understanding**	Evidence within the text that implies a desire to understand or the learner has perceived they understand *'Thank you. I really understand 2 now. Number 1 is coming to me too ... taking a little more time'.* [also LRP] **ExThd1**
LRW	**Showing working**	As with LRO, including current work is required by the posting protocols *'I got as far as this ……. and it didn't factorise'.* **ExThd1**

Table 9.4 Theme One: Features in a Learning Role

Instances within posts where any of these eight features occur will be explicit within the text. The same cannot be claimed for interpreting content of text to measure the internal process of understanding. Nevertheless, anyone using AskNRICH knowing that no direct solution will be given must automatically be seeking some degree of understanding to a problem that is more than just being told the answer. Although it might be inferred that the type of understanding sought is beyond a superficial instrumental understanding, it is not always possible to ascertain whether the way that the problem is worked through is more than using a rote technique. Nonetheless, the feature **'developing signs of** *(deep/relational)* **understanding'** [LRU] has been assigned to parts of the text that revealed a desire to understand, or the learner perceived that they now understood – so for example where working presented *showed* understanding, or there were statements such as 'Got it!' or along the line of 'I understand now'.

9.3.2 Theme Two: Features in a Teaching Role [Prefix TR]

This theme was assigned ten features as depicted in Table 9.5 [next page]. Five are used to define different teaching strategies, all of which should have been a result of the Posting Protocol expectation that only hints and explanations should be given that would help the person asking for help to understand. Four of the strategies employed: *a* **'worked solution to a different example'** [TREG]; **'anticipating difficulties'** [TRAD], *providing* **'specific method'** *to adopt* [TRSM] and **'alternative methods offered'** [TRAM] have names that are self-explanatory. **'signalling error'** [TRSE] relates to instances where the helper indicates errors either in the working presented or where the learner is showing a misconception. Although the Posting Protocols ask helpers not to provide the solution, there are instances of explicitly providing '**direct explanation /working through the problem'** [TRDE]. Providing **'mathematical advice'** [TRMA] relates either to some aspect of a particular mathematical problem or on the process of working mathematically. Both cases provide the opportunity for tools to be added to the mathematician's toolbox [see Section 11.5.1 p258]. The feature **'open discussion'** [TROD] refers to general exchanges that remained mathematics focused e.g. discussion on a particular textbook or area preferences. Anyone can offer help, not necessarily correct and/or limited in what it will achieve; and at any time, which might not always in the most logical sequence: hence the features **'restricting response'** [TRRR] and **'overlapping help'** [TROH] respectively. These two latter features are a consequence of the open access, asynchronous nature of AskNRICH, though they are not necessarily disadvantageous as the labels might seem to imply.

TR – Features in a Teaching Role		Explanation and/or Examples from the data
TRAD	**Anticipating difficulties** in the current problem	A teaching strategy that anticipates a common viewed difficulty [or a known misconception] and seeks to highlight within the help offered *'For 2, in a few steps you have divided/multiplied by x, which means that you have to check the case x=0 extra'*. **ExThd1**
TRAM	**Alternative methods offered**	Offering a different way in which the problem could be solved *'Another method then the one given above: to find that 2^m-1 is in the form 4k+3, just factorise it over four, to give: …'* **ExThd2**
TRDE	**Direct explanation/working through the problem**	Although 'against' the posting protocols advice not to give solutions there can be occasions where it might be appropriate to directly work through the solution to the problem or spell out relevant facts *'For 2 we consider the following cases:'[and then gives full solution]* **ExThd1**
TREG	**A worked solution to a different example**	A teaching strategy that allows a poster to adapt the solution given to a different problem to the one that they are attempting to solve *'Basically, find y in terms of x (or vice versa) … Here's an example… See if you can do it for yours now.'* [also TRSM] **ExThd1**
TRMA	**Mathematical Advice**	Instances where advice is given either for a particular mathematical problem or a process '*I know what you mean though about not even thinking along the right lines. Often it's tempting when faced with a solution to think "Wow, I never would've thought of that", but it's best not to think in that way. Instead…'* [also SPC] **ExThd2**
TROD	**Open Discussion**	General exchanges that remained mathematics focused '*Out of interest, how do you find the inequality problems, and if you have the geometry book, how do you find those problems?'* [also SPC] **ExThd2**
TROH	**Overlapping help**	Different helpers involved, focusing on different aspects. In **ExThd1**, (i) different worked example, (ii) direct explanation and (iii) to highlight original error, suggestion to expand two different expressions
TRRR	**Restricting Response**	Inherently limited help *For 2 we consider the following cases:…'* [also TRDE] **ExThd1**
TRSE	**Signalling error**	Instances where the helper has spotted the error or misconception *'you've just made a mistake in expanding the expression… '***ExThd1**
TRSM	Providing **specific method** to adopt	Informing the poster of a specific method, even if there are alternatives) to adopt *'The method you want to use here is substitution'* **ExThd1** [although in this case a graphical method would be a viable alternative method to adopt]

Table 9.5 Theme Two: Features in a Teaching Role

9.3.3 Theme Three: Social and Personal [Prefix SP]

Table 9.6 shows the six features allocated to this theme. Two distinguish **‘banter’** [SPB], where there is obvious humour but delivered with light-hearted teasing, from **‘humour’** [SPH] the genuine neutral witty remark. The Posting Protocols expect respect to be a pervasive feature of AskNRICH. The feature **‘politeness’** [SPP] is used where the text explicitly shows what would be considered good manners. However, asking for politeness and respect does not automatically engender a sense of care, thus an additional feature **‘care for others’** [SPC] is used to indicate for example kindness to, empathy with, or nurturing of, other AskNRICHers. **‘non-mathematics talk’** [SPT] is self explanatory. **‘opinion’** [SPO], is reserved for critical comments/judgements, whether about mathematics or not. The latter two features are far more prevalent in the NRICHtalk section on the private part of the web-board. Indeed this was the purpose for which NRICHtalk was set up.

SP – Social and Personal		Explanation and/or Examples from the data
SPB	**Banter**	Light-hearted teasing In **ExThd2**, a numerical slip is pointed out: *‘$24=2^3x3$* ☺’ receiving the reply: ‘*Yes, all right, fair enough’*
SPC	**Care for others**	Showing consideration *‘See if you can do it for yours now. If you can’t, post your working and we can see where you’ve gone wrong … ’* **ExThd1**
SPH	**Humour**	Distinguished from SPB as the genuine neutral witty remark *‘factorisation mess….’* [also SPP] **ExThd2**
SPO	**Opinion**	Personal, critical judgments *‘but if they are stretching you that's always a good thing because unfortunately i doubt Alevel will or does’* **ExThd2**
SPP	**Politeness**	Good manners *‘Any help is greatly appreciated. … Thanks in advance*’ **ExThd1**
SPT	**Non-mathematics talk**	Useful information but not strictly mathematical *‘By the way I am in year 10’* **ExThd2**

Table 9.6 Theme Three: Social and Personal

9.3.4 Theme Four: Temporal Aspects [Prefix T]

Four features potentially present in every thread and pervasive throughout AskNRICH were allocated in this theme [see Table 9.7 below]. Although these features are common to CMCs in general [Henri 1992, Rennie & Mason 2004], nevertheless it remains important to include these when building the characterisation of AskNRICH. An inherent facet of AskNRICH that has a major liberating effect is the removal of time boundaries, captured by two features: **‘mathematical study present beyond the school day’** [TM] and **‘working on a mathematical problem sustained over an extended length of time’** [TE] where the duration of time spent working on a problem spreads over a longer uninterrupted period. Where the time gap between posts, for example noticeable speed or a long measured reply posted within a short time, is significant to its interpretation, the feature **‘time between responses’** [TB] is assigned. The feature **‘significant influence of asynchronous communication’** [TA] is used for various specific instances where the medium of AskNRICH as a web-board is interpreted to have an effect on the thread, for example multi-helpers simultaneously posting or any new poster instantly offering help.

T – Temporal		Explanation and/or Examples from the data
TA	**Significant influence of asynchronous communication**	Incidences of ‘technical’ effects. For example in **ExThd1** three helpers have become involved within the first seven messages [also TROH]
TB	**Time between responses**	When the speed/time between Posts is deemed worthy of note. For example in **ExThd1** the detailed worked-through example matching the structure of the original problem arriving within three quarters of an hour of ***Plea1*** requesting help
TE	**Working on a mathematical problem sustained over an extended length of time**	For example: the four hours that ***Plea1*** in **ExThd1** spent on a Friday evening ‘*I really understand 2 now. Number 1 is coming to me too …*’
TM	**Mathematical teaching present beyond the confines of the school day**	Posts made to help a learner, posted outside of the normal school day, evidence by posting day/time

Table 9.7 Theme Four: Temporal Aspects

This section has provided an explanation and illustration of the 29 features with the derived code index presented alongside. The next section is a discussion on the general practices revealed by these features and themes.

9.4 Discussion

To demonstrate how the findings above will contribute to the later overall characterisation of AskNRICH, this section discusses the common practices in terms of teaching and learning interactions under three headings: the medium of AskNRICH in which these interactions take place; the conversational tone of the interactions (a precursor to an in-depth consideration of conversation-for-education in Chapter Eleven), and Socratic-Style Dialogue and Scaffolding, taking place within the interactions. Although illustrative examples are taken predominately from the two exemplar threads, occasionally additional material from other threads is included.

9.4.1 Medium

Some key features of AskNRICH important to this study are inherently due to the asynchronous, temporal nature of the web-board. Being freed both from the finite time limits of a school lesson and from the confinement of accessing 'teacher' help only within school opening hours is crucial to enabling the AskNRICHers to pursue their studies. For example **ExThd1** was started on a Friday evening at a time when the most likely next school contact would be Monday. ***Plea1*** clearly wanted to solve the problems there and then and has help arriving within the hour. The time interval before help arrives is reliant on a sequence of three events: someone prepared/able to answer has to log on and read the message; the necessary help needs to be compiled, and thirdly, the help post composed and sent. The interval between posts in this thread is short and this is typical of AskNRICH. In this instance the first nine posts span four hours (on a Friday evening no less) and, after the first reply, there is a flurry of posts to-and-fro, for example three helping posts from two contributors (in response to and being responded to by ***Plea1***) arrive within a time span of 18 minutes. This is followed by other flurries punctuated by longer periods of quiet (in this case overnight and into the next day) until all is resolved to everyone's satisfaction.

In AskNRICH postings are made on all days of the week and at all times of the day and night, albeit predominately out of school hours [further evidenced in Chapter Ten]. Indeed, exchanges near the beginning of **ExThd2** are taking place during the Christmas vacation, including Christmas Eve.

The apparent amount of time that the person asking for help is prepared to work on trying to find the solutions can be substantial, as both these thread show. In **ExThd1**, ***Plea1*** was involved for over four hours on the Friday evening, making a final post at the relatively late hour of 11.33pm. Although it is not possible to know what other things ***Plea1*** might have been doing during this time, the number of postings and the work that ***Plea1*** had needed to do in order to make the next post implies that a substantial proportion was given to working on the problem. Furthermore, ***Plea1*** has 'stuck' at the problem for quite some time. Full resolution on ***Plea1****'s* part is early Saturday afternoon, well before the next school-day. The working-outside-of-'normal'-hours and the speed at which help can be offered are natural aids to 'perseverance'.

In addition to all of the above, the asynchronous nature provides time for reflection, the "slow-down time" [Kyriacou & Issett 2008: 10] or the "Start-Stop-Go" [Tanner & Jones 2000b: 29] sequence advocated for classroom practices but, as the authors infer, not always observed. Thus the presence of metacognitive knowledge and skills [discussed further in Section 9.4.3 below] can flourish naturally within the environment of AskNRICH.

As threads can involve a number of individuals deciding to participate, posts can become 'entangled' and the sequence of posts appearance might result in a 'jumbling up' of help [see **ExThd1-P2**, **P5**, **P7**, **ExThd2-P25** to **P42**] as envisaged by Posting Protocol 5 [see Table 8.2 p170]. In **ExThd1**, at around 10pm there are three posters involved concurrently, ***Plea1***, ***Help2***, and the ***Moderator***. Posts are coming in quick succession and there is some inevitable asynchronous overlap in the posts. Although the posts appear in a linear time sequence the relevance of message may not necessarily follow this simple timetable [Chapter Eleven addresses this in depth]. In addition anyone can make a post that offers help. In this thread, as evidenced by no comments to the contrary, ***Plea1*** appears to be unfazed by the number and focus of the helpers and any overlapping of posts. Indeed, when a participant offers help for a question now solved or serendipitously an alternative method, these can be compared against the original for elegance, accessibility etc.. In the case of **ExThd1-P7**, the ***Moderator*** is taking ***Plea1*** back to look at the original error some time after strategies have been offered. However, there are no later posts indicating whether ***Plea1*** did so.

Although the help given can usually be considered of good quality, it is totally reliant on or restricted by the person offering it (i.e. it might not be universally excellent or correct). Obviously the methods proposed for solving the problems also depend on the people posting and the experiences they have had in solving similar problems in the past. Thus, for example in **ExThd1-P2** ***Help1*** provides an algebraic method that is continued throughout the thread although a graphical method of solution would be possible. In other words the help offered may not be all embracing in terms of methods available. Moreover, although it might be easy to criticise the quality of some of the help posts in **ExThd2** (e.g. **P2,17,27,28,33**), consideration of the quality of help offered across AskNRICH generally demonstrates that this would be unfair. Overall, the whole system is sufficiently robust to overcome any difficulties, any errors will be politely corrected by other posters, or even, in the last resort, by the ***Moderator***.

9.4.2 Conversational Tone

The word conversation takes on a specific meaning as further analysis of AskNRICH reveals [see Chapter Eleven], but the tone of the 'talk' discussed in this chapter illustrates the varied practices of AskNRICHers, in part revealing their 'human' side.

ExThd2 was specifically selected for the quality of advice that one peer gives to another [see for example **P3**] in a situation where the giver can feel an empathy with the receiver, having been in that position only a short while earlier. Although a teacher might offer the same advice, in that instance a distance (power relationship) would inevitably be present and thus the empathy reduced or lost.

HelpA's first response [**ExThd2-P3**] directly back to ***Plea2***'s question *'how much prior knowledge is assumed'* [**ExThd2-P1**] opens the discussion on the difficulty, or not, of the book. Later in the post ***HelpA***'s comment *'Out of interest, [other topics] ... how do you find those problems?'* has 'opened up' the conversation to include more than the original.

The two examples above have a mathematics focus, the next one carries with it an added critical, personal opinion on the state of school mathematics *'if they are stretching you that's always a good thing because unfortunately i doubt Alevel will or does ...'* [**ExThd2-P7**]. As would be expected the more critical comments are generally made within the private part of the board [NRICHtalk], but even there they are always delivered with politeness if not without some understandable frustration.

Other talk is ‘looser’, for example: *‘I am doing the same section of the book’* and *‘By the way I am in year 10’* [**ExThd2-P4**]. These are essentially ‘snippets’ that one might find out about the life of someone within the open access areas of the board, perfectly illustrated by Peter’s explanation [next chapter] as to why he had not replied for a while as *‘the family had been burgled and the computer stolen’* [**CS-P126**].

The Posting Protocols counsel careful consideration of the use of humour [Table 8.2 p170]. Similar care needs to be taken on interpreting whether what appears as a humourous remark (often noticed by the addition of ☺) was intended as such. Given that many of the AskNRICHers ‘compete’ in National Competitions there is always the possibility that some comments are delivered with a natural arrogance. There is, however, no compelling evidence for this in the content of the many thousands of posts read in this study. Moreover, certainly within the more personal private posting part of the board, everyone seems supportive of one another whether the competition scores are high or low. Nevertheless, the features incorporate a distinction between banter, where there is obvious humour but perhaps delivered with some frisson and/or light-hearted teasing, and humour which is reserved for the genuine neutral witty remark. [See also Chapter Eleven].

In giving a clear explanation in **ExThd2-P22**, ***DM*** writes (incorrectly) ‘$24=2^2x3$’ which by way of banter receives the response ‘$24 = 2^3x3$ ☺’. One can imagine people ‘laughing’ at this in a light-hearted way, no-one can seriously believe that ***DM*** has made a real error but people find fun in pointing out such ‘howlers’. This can be slightly annoying for the person who has made the error and the only course of action is to take it ‘with good grace’: *‘Yes, all right, fair enough. Hopefully [**Plea2**] will understand what I was trying to say despite that. (Curiously, I thought that something was a bit odd when I wrote it, but still didn't spot it!)’*. ***Plea3***’s first post in **ExThd2** ending with *‘and I’m in year 9 ☺’* [**P20**] is a further example of banter, implying being ‘better’ as they are one year younger. These examples are different to genuine (bringing a smile to one’s face) humour. Reading *‘yours was a very helpful hint which made the problem break down much more quickly in this factorisation mess’* [***Plea2*** in

ExThd2-P41] one can imagine how much of a mess, metaphorically, there had been in working through the problem. Indeed reading the comment for the first time made me laugh-out-loud or as the AskNRICHers write 'lol' [See Chapter Eleven later]. The post: '*Now ive got the first one im motoring through the exercises. who would have thought trigonometry could be this much fun'* used in the opening pages to this thesis is indicative of AskNRICHer's humour.

9.4.3 Socratic-Style Dialogue and Scaffolding

LRIII set out the understanding of the terms Socratic-*Style* Dialogue and Scaffolding [see Sections 7.5.1 and 7.5.2 respectively, pp151-154] used for this study. The Posting Protocols' entreaty to avoid simply giving a solution encourages AskNRICHers to find teaching strategies other than teacher exposition. Moreover, within AskNRICH, given that anyone can choose to offer help, there is a difference from the classroom situation where essentially just one teacher with their own way of working is available. The classroom teacher may offer different ways of understanding and solving a problem and other classmates may try to help. However, for AskNRICHers, the only means of proceeding is being helped by peers that brings the distinct advantage that multiple helpers may bring multiple strategies and perspectives to understanding and aiding solution. The dominant teaching and learning strategies both invoke the questioning stance of a Socratic dialogue as a means for helpers to scaffold the learner's learning, with the aim that, once the problem has been completed, the learner is in a position, the next time such a problem arises, to undertake the work with less or no help.

Help1's reply [**ExThd1-P2**] in providing a relevant, worked through, related example could be seen as scaffolding ***Plea1***. Finding an example was not necessarily trivial, as it required integer solutions i.e. the quadratic equation that will factorise. As previously mentioned [Section 9.4.1 p204], by providing this example, ***Help1*** has by implication suggested *the* method required, though incidentally in this instance although it is probably the most common it is not the only method that could be used. [In **ExThd2**, ***Plea3*** offers an alternative method [**P20**] to the one ***Plea2*** shared (though incomplete)]. Whether it was just fortuitous or not, ***Plea1***'s error [**ExThd1-P3**] sets up a cubic equation for which it looks possible that each term can be divided by x and the equation reduced to a quadratic. The correct equation will allow a similar division and ***Help2*** is anticipating a universal common error (misconception) [Swan 2001] of doing the division in both these circumstances and forgetting that the equation would also be true if x=0. In **ExThd1-P5**, ***Help2*** a brand new poster, offers help towards the solution by signalling the errors. The Posting Protocols' advice to show working ensures that signalling errors is a common way of offering help, but simply pointing out the error is not the only means. Later in the **ExThd1**, for example, the AskNRICH ***Moderator*** sets out two expressions to be expanded [**ExThd1-P7**] that, akin to Socratic dialogue, provides ***Plea1*** with the opportunity to realise their algebraic error.

The use of teaching strategies that offer hints, nudges and advice, that is various degrees of funnelling and focussing [Wood 1996], by necessity help a learner to progress both in the present and providing the means to attempt similar questions, unaided, in the future. That is, the scaffolding help offered enables the learner to move forward within their Zone of Proximal Development [Vygotsky 1978]. The post *'What do you get if you multiply together two numbers of the form 4k+3 (call them 4k+3 and 4l+3)? What form does it take? What if you multiply together four such numbers. Or six?'* [**ExThd2-P12**], scaffolds ***Plea2****'*s learning. These direct questions offer an idea and suggest a way forward, initially *prompting* questions, specifying/telling (*funnelling*) the format of multiplying the two numbers and other even number of terms, reminiscent of a step-by-step scaffold [Bliss, Askew & Macrae 1996]. However the problem itself requires working with an odd number of terms of the form 4k+3 (not dealing with 2, 4 or 6 terms) so ***Plea2*** is required to make their own connection and adapt the number of terms to be multiplied to solve the problem; eventually the *probing* questioning becomes a means of *focussing* on structure. ***Plea2****'*s next reply [**ExThd2-P13**] indicates that they have been able to complete the problem and *'I think I've got it'* indicates that next time they should be able to do so unaided. Thus the support offered is then *faded* [van de Pol et al. 2010] and the *knowledge transferred* to the learner within a *responsive/contingent, discussion* [see Section 7.5.2 p153]. Just within this one thread, **ExThd-P22**, **P33** and **P38** contain further examples where the nudge is sufficient for ideas to be taken on board and used in future. Furthermore, anyone just reading some of the posts on AskNRICH (lurking) and doing the mathematics that others are sharing/doing, is presented with a scaffold that can lead to future unaided work [see description of Julia in Section 8.4 p171].

Despite the Posting Protocols' stance on not simply providing the answer, such instances do occur. For example, ***Help2*** [**ExThd1-P8**] gives the final part of the solution. It is conceivable that, following ***Plea1***'s posts, ***Help2*** made the judgement that ***Plea1*** is not familiar with taking special care with x=0 and this could be understood with the direct explanation of a worked solution. Thus there can be some acceptable reasons for such 'telling'. *'I really understand 2 now'* [**ExThd1-P9**] suggests that ***Plea1***'s learning has implicitly been scaffolded by this direct intervention. However, there is simply no way of knowing whether ***Plea1*** will now be able to do such problems unaided. **Peter**'s direct help with modular arithmetic to R [**3Thd2** Chapter Eleven] provides a similar example.

LRIII has discussed the role of metacognitive knowledge and skills in achieving effective scaffolded learning. Features resulting from coding demonstrates the presence of such knowledge and skills in the AskNRICHers, as evidenced by for example (i) taking responsibility for, and persevering in [LPR], own learning; (ii) the desire to understand[2] [LRU] the mathematics involved (iii) pursuing the notion of proof [LRC], and
(iv) discussing the quality of solutions [LRB].

The teaching and learning roles features imply the AskNRICHers' metacognition leads them to be reflective both *in-action* [Schön 1983] and, aided by the asynchronous nature of the web-board [discussed above], at the higher level of *on-action* [Schön 1991]. The AskNRICHers awareness and reflection may also be seen both in terms of the dynamic and reflective scaffolder teachers of Tanner and Jones' [2000b] study and Anghileri's [2006] theorising of scaffolding in terms of a transference of a teacher's reviewing and restructuring to the learner.

Overall, the help offered by AskNRICHers to their peers must be considered to be impressive. The Posting Protocols were designed to encourage strategies that evoke Socratic dialogue and scaffolding and the analysis of AskNRICH clearly shows that they are being used and, furthermore, that AskNRICHers' metacognition means that they have both the propensity and the capability of interacting with each other in such a way. The interactions are as pedagogically sound as they are because the AskNRICHers have the ability to reflect

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[2] See LRI Section 2.7 pp51-52 for definitions of mathematical understanding.

on their own learning and, it might be inferred, wish to offer strategies based on how they prefer to learn and perhaps be taught [see Afterword at the end of this chapter].

9.5 Features Summary 2

The Features Catalogue for this chapter, relating to Teaching Interactions and Learning Interactions, is presented in Figure 9.1.

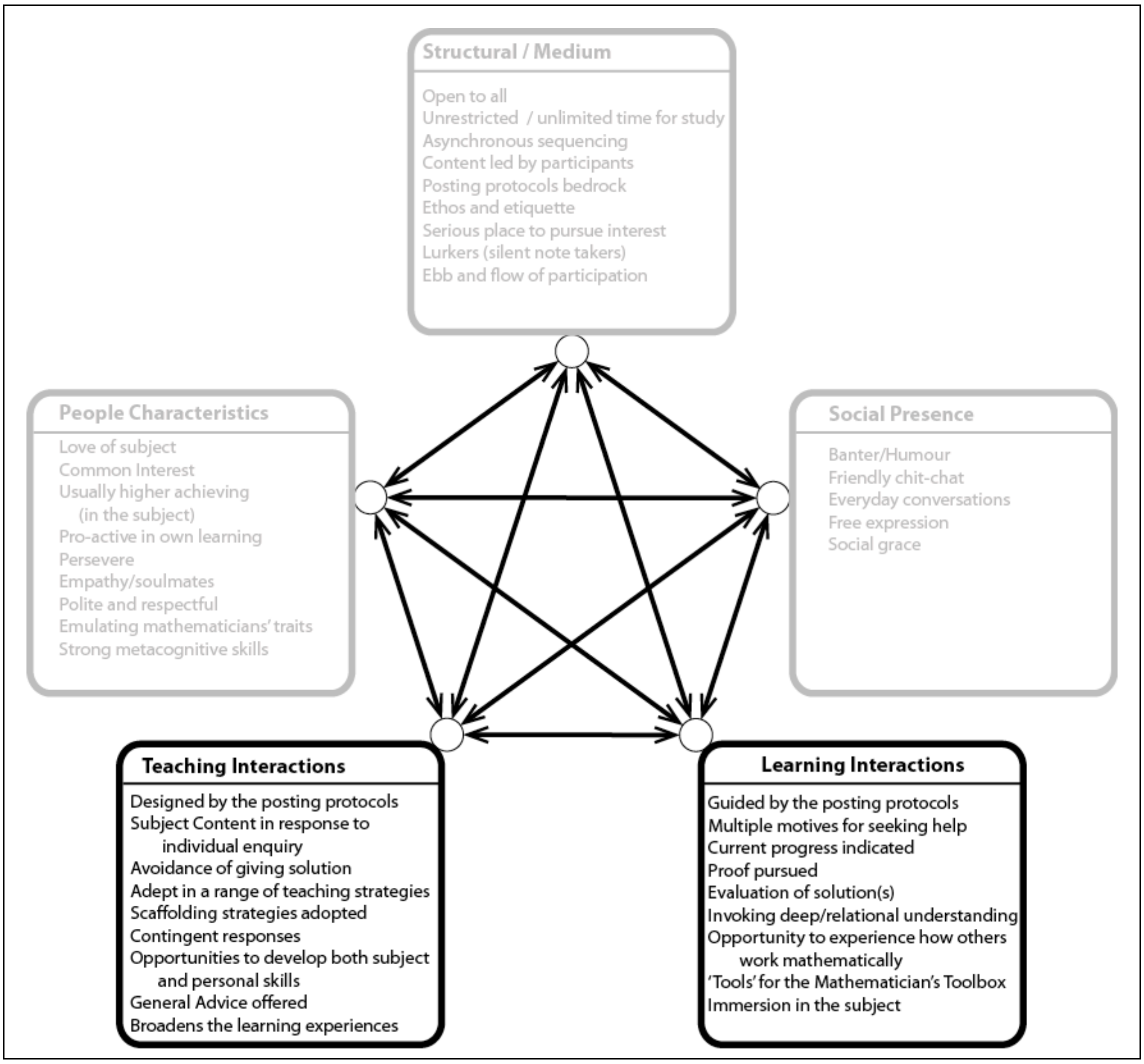

Figure 9.1 Features Catalogue: Teaching Interactions and Learning Interactions

9.6 Conclusions

This chapter has focussed on practices that are 'general' within AskNRICH using the Perspective of Two Exemplar Threads, although as the analysis reveals these practices might well be considered 'remarkable' in the world at large. An overview of the topic and content of the two threads has been presented, accompanied by the outcomes of applying open coding to the interpretive commentaries constructed from the posts. The features resulting from the coding have been explained in detail under the four themes of: teaching, learning, social and temporal. The general practices exposed by this examination of findings are then discussed, in terms of teaching and learning interactions, in three sections: medium; conversational tone, and Socratic-Style Dialogue and Scaffolding.

The asynchronous nature of the web-board ensures freedom from the constraints of finite lesson time to pursue study outside-of-normal-hours. This enables an individual, within the home environment, both to persevere (and be supported) for an extended period of time and to pursue challenging problems. The nature of a web-board also inherently allows time for reflection at any stage before, during and after interactions. AskNRICH provides opportunities for encountering like-minded peers who, in that moment at least, live and breathe the subject. The consistently high-quality exchanges between equal peers, with evident absence of power relationships, are characterised by a conversational tone of respect and consideration, interspersed with a scattering of witty remarks. The Posting Protocols not only form the foundations for such well-mannered conduct but, in prescribing the way that help should be asked for and provided, foster a Socratic-Style Dialogue. The AskNRICHers', albeit untrained, pedagogical skills are shaped into implementing teaching strategies that scaffold others' proactive, reflective and receptive learning.

This chapter has presented a first Perspective on how young people are using the Internet in their proactive, independent pursuit of mathematical studies beyond the confines of the classroom. The next chapter continues the exploration of AskNRICH by tracking one representative participant as a case study over an eighteen-month intensive period of posting.

Afterword

The AskNRICHers' commitment to the ethos of the Socratic style of interaction and their self-moderation in maintaining the Posting Protocols and seemingly unconscious pedagogy is aptly encapsulated in the following post:

> *the purpose of posting on this board isn't to give you an opportunity to do whole questions and deny the poster the right to do it themselves. A response like mine, which perhaps nudges the poster into solving the problem, is probably more useful to them than a post telling them exactly how to solve the problem (from which they learn basically nothing). I don't like to nag, but in posting a response like the above you were inviting the (hopefully constructive) criticism*
>
> *[Posted October 2008, co-incidentally the poster of* ***ExThd2-P17****]*

Chapter Ten

Doing Mathematics in Local Isolation
Perspective Two: Case Study of an AskNRICHer

> *I love maths that makes me think and being able to go into a world inside of my mind and then the feeling of satisfaction when I have solved a problem I have been trying for ages.*
>
> *[Peter aged 16, email communication]*

10.1 Introduction

In the previous chapter the common practices and use made of the web-board by the AskNRICHers were determined using the Perspective of analysing two exemplar threads. This chapter is the second of three Perspectives reporting the findings of interpretative analyses of a selection of threads. This Perspective is a single case study of one representative user, Peter, through the 151 retrievable threads in which he participated over an eighteen-month period. The analyses of Peter's interactions are based on an in-depth examination of the 1875 posts in those threads, including the 484 that he made. AskNRICHers like Peter work in isolation, confined to a local environment and thus generally unable to physically meet with peers with similar ability and enthusiasm for the subject. As the concluding remarks of Chapter Eight explain [pp181-182], the AskNRICHers find, remotely, like-minded others that they can engage with and enjoy rich mathematical experiences, no longer alone.

The purpose of this chapter is to:

i. present a statistical analysis of Peter's posts that reveals his pattern of use of AskNRICH
ii. use two threads to examine Peter's engagement and interactions when in a learning role
iii. reflect on eleven threads on Mathematical Induction [MI] to follow Peter's progress in mastering the topic, from starting out in a learning role and moving onto a teaching role
iv. use a variety of selected threads to scrutinise a number of ways that Peter interacts in a teaching role

Thus the findings of this case study contribute to further addressing sub-questions of **RQ5**: *Who are the participants? Why do they participate? How is AskNRICH typically used?*; the whole of **RQ6**: *What characteristics do participants of AskNRICH exhibit as they pursue their interest in mathematics? What mathematics teaching and mathematics learning roles are manifested within AskNRICH?* and the first sub-question of **RQ7**: *What types of interactions are shown between the participants as they engage with mathematics?* [see Table 8.1 p165].

van Lier's [1996] conceptualisation of an individual's four-part ZPD [see LRIII] was selected as the theoretical underpinning in the reporting for this chapter. The case study focuses on the participation of one AskNRICHer who at times is seeking help whilst at other times is offering help. Within AskNRICH, help may be offered by more experienced, equal, or by less experienced peers. Analysis of threads involves both the case study subject and all the other AskNRICHers participating in the thread. Thus testing van Lier's four-part model for the case study subject provides the opportunity to consider how each of the four parts of the individual's ZPD may be exemplified in the individual's multiple activities.

The remaining part of this chapter is in four sections that mirror the division used above in setting out its purpose. Thus the section that follows introduces Peter, including some background generated by an email interview with him, and then examines the patterns of his participation in AskNRICH.

10.2 Background Information for the Case Study

Although school pupils can come to AskNRICH, post a query but hardly stay, there is a core of prolific and veteran posters who do participate over a period of time – in some instances for years [see Section 8.4 pp171-176]. Although there are other participants who have contributed a greater number of posts, Peter's posts were a manageable number to study in depth. As previously explained, this was just one of the reasons why Peter was chosen as the case study subject [see Section 6.3.3.2 p119].

In this chapter Peter's involvement with AskNRICH over an eighteen-month period, is analysed. During this time, November 2006 to May 2008, Peter made 501 posts across some 150 different threads. Towards the latter part of 2007, email communication with Peter (preceded by telephone contact with his parents to gain informed consent) provided additional background material to this case study. Peter was contacted by email in November 2007 [Appendix 10.1] and agreed to answer some questions.

This section continues by using Peter's responses to the email correspondence just mentioned to describe his approach to, and motivation for studying mathematics. This is followed by numerical data on Peter's postings to indicate the volume and type of posting (asking for or giving help) and the days and time of day the postings were made.

10.2.1 Introducing Peter in his Own Words

Peter[1] reported that he started to use the NRICH site[2] as he wanted more out of his mathematics studies than he was then able to have at school. Moreover, as his comment below implies, he was experiencing a degree of frustration but had the motivation to be proactive in searching for an alternative resource that would suit his needs [like Adam in LRI p49]. His comment below additionally suggests that when he lost the website's address he had determination, in attempting to relocate it. Peter was thus interested in the subject and wanted to do more:

> *... about a year and a half ago when I first became interested in maths. I was bored and wanted to further the level of maths I did. I then temporarily stopped using the site and forgot the name and then spent a couple of weeks typing enrich*[3] *into Google and searching all of the pages. At the end of the summer a year ago I then finally found the site again and began to use site regularly. I completed a load of problems of the site.* *[email communication]*

From the comment below, made during his first term of his final year of compulsory school, it can be deduced that Peter sat his GCSE mathematics examination a year early (then

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[1] In November 2007, Peter was aged 16.

[2] The part of the site where new challenging mathematical problems are posted each month.

[3] The addition of the 'e' caused the difficulty – though in 2011 a google search on enrich would have nrich as the first hit.

aged 15) and gained an A* grade. Peter described his school mathematics experience as one where the pace was too slow and the work unchallenging. His own learning was taking him far beyond the school syllabus and he was now involved in self-teaching. Such experiences have much in common with other participants, evidenced by similar remarks appearing in various postings.

> *My maths lessons were last year really boring for me as I found them too simple and moving at too slow a rate. At the end of year 10, I sat my maths GCSE and got an A*. I also took core 1 maths and achieved 95%. I taught myself all of the core modules in my spare time and am now working on mechanics. In lessons at school I am teaching myself the rest of the modules for maths A-level. I shall also teach myself further maths after this.*
>
> *My maths teacher is very helpful and helps me with things I am stuck on whenever he can. The math department as a whole is willing to be very flexible to allow me to further my mathematics education. For example, I was allowed to drop ICT and instead sit in the maths department teaching myself maths from a text book.* *[email communication]*

Although Peter's special needs do not appear to be directly addressed by his school, Peter is sympathetic towards the efforts of his teacher and has gone to some lengths to negotiate his own timetable.

Peter had been asked to describe how he learnt his mathematics, and, if he was teaching himself new material, how he did this. Three interesting points stand out in his response below. First is his wish to let things settle, a key strategy frequently adopted by mathematicians encountering either challenging problems or new work [e.g. see Rowland [2003] for an account as to how he solved the handshakes problem, and also further discussion on this strategy in Chapter Eleven]. Secondly, Peter shows maturity in metacognitive self-reflection in realising what helps him study. Finally, Peter's revelation that the need to let things settle is generally only necessary with harder material beyond A-level, he finds the latter causes few problems in his overall understanding:

> *I do teach myself all the new material that I learn. I do this through reading through text-books, leaving it to settle for a week or so then doing as many questions I can. I don't always leave it to settle but find it most helpful when learning harder material, for example. When learning A-level this is not necessary since I can normally understand it pretty much straight away.* *[email communication]*

Indeed in another part of the same email Peter remarked:

> *most A-level work is just solving the same problem with different figures.*
> *[email communication]*

For Peter, studying work beyond A-level appears to be a natural state. In the email below he reveals that 'Analysis' and 'Number Theory' were amongst the topics he was most interested in, which are usually first met at undergraduate level. 'Olympiad maths'[4], as its names implies, is formed of the most challenging problems, far beyond A-level standard, many of which are on number theory and geometry. Peter, like many of the AskNRICHers, rising to the Olympiad's challenge, uses the web-board to discuss past problems[5]. Peter is not alone in his 'passionate' hatred of geometry[6], an emotion which probably arises because formal Euclidean geometry is substantially unfamiliar territory in UK schools [Jones 2002].

> *When I'm not doing A-level school maths I teach myself Olympiad maths except for geometry which I hate with a passion. I also look at some analysis though not much since I have not completed enough maths in other areas to get to the really interesting stuff. My favourite type of maths is number theory closely followed by algebra.*
> *[email communication]*

The comment above also demonstrates Peter's awareness of his own limitations. Although obviously a high-attainer in the subject, Peter can nevertheless still recognise and accept that he has yet to experience the pre-requisite mathematical topics to be able to succeed at the level of Analysis he aspires to. Here again there is further evidence of strong metacognitive skills providing the basis for 'deep learning' to flourish [see LRIII].

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[4] National school-age mathematics competitions culminate in the most successful scorers being invited to be part of the British Mathematics Olympiad [BMO] Team.

[5] Three Threads in next chapter provides an example of AskNRICHers doing this.

[6] From first publication in 1989, The Mathematics National Curriculum (England) had a Programme of Study and Attainment Target entitled Shape, Space and Measures. This was changed to Geometry and Measures for use in Schools from September 2008.

Having shown Peter's avid interest in pursuing his mathematical studies this chapter now turns to a quantitative account of his postings to AskNRICH.

10.2.2 Numerical Data on Peter's Postings

Given an individual's posting name, the AskNRICH search facility returns all their posts that are still-retrievable, grouped by web-board section and then thread title, but listed in an apparently random order. Clicking on a specific post will display the entire thread, at which point it is possible to extract further information on the post including: post number, date, day and time. Only at this stage can the posts be sorted into chronological order and the only means of doing this is manually. Appendix 10.2 provides a table of all Peter's posts in chronological order giving web-board section, thread title, post number(s), date, day, time and Peter's teaching/learning role in it. In this case study the earliest retrievable post was Peter's ninth[7] on 25th November 2006, the last was number 501 on 1st May 2008, the intervening period spanning the final two years of Peter's compulsory schooling. These posts appeared in 151 different threads.

For ethical reasons only postings on the open-access mathematics sections are included in the reporting of this case study. Table 10.1 provides statistical data on threads involving Peter.

November 25th 2006	Earliest retrievable message (9)	Year 10 (14 to 15 years old)
May 1st 2008	Last message (501)	Year 11 (15 to 16 years old)
Total number of threads retrieved (involving Peter)		151 (including 16 'private' threads)
Total number of threads retrieved in mathematics sections		135 (89.4% of total threads)
Total number of Peter's posts retrieved		484 (96.6% of total posts)
Total number of Peter's posts in one single private thread		104 (20.1% of all retrieved posts)
Total number of Peter's posts retrieved in mathematics sections		345 (90.1% if thread above disregarded; 71.3% of all retrieved posts;)

Table 10.1 Duration and Counts of Peter's Participation in Threads

Of the 151 threads, above, 135 (89.4%) appeared on the open-access sections. 484 of Peter's 501 posts were retrievable of which 345 (71.3%) were in the open mathematics sections. This percentage figure is misleading as Peter spent 104 posts participating in the Private Section of the board, in a fun, not mathematics related, word-association game. If this thread

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[7] See earlier quotation above on losing the website's url, Message 9 is the beginning of Peter posting regularly.

is disregarded then the study is working with 90.1% of all Peter's retrievable posts within the mathematical sections spread across 135 different threads. These 135 threads contained a total of 1875 posts by all participants, including Peter, though similar to above, ignoring one outlier thread, this becomes 1474 posts, all of which were considered when undertaking the analysis that formed this case study. Table 10.2 enumerates the number of threads and posts, according to Peter's role, assigned one of four categories, determined by examining the post and thread. Crucially, although the number of *threads* (85) in the category where Peter only offers help is nearly twice that of the other three categories added together, examining the number of *posts* involved shows a much more even division. Peter's involvement in threads where he is in a learning role involve more posts per thread; the number of Peter's posts in the threads where he is asking for help (176 at least), just exceed the number of posts where he is offering it (169 at most).

Type of thread	Number of Threads	Number of Posts	Percentage of Posts
Thread started by Peter requesting help	37	159	46.1%
Thread within which Peter asks a subsidiary question	9	17	4.9%
Thread where Peter offers help	85	156	45.2%
Thread where Peter offers help and asks a subsidiary question	4	13	3.8%
Total	135	345	

Table 10.2 Breakdown of Peter's Participation in Mathematical Threads

Table 10.3 below summarises Peter's posting pattern in terms of day and time of the first appearance of a post by Peter within the thread. The results demonstrate that Peter's use of AskNRICH was essentially an out-of-school activity. There was greater activity per day at weekends accounting for just over half of all his postings. The majority (85%) of weekday postings were made early morning, late afternoon or evening i.e. outside of normal school hours. Where there is activity during the day, this was mostly on dates that were likely to be during school holidays. Table 10.3 also shows only three first posts made after 10pm, indeed an examination of Peter's subsequent posts in threads reveals that Peter tended not to post much after 10.30pm. However this should not be taken as implying that no work is undertaken later than this; for example, ***Peter-Post318*** mentions being up until 1am the previous evening, trying to solve a problem started at 10pm.

Day of posting (all 484 posts)	Sunday (113 posts) 23.3%	Saturday (98) 20.2%	Weekdays (273) 56.5%
Time of first post in the 62 mathematics weekend threads: Earliest 8.37am Latest 11.12pm 54 threads were between 10am-10pm: 23 between 10am-2pm; 31 2pm-10pm. 6 threads were earlier than 10am & 2 later than 10pm			
Time of first post in the 73 mathematics weekdays threads Earliest 7.16am Latest 10.14pm 65 threads were between 10am-10pm: 11 between 10am-4pm; 23 4pm-6pm; 31 6pm-10pm 7 threads were earlier than 10am & 1 later than 10pm			

Table 10.3 Peter's Posting Patterns (Day and Time)

This section has introduced Peter through his own words and by quantitative analysis of his participation using a specially created catalogue of Peter's posts. The remaining sections of the chapter portray Peter's mathematical experiences as an AskNRICHer by presenting the findings of the analysis of all retrievable threads that contain his posts.

The following section analyses Peter's experience when engaged in a learning role and begins with reporting the findings from a detailed analysis of two sample threads where Peter is seeking help. Although within AskNRICH anyone can join in to offer help, the threads used to portray Peter in a learning role involve helpers who are more experienced, either peers or older people. In this respect then Peter's position is in the zone of van Lier's four-part ZPD [see Figure 7.3 p159] which is labelled '*assistance with more capable peers or adults*'. Equally, Peter's declared 'self-studying' as reported above, intrinsically also places him in the zone labelled '*inner resources*'.

10. 3 Peter Engaged in a Learning Role: Viewed through Two Sample Threads

Although all 46 threads where Peter is asking for help were read, the same arguments as presented in Chapter Six [Section 6.3.3.1 p118] concerning selection of threads for **ExThds** could be applied here, i.e. any two could potentially have been selected as samples. Nevertheless, the first selected was Peter's first retrievable thread (November 2006), a deliberate choice as it was the first. The thread was based on an Olympiad question, different to his school studies. The choice for the second sample was based on it being around the time when Peter had become established at asking for help, and in contrast to the Olympiad questions, based on A-Level mathematics, ‘normal’ school mathematics, even if Peter was studying this earlier than usual. In the event it was about the twentieth that Peter had initiated asking for help (May 2007). The two threads are précised here in Tables 10.4 and 10.5 respectively [pp221-222]. The complete text of all the posts in both threads and their accompanying interpretive commentaries, made in preparation for analysis, are presented in Appendices 10.3 and 10.4 respectively. Appendices 10.5 and 10.6 present the early attempts at coding, referred to in Section 6.3.4.3 [pp124-126] and Figure 6.2 [p117], using these two threads.

10.3.1 The Sample Threads

Peter was around 14-years-old at the time he initiated the first thread [**T1**] and at the start of a period of sustained posting. Peter’s opening message, on a Saturday, had him explaining that he is reading a Number Theory book and, using the definition given for a prime number[8], he had come up with an idea that he knew could not be true but could not see why. This situation might be considered as a self-aware misconception or self-induced cognitive conflict [Swan 2001]. During the exchanges, Peter receives a comprehensive explanation of the rigorous definition with links made back to other less rigorous definitions used in school [e.g. see DfEE 1999, Daintith & Nelson 1989].

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[8] Not the usual, simplified ‘layman’s’ prime number definition of a number only being able to be divided by itself and one, or the ‘better’ definition which clearly rejects the number 1 from being prime that a prime number that has two and only two different factors.

Post Number	Précis of message thread [Comments in square brackets is additional commentary relevant to analysis]
P1	***Peter-Post9*** **Saturday 1.39pm** *hi ive*[9] *just began to read an introduction to number theory and inequalities and i seem to have fallen at the first hurdle.* [Book defines a prime number] *later* [the author] *proves that this also means that p only divides by ±1 and ±p but i am still confused. surely this means* [using the book's definition] *that 6 is a prime as 6|42 and 6|6 but not 7. clearly i have over looked something. please explain. thank you.*
P2	***HelpA*** **1.43 pm [4 minutes later]** *Yes, but if we choose m and n to be 14 and 3, 6 goes into neither 14 nor 3,* *but goes into 14*3 = 42.*
P3	***Peter-Post10*** **1.46pm [4 minutes later]** *Thanks. i new that it would be something basic like that i had overlooked. i thought it meant for any choice of multiples in which case all numbers would be prime. i get it now. cheers.*
P4	***Deputy Moderator [DM]*** **1.47pm [1 minute later]** [Writes a very full explanation, only eight minutes after Peter's first post, as to the reason why the definition does not imply that 6 is a prime number. Extends the examples to illustrate the mathematical sense of 'or' which can imply 'both']. *Does that help? Do post back if not!* *By the way, it's integer, not interger. I don't want to be picky, but it's sometimes useful to know the correct spelling as it makes looking things up easier!* [Continues to discuss the formal, rigorous definition in the book in relation to school definitions of prime numbers].
P5	***DM*** **1.52pm** *Oops, it took me so long to write that that **HelpA** got there first!*
P6	***HelpA*** **1.52pm** *Better to get a full explanation after 10 minutes than a one-line sentence after 5!*
P7	***Peter-Post11*** **3.14pm** *thanks to both of you. i get it now. just out of interest, others who have read the book how did you find it. ive never looked at number theory before but im finding it a bit harder to understand than other maths i have looked at. thanks to all*

Table 10.4 Synopsis of First Sample Thread [T1]

The second thread [**T2**] results from Peter attempting to teach himself A-Level Mathematics and meeting trigonometrical equations for the first time. The help provided is 'light touch' in response to Peter asking only for a 'few subtle hints' [**T2-P1**]. During the exchange, Peter is given additional explicit technical help over how to post mathematical expressions so that they appear as 'normal' text [see **T2-P7** below].

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[9] Apart from interesting spellings, Peter does not use capital letters, which in a modern-day texting life is common and normal for some people. Peter's spelling and punctuation will be normally left as it appeared in the posts.

Post Number	Précis of Message Thread [Comments in square brackets is additional commentary relevant to analysis]
P1	***Peter-Post385*** **Saturday 4.25pm** *can some one help me with this problem. this is the first trigonometrical equation i have done so please take it slowly and drop me a few subtle hints. prove that: tan(45'+A/2)=(1+sinA)/cosA = cosA/(1-sinA) where 45' means 45 degrees* *sorry for the lack of formatting but i tried to put it in latex and it didn't work.* *thanks for any help.* [Here formulae text written using only standard keyboard that can be open to confusion. The AskNRICH board has instructions on how to use a mathematical text (LaTeX)].
P2	***Help1*** **[Team Member] 4.49pm** *Hi Peter: For the first equality, do you know the formula for tan (x +y)? Do you need help with the second equality?* *To write in LaTeX, start your line with \[, end with \], and write maths in the middle! (There's a slightly more comprehensive guide here.)*
P3	***Peter-Post386*** **5.12pm** *if i sort the first equality then ill give the second a go.* *Yes I do know the formula to expand tan(X+Y)* *I have tried doing this and meant to post my workings here but forgot.* ☺ [Provides workings – all correct]. *from here i tried a variety of things but each one has failed, quite possibly because of a lack of competence on my part. can you nudge me from here please.*
P4	***Help2*** **5.20pm** *Might help if you write sinA and cosA in terms of tan(A/2).* [A succinct but key hint].
P5	***Peter-Post387*** **5.50pm** *as i guessed i failed because of a lack of competence on my part when trying the correct option. i did this before but I think I must have gone wrong short of the mark. ill put it down to experience. if anyone is interested i did the following:* [Shares solution although there is a small error writing 1-t not t-1in the final line]. *thanks* ***Help2*** *and* ***Help1***. [Misspells latter's name]. *ill post back if i can't get the second one*
P6	***Help1*** **6.10pm** *Almost - have another look at your very last line.* *Great stuff otherwise!*
P7	***Help1*** **6.13pm** [Additional technical advice on even better use in marking-up mathematical text distinguishing between ordinary text and italicised script for variables].
P8	***Peter-Post388*** **6.37pm** *i put t-1 not 1-t like it should be ... and ... spelt your name wrong.* *Now ive got the first one im motoring through the exercises. who would have thought trigonometry could be this much fun. thanks again*
P9	***Help1*** **6.56pm** *Lol, I was referring to the 1-t, but that too!* *Good luck with the rest of the problems*

Table 10.5 Synopsis of Second Sample Thread [T2]

10.3.2 Observations on Learning Opportunities

The discussion below of the analyses of the two sample threads focuses on drawing out Peter's learning opportunities as an AskNRICHer, presented in four sub-sections.

10.3.2.1 Mathematics Challenge

In both threads, all the work being undertaken is far in advance of the syllabus/curriculum intended for school pupils of Peter's age. For example, as already stated, **T1** requires a more rigorous definition of a prime number than is usually found in school. Furthermore, the underlying principles contained within the definition are also beyond school study. The further contributions [**T1-P4**] by **DM** provide connections with known school definition of prime numbers thus extending general mathematical knowledge. Thus in relation to van Lier's [1996] types of Pedagogical Interactions, [see LRIII Section 7.4 pp145-147] this fits on the cusp of transaction/transformation. The topic of **T2** is most likely to be met during A-level studies, two years later than Peter's school year. Here Peter is trying to learn how to manipulate trigonometrical identities/equations. He is gaining mathematical knowledge through knowing formulae [**T2-P2**] and the hint to rewrite in terms of half angles [**T2-P4**], a common technique that facilitates algebraic manipulation across a range of similar problems. Therefore, in this instance the thread fits a less contingent, more restricted, type of pedagogical interaction somewhere between (good) IRF Questioning and Transaction [see LRIII Section 7.4 pp145-147].

10.3.2.2 Experiencing Other People's Mathematics

The emphasis in this sub-section is on the opportunities for Peter to be immersed in an wholistic mathematical experience through the interactions with others who participate in offering help. Such 'one-step removed' experiences are a variant of Sawyer's [2006: 4] contention on enhanced learning opportunities through engaging in activities similar to professionals within the field. This is a theme that is returned to in the next chapter.

In the first thread, Peter immediately gains a mathematical experience through ***Help1***'s comment providing an example that counters Peter's idea and demonstrates the definition [**T1-P2**]. Just four minutes after ***Help1*** has replied, Peter is introduced (by ***DM***) to the need for more rigorous mathematics [**T1-P4**]. The ensuing exchange, a contender for a contingent conversation [van Lier 1996], a focus of Chapter Eleven, between these two helpers [**T1-P4-6**] about speed of reply versus depth of definition, provides Peter with an unplanned learning opportunity to consider relative merits of ways of 'doing mathematics'. The discussion on the merits of both the 'quick-fix' response and a more measured relational deliberation, connects the common non-rigorous definition with the mathematically rigorous. There is, however, no evidence to indicate whether Peter has noted this. Nevertheless, the ideas conveyed in this exchange would have a place in a mathematician's toolbox [see Section 11.5.1 p258]. In the second thread, the advice [**T2-P7**] about italic and non-italic font being intrinsic to assuming variables and ordinary text respectively highlights, at least to mathematicians, an important difference. In this instance, the advice is explicit and thus it can inferred that Peter should have noticed it and potentially have a new tool. ***DM***'s message [**T1-P4**] asking for the word integer to be spelt correctly is not strictly experiencing mathematics and could be judged as a reprimand, though it is gently accomplished and accompanied with a firm, precise, (mathematician's) reason as to why the correct spelling would be useful!

10.3.2.3 Exploiting Thinking and Understanding

This sub-section highlights instances where Peter's current thinking and understanding can be exploited by others, that in turn, provide him with the opportunity to develop his thinking and understanding further. At the start of **T1** there is clear evidence in the way the message had been phrased that there has been careful thought prior to posting. Having met in a book a new and rigorous definition of a prime number, Peter had realised [**T1-P1**] that the interpretation he is making could not be correct. Hence Peter was thinking and understanding that he had a misconception that led to a contradiction [see earlier reference to a self-inflicted cognitive conflict]. Even when the misunderstanding had disappeared, Peter continued to think about the principles involved by acknowledging that his (initial and incorrect) idea would mean every number being a prime [**T1-P3**], rather than quickly moving on with an unquestioning acceptance. This is an example that can be categorised as conceptual (deep) rather than surface thinking [see LRI p51]. The detailed definition [**T1-P4**] has provided the opportunity for relational understanding[10] [Skemp 1987]. In the second thread there is the some evidence that Peter is determined to understand, in the 'work-things-out-for-himself' sense, as he asks only for a hint as he encounters a new topic [**T2-P1**]. By experiencing/doing similar questions there is provision to make gains in understanding, though with the evidence available, the understanding gained can only be

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[10] for a fuller discussion on the definitions of mathematical understanding see LRI.

claimed to be at least instrumental [Skemp 1987]. Nonetheless, Peter's actions in each of these threads map neatly onto the elements of 'Learning Knowledge Deeply' listed by Sawyer [2006: 4] [see Figure 2.1 p50].

10.3.2.4 Reaching out to other AskNRICHers: following Ethos and Etiquette

This sub-section focuses on instances within Peter's posts conducive to his and others' learning, rather than just his own, and relates to the ethos and etiquette of the web-board. Although Peter's direct interactions in the two threads considered above was with more capable peers and adults, given the open-access to the web-board, his interactions could be considered additionally related intrinsically to two other zones of van Lier's multiple ZPD labelled: 'interaction with equal peers' and 'interaction with less capable peers'. The relation to these two zones is explored fully later in Sections 10.4 and 10.5 where Peter takes on a teaching role, but some parts of posts resulting from Peter's adherence to the Posting Protocols [set out in Appendix 8.1] provide some initial indirect examples.

So for example:

- giving a clear exposition of the problem and asking for an explanation [**T1-P1**]
- showing what he is able to do by being open in sharing his current confusions [**T1-P1**] and limitations [**T1-P7 T2-P3,5&7**]

ensures that Peter articulates his current state, both to himself and to others that will come to help or 'lurk'.

The following three examples illustrate adherence to the protocols creating a pleasant, sharing atmosphere within AskNRICH:

- apologising for forgetting to share his work in the first message [**T2-P3**]
- always being polite throughout, with a constant stream of '*please*' and '*thank you*', [**T1-P1&7**, **T2-P1&8**] and a more contemporary expression of gratitude of '*cheers*' [**T1-P3**]
- sharing his solution with others who might be looking at the exchange [**T2-P5**]

There are further noteworthy personal touches, falling outside the protocols, which result in AskNRICH being a 'happy place' in which to learn:

- suggesting that it his own lack of competence that is causing the problem [**T2-P3**]
- attempting to draw other people in by asking if anyone else is reading the book [**T1-P7**]
- a further relaxation with 'friends' with the use of the ☺ emoticon [**T2-P3**] and humour – '*motoring through*' and '*who would have thought that trigonometry could be this much fun*' [**T1-P7**][11]
- a light hearted (lol) exchange with ***Help1*** [**T2-P9**] whose intention had been to focus Peter back to 'the last line' of the mathematics, not the mis-spelling of ***Help1***'s name

The observations made here and in the preceding three sub-sections all add to and further exemplify the features and discussions presented in the previous chapter.

So far, Peter's learning role has been considered through two sample threads. This chapter continues using a series of threads on Mathematical Induction [MI] to investigate Peter's transition from learning the topic to taking on a teaching role, helping others who are subsequently encountering it.

10.4 From Learning Role to Teaching Role: Experiences of using Mathematical Induction

During examination of all threads involving Peter, those involving MI stood out because of both the number of threads and the quality of the learning and teaching evident in the posts, especially for someone of Peter's age. This bounded set of threads provided the opportunity to track Peter's mathematical progress in learning the topic and follow Peter's transition from a learning role to a teaching role. These threads, which again also typify AskNRICHers engaging in contingent conversations [van Lier 1996], can be related to all four parts of van Lier's multiple ZPD through Peter's interactions with more, equal and less capable peers and Peter's observable inner resources.

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[11] the quotation selected as an introduction to the entire thesis.

Rigorous MI proofs are seldom studied in detail at A-level[12]. My own experience is that even at undergraduate level, voluntarily pursuing it beyond the standard series proofs is not the norm[13]. As will become apparent from the threads, it could be said that Peter [aged only 14-15 years] appeared in the nicest sense of the word, *obsessed*, with this topic. Additionally, Peter recalled, unprompted, his MI experiences in his later email interview:

> *Without the Internet I would have struggled to learn new maths as I wouldn't have been able to find the most interesting areas of maths to buy books on and study further. For example, I taught myself a lot of number theory from the Internet before realising that I was very interested in it. I also use a lot of Internet articles and e-books to learn new maths, for example I learnt [mathematical] induction of the Internet (using Vicky Neale's[14] article on NRICH).* *[email communication]*

Eleven separate threads were used for the analysis. Table 10.6 [pp228-229] provides a précis of these threads based on an interpretation of the texts. The second column of the table indicates Peter's progression that can be related to van Lier's four-part ZPD: starting with the self-study of the subject that brought him to learning from more able peers, to working with equal peers and on through to teaching less experienced peers, gaining increased inner resources in the process.

10.4.1 Threads Involving Mathematical Induction

As mentioned above, Peter is considerably younger than the normal age for meeting Mathematical Induction – some five years before it is expected to be part of a repertoire of proof strategies. However, as soon as the word mathematical induction is mentioned Peter is proactive in finding out more.

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[12] e.g. at the time of writing the OCR specification had Mathematical Induction in Content Summary of Further Mathematics 1 – "Candidates should be able to Use the method of mathematical induction to establish a given result (not necessarily restricted to summation of series)".

[13] similar sentiments were expressed by colleagues in personal communications.

[14] ***DM*** who also made the final exchanges in the first of the Mathematical Induction threads.

Table 10.6 Peter's Progression through Eleven Threads on Mathematical Induction

Thread	Progression	Interpretive summary of events evident within the thread
One November	The term Mathematical Induction is introduced	The first time the term is mentioned to Peter is in response to a thread started by his 29th post where a helper asks the question: *Do you know Induction? I'll start you off ….*
Two November	Peter's first attempt at using MI Posts 47-65 excluding 49	Four days later Peter begins a new thread [see Appendix 10.7 for full text] calling it mathematical induction. Has been shown a proof (which he gives) but at a particular stage stops understanding it. Help quickly came, enabling the comment: *I'll sleep tonight now*. The next day asks if anyone can recommend a site he could visit and whether others have found it difficult when it comes to constructing one's own proof (rather than reading someone else's). The latter receives some nine different people helping. Two people set questions to practice whilst another offers the three steps always required in the formal proof. The ***Deputy Moderator*** [***DM***] points Peter to an article on the NRICH site [see email communication above].
Three January	Peter's second attempt at MI, eight weeks later Posts 160-164	Peter wonders whether he has covered the relevant material to be able prove an inequality using the technique. Receives an algebraic hint and a reminder that just needs induction arguments. Peter sends a solution wondering whether it can constitute a proof and a new helper replies *not quite* and again lays out the three steps. 15 minutes later Peter returns having done it though the working out is spasmodic with some gaps. The thread concludes with Peter recommending a web address that he had found useful *'especially if you have taught yourself'*
Four February	A non-standard use of MI and debate between two other AskNRICHers on visual explanation versus MI Posts 169,171&172	A week later Peter starts the thread asking for a hint on a chess-board problem. After someone suggests simply looking at a chessboard, the answer is obvious. Two undergraduates discuss proving the problem using mathematical induction, with Peter 'lurking' – evidenced by a final comment.
Five February	Approving someone's solution Post 173	Again this shows Peter 'lurking' as he offers congratulations to someone else who is in the early stages of trying out doing the proof: *yes that's correct, well done*.
Six February	MI is not strictly needed Posts 183,184&187	Having started a thread on a four-part sequence question, one helper suggests that one way of solving it might be to use Induction. Peter admits that he is a bit confused on using it if not in the usual format, though has remembered the three necessary steps.
Seven February	Peter offering help (for the first time) to a newcomer wishing to know about MI Posts 255,257,259&260	Three months later Peter succinctly gives the three steps to a new poster. He also provides an example of the proof concluding with an explanation behind the principle of MI. ***DM*** offers same article link as to Peter earlier. The new poster remains unsure so Peter reiterates the reasoning behind the three steps and promises: *to try and find a good exercise that [he] had used when he was leaning about the topic*. Four minutes later he posts the web reference, prefaced with the words: *here we go*.
Eight March	Investigating an alternative proof which uses MI Posts 314&315	Another month later Peter starts a thread stating he has solved a problem using modular arithmetic but wanted to try it with mathematical induction. He shares his incomplete proof, using the three steps. Asking for a *'gentle push'* two people offer a little help and Peter realises his proof.

Thread	Progression	Interpretive summary of events evident within the thread
Nine April	Peter voluntarily using MI Posts 346-348	A regular poster poses a problem and though help has been given after a week the problem is unresolved. But the words mathematical induction have been introduced seemingly from nowhere. In cross-posting Peter offers to try a proof by induction at the same time as a team member suggests: *staying away from induction for now (for several reasons)* [**AskNRICHer-Post421**]. However Peter decides to try it out and having sought confirmation in recognising an error, eventually succeeds, gaining praise from the team member: *Peter, Your proof by induction is great – well done. I don't usually like counting rectangles, but you have done it in a neat way* [**AskNRICHer-Post439**].
Ten May	Using MI on de Moivre's Theorem[15] Posts 399-405	Peter posts a query[16] connected to using the binomial expression within a trigonometrical formulae proof and has done the base case (step one of the three required) but is unable to move forward. The first person offering help mentions using de Moivre's theorem. Peter admits that the book actually had given two hints – not only induction but also de Moivre's theorem but as he had never heard of the theorem wants to stick with using induction. Help returns with two further hints and stating de Moivre's theorem. 15 minutes later Peter begins his next post: *proof of de Moivre's Theorem*[17]*: (a lot easier than I thought it would be ☺).* At the start of the next thread on a different topic, Peter indicates that he has solved the original problem.
Eleven May	Using MI to prove a pattern spotted Posts 410&411	The thread has been started by someone asking about how to find the formula for the sum of n rows of Pascal's Triangle. Peter's response begins with a comment that he has spotted a pattern: *summing the first few rows i noticed that the sum is* 2^{n+1}*-1. now we want to prove this formula for all n. using induction there is a simple proof and i havent attempted any other method.*

10.4.2 Analysis of Peter's Progress in Studying Mathematical Induction

The following analysis has been made based on a consideration of episodes evident within the sequence of eleven threads showing Peter progress as he engages with a new topic. Comparisons with what might happen within a classroom setting when learning any new topic is made where appropriate.

In **MI-T2**, Peter realises (through thinking and practice) that he lacks a full understanding of the proof. When he asks for help, he receives help from no less than nine people, all more experienced AskNRICHers, on what to do, is given further problems to try and is steered into completing the three step formal proof. These same processes would occur when the

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[15] Understand and use de Moivre's Theorem is within the specification for Further Mathematics final level [OCR nd :54]. *Wanting* to prove it would seem to be impressive at school level, *wishing* to prove it would be more appropriate at undergraduate level.

[16] During this thread Peter achieves veteran status [see p172].

[17] Later in the thread the helper suggests that Peter has only proved it for the positive integers.

subject was taught in the classroom, but Peter has a fuller, more enhanced experience garnered from help from many teachers, not just one. Peter's posts at the end of the thread suggest that he is successful in solving the problem, from which it might be inferred that the topic has been learnt and understood. However, as **MI-T3** shows, after an interval of some eight weeks, the three steps have to be given again. It would be reasonable to assume therefore that the topic has neither been learnt nor fully understood and to some extent forgotten. Peter's comment at the end of the thread supports this inference:

> *because I have only done a few and it's been quite a while since I last did [one].* ***[Peter-Post164]***

In the mathematics classroom this would be addressed or pre-empted by the teacher referencing previous work. However by the end of the thread [**MI-T3**] the posts show progress in both learning and understanding the topic, although even as late as **MI-T8**, when he provides an incomplete proof, Peter has not yet fully grasped it. Consolidation and practice, a strategy recommended by the Cockcroft Enquiry [DES 1982 paragraph 243], comes, for example, in **MI-T4**, **MI-T6** and **MI-T8**, when re-imagined as exercise questions.

The discussion within **MI-T4** is of particular interest in two ways. Firstly, it provides further strong evidence of experiencing other people's mathematics as in Section 10.3.2.2 above. As the thread develops, two team members choose to employ MI to solve the problem. Secondly, it provides an authentic but unusual situation, rather than the normal set of routine number based exercises from a textbook, in which mathematical induction can be used to solve the problem. Thus the posts initiated within the thread provide Peter with an alternative and additional viewpoint of how MI can be implemented in problem-solving; a further strategy to place in the 'tool-box'. Later **MI-T6** and **MI-T9** show MI being used by Peter in different contexts and although in **MI-T8** an alternative proof has already been found, Peter is seeking a MI solution.

MI-T5 marks a temporary departure for Peter from only asking for help as he offers congratulations on another AskNRICHer's successful solution. However, it is **MI-T7** that clearly sees Peter taking on the teaching role and offering resources that he had previously found helpful. It might therefore be inferred that the topic has now been learnt and understood, but given Peter's later requests in **MI-T8** and **MI-T9** for help, further learning on his part has still to take place. Nonetheless, the final thread in the sequence **MI-T11** again has Peter totally in a teaching role. Peter provides a pattern spotting formula and assures the person asking for help, a less experienced AskNRICHer, that it can be proved using MI (as he has done it!).

The posts within **MI-T10** suggest that Peter is moving towards ‘mastering’ the topic. At first glance Peter appears not to have mastered the topic since he asked for help, unable to move beyond the first step using the base case. However when a helper suggests using de Moivre’s theorem, which Peter has not heard of, as an alternative method, Peter first proves the theorem using MI rather than applying it to the problem. He then completes the original problem using MI:

> *btw incase anyone is bothered I solved the question i posed earlier.*
> *Thankyou very much to anyone who helps, your all great resources ☺*
>
> ***[Peter-Post406]***

It was this thread that led to my earlier portrayal of Peter’s interest as ‘obsession’ with the topic.

This section has used a bounded set of threads that involved a sequence of MI related problems in which Peter’s increasing inner resources of knowledge, experience and memory enabled him to make the transition from asking for help to offering it. He has involved newcomers and offered ‘old hands’ an additional insight into the topic [**M1-T9**]. Thus all four parts of van Lier’s multiple ZPD [Figure 7.3 p159] have at some point been evoked within these threads. Furthermore, however, the threads illustrate that Peter’s pursuit of understanding and a quest to understand underlying principles connects with ‘Learning Knowledge Deeply’ [Sawyer 2006: 4], ‘Making Connections’ [Ofsted 2008, Upitis et al. 1997], and the portrayal of ‘Adam’ in Anthony [1996] [see LRI pp48-49]. By way of further example, in **MI-T10** having proved de Moivre’s theorem using MI, Peter then asks how de Moivre’s theorem is applicable to the original problem. Moreover the linear progression of understanding through the sequence of threads emulates that postulated by Byers and Herscovics [1977: 26] in their four-part model of understanding: informal knowledge, initial conceptualisation, gaining precision and finally formalisation [see LRI p52].

The next section continues to follow Peter focusing on his participation when the primary purpose of posting is to offer help.

10.5 In the Role of Helper

Peter offered help almost from the start, with his seventh retrievable post [***Peter-Post15***], five days after re-establishing contact with AskNRICH. Peter made contributions to all three mathematics sections, offering varying help to AskNRICHers with less, the same and more experience. This section presents findings resulting from studying and analysing all 89 threads that included Peter in a helping role. The findings are reported under three main sections: Teaching Strategies; Helping but Learning, and paralleling Section 10.3.2.4 on the learning role, Reaching out to other AskNRICHers but this time *perpetuating* ethos and etiquette.

10.5.1 Teaching Strategies

Analysis of Peter's helping posts showed that, when he had expert knowledge that he could pass on, he engaged in many of the teaching strategies that (may) result in scaffolding the learning, funneling and focusing [See LRIII Section 7.5.2 pp153-154] found in the **ExThds** discussed in Chapter Nine earlier. Peter's strategies include: offering hints, using a different example to explain a technique and direct explanation. Examples of each are briefly reported below.

10.5.1.1 Offering Hints

Just as Peter abided by the posting protocols when asking for help [see Section 10.3.2 earlier] he follows the protocol of offering some advice/hint on what to do next but not offering a solution. For example responding to a first time poster, Peter and one other offer help over one and a half hours. During the exchanges Peter engages in Socratic-Style Dialogue [LRIII Section 7.5.1 pp151-152] by posing a question back that implicitly includes the hints:

> *now that you know that the difference is 2, how do you write that in a formula involving n?* ***[Peter-Post306]***

… and later shows some partial working i.e. providing further but more explicit hints, ending with the remark:

> *i'll leave it to you from here* **[Peter-Post308]**

Offering hints was normally Peter's initial strategy although he adapted this when appropriate.

10.5.1.2 Offering an Alternative Example

Some five months in, Peter offers help to another first time poster on how to solve simultaneous equations. Near the beginning he posts:

> *multiply the equations by the number x is multiplied by and then subtract one equation from the other. since i havent explained very well i shall give another example not one of the questions you asked so you can still do the same question.* **[Peter-Post374]**

and then does the example clearly and fully. He chose a different problem from the three the poster asked about, but ensured that, like the ones given, one equation had a negative coefficient rather than presenting the simplest type. The effective tactic of ensuring that the example offered maintained the same structure as the original is the same as that adopted by ***Help1*** in **ExThd1** in the previous chapter. Two of the three questions posted and Peter's example only required one of the equations to be multiplied throughout before addition of the two equations, but Peter's final, anticipatory line of advice [see also **ExThd1** and anticipate difficulties code **TRAD**] made reference to at times needing both equations to be multiplied. This was a carefully thought through reply with the potential of being of great help to anyone embarking on this topic. Peter finished with the oft-used sign-off sentence …

> *Post back if you dont understand or get stuck.* **[Peter-Post374]**

… in order to ensure that if this example was not successful then the exchange could continue[18].

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[18] The Moderator, herself a teacher, did add a further response, beginning work on the first of the three questions posted by the originator. The reply, *'okay, thanks guys'* [**AskNRICHer-Post2**] came back, the plural implying help from more than one person had been useful.

10.5.1.3 Direct Explanation

In the second of the three threads [**3Thd2**] forming the third Perspective [see Chapter Eleven], Peter was in a sustained exchange with a poster who could not solve a part of the problem that Peter had earlier successfully solved unaided. Peter first attempted to help through offering a range of hints starting with the leading question:

> *what form do primes greater than 6 take* ***[Peter-Post267]***

… which if known, would lead neatly towards the solution, or, as Peter added:

> *as soon as you see what to do this is very simple so I shall leave the hint at that.* ***[Peter-Post267]***

However these hints proved insufficient and Peter continued trying to help. At one stage Peter mentioned modular arithmetic[19], which is essential to a solution, but it became clear that this would be a new topic for the poster. Peter then posted:

> *since I don't think that you understand modular arithmetic (don't worry about this) I shall write it in basic algebraic form.* ***[Peter-Post270]***

… and after several further essentially didactic posts [see discussion of direct explanation in Section 9.4.3 p208], that nonetheless produced fruitful interactions, the poster arrived at the solution, leaving Peter to comment:

> *yes, well done this completes the proof. i remember fondly this question.*
> *this was my first bmo question i completed. arrr memories ... yes anyway.*
> *well done* ***[Peter-Post271]***

The examples given in this section have been ones where Peter is entirely in command of the mathematics; the next section discusses episodes where he may not be.

10.5.2 Helping but Learning

Peter's enthusiasm for both the subject and AskNRICH sometimes led him to enter a thread in a helping role, but subsequent interactions provided him with the opportunity to also increase his own learning. van Lier [1996: 193] quotes the Latin dictum, *docendo discimus* and indeed many AskNRICHers openly subscribe [see John p174] to this dictum which

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[19] see Houston [2009: 208] for the importance of this for number theorists.

translates as *we learn by teaching* [ibid], hence this section's title 'Helping but Learning'. In this respect Peter's interactions relate to the part of van Lier's multiple ZPD [Figure 7.3 p159] labelled: 'interactions with less capable peers'.

This notion of 'helping but learning' is introduced using the first few posts of one particular thread [**H**], presented in Table 10.7. The thread occurred four-fifths of the way through Peter's contributions, thus at a stage when Peter had gained experience in helping others and was well settled into the helping role. The mathematics is unimportant, the thread is merely illustrating an exchange between Peter and someone he is trying to help.

	Précis of Text	Critical Observations
H-P1	***O*** - Brand new poster. **Friday 8.20pm** *I have read in several places that the algebraic numbers are closed under addition, subtraction and multiplication, and that this "could be easily proven", though I have not seen this done. .....* *.....If x and y were algebraic numbers, what polynomial would $x+y$ or xy be a root of? How can it be constructed? e.g. $\sqrt{2}+\sqrt{3}$.*	This is the very first post that ***O*** has made – or to be exact at least the first post made under the posting name.
H-P2	***Peter-Post390*** **9.18pm** *im afraid i dont know what closed under addition and subtraction means but the second question i can help you with.* *let $x=\sqrt{2}+\sqrt{3}$now eliminating the square roots gives a polynomial with $\sqrt{2}+\sqrt{3}$ as roots.* *i think this is what you wanted. somebody with more expertise shall be along soon though and give you more help than i can*	Peter will always admit when he does not know something but nevertheless selects the second part of the query and offers some help, again making clear that the query may be in need of more expertise than he is providing
H-P3	***Peter-Post391*** **9.20pm** *when i say eliminated i mean by squaring*	Shortly after posting instructions, Peter posts again to make instructions clearer – he has been thinking things through further or checking that his message makes sense.
H-P4	***O*** **9.33pm** *By closed I mean the sum or product of any two algebraic numbers is another algebraic number.*	The original poster turns helper in explaining what closed means in this context – Peter is thus learning something new too.
H-P5	***Peter-Post392*** **9.59pm** *if the two algerbraic numbers are expressable as the sum of roots of rationals then i think that it is quite easy to create a terminating algorithm to show there is a polynomial with that root. does this cover all algerbraic numbers? if not i'll leave it to some one else who knows there stuff.*	Peter continues to help even though the overall topic is beyond his experience. He then asks his own question about generality. He is still suggesting that someone more expert will help out …
H-P6	***Expert*** **10.03pm** *Yes .....*	… as they do here

Table 10.7 Start of Thread [H] illustrating 'Helping but Learning'

Although in this thread, **H-P2** and **H-P5** exemplify Peter using existing knowledge to help as described in the previous section, analysis of the helping posts, finds examples in which Peter is:

- admitting if he is unsure of his help [**H-P2**, **H-P5**]
- making clear what he does not know [**H-P2**]
- gaining knowledge from originator or other helpers [**H-P4**, **H-P6**]
- posing his own question [**H-P5**]

The first three of these actions indicate that here Peter appears to be giving help in areas beyond his current knowledge and expertise, which he always acknowledges. The fourth, where he poses his own question, is a variant on other incidences in other threads where Peter picked up the problem posed and tried to find a solution not only for or with the person whose problem it originally was, but also for himself [see code **LRJ** Table 9.4 p197]. In all four Peter was essentially attempting to offer help, but the exchanges provided him with an opportunity to learn new work.

The notion of Helping but Learning is now elaborated using additional examples from other threads.

In some of his posts Peter appeared to be picking up a problem, trying it out and sharing his ideas, which were not necessarily always correct. In the extract below, Peter attempted to help with a problem posted[20] in **HD** (for university mathematics and thus well beyond the norm for his age) seemingly not to mind being told he was wrong:

> *yes, i realise ... sorry to anyone I mislead. ... sorry i seem to have led you down the wrong path. You are correct.* ***[Peter-Post74]***

Shortly after, ostensibly offering help Peter posted his workings for a new example and feeling that final value was too small, asked for someone to check. When the person who posed the problem in the first place whom he was meant to be helping responded suggesting an error in the first line, Peter replied:

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[20] Peter is simultaneously asking for help on one of his own questions.

> *yeah, sorry i'm [worn out] and not thinking properly, that's what I meant by checking my answer.* ***[Peter-Post77]***

Between them, they never get it correct. Eventually an 'expert' comes in: '*Right well, I think its just that you* [suggests the mistake] *…… or I have [got it wrong] and you guys are right'* [**AskNRICHer-Post1251**], a kindly let down perhaps. The thread was started from someone who is *'practising some questions for my interviews at Cambridge on Tuesday and thought it best to ask someone who is very good at maths!*' [**AskNRICHer-Post17**]. The impression gained from the friendly exchange was that it was potentially a valuable experience for the prospective interviewee. Working through things together, errors, misleads and all, as Peter was doing here in an attempt to help someone else, could actually help the originator clarify his/her own learning and understanding.

In another thread some seven weeks later, after others had offered suggestions, Peter, even though as the final sentence reveals he had actually never met the topic(!), joined in with his own idea posed as a question:

> *would*[21] *the best strategy if A started be for A to bid £9.99 then it is not worth while B bidding so B gives up and loses nothing and A wins 1p? i have no idea of game theory though.* ***[Peter-Post155]***

In a later thread Peter was again explicit in signalling his limitations:

> *i'm not totally sure so dont take this as gospel. but if you try ...* ***[Peter-Post290]***

… followed by:

> *somebody else will come along some and give you more solid advice. sorry i cant be too much help.*[22] ***[Peter-Post290]***

… the first part of which reiterates the official posting protocols that someone will always correct if necessary. However it might be argued that Peter in his enthusiasm is less heeding of the protocol about not offering help if unsure!

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[21] Reading all of the thread the word 'would' in this context is being used in the sense of 'I feel that the best strategy would be'.

[22] … and someone does.

In the preceding example *'but if you try ...'* Peter had acted in part as teacher. In the next example Peter provided a method for solving the problem and was thus more fully in a teaching role. The problem was a relatively basic question, could the specific quadratic equation be found given two roots (solutions). Peter's post appeared some ten minutes after another AskNRICHer had easily addressed the problem two minutes after the question was posted. Peter's method was correct but tortuous, maybe a signal of not yet having full mastery of quadratics. Peter's post was greeted by one word: *'Um...'* [**AskNRICHer-Post973**] made by the person who had quickly solved the problem. Peter appeared content to accept this criticism with good grace:

> *Yes my method is not particularly elegant but i didn't see your solution when i posted mine. O well ☺* ***[Peter-Post379]***

… and in the process had been made aware of an alternative more elegant (efficient) solution[23].

This section has focused on Peter's posts where he has entered a thread in some form of helping role but the interactions provided him with the opportunity to increase his own learning. Peter undoubtedly has a mathematical attainment well in excess of his chronological age. For those occasions where a lack of experience appeared to show through, Peter was at the very least an enthusiastic 'amateur', with an apparent keenness to fully participate in AskNRICH. This extended yet further to Peter acting as teacher (or moderator) if standards slipped as demonstrated in the next section. Peter was helping to uphold the ethos and etiquette of AskNRICH as elaborated below.

10.5.3 Reaching out to other AskNRICHers: Perpetuating Ethos and Etiquette

Two posts selected in Section 10.5.1.3 above to illustrate direct explanation also demonstrate Peter showing care and consideration [***Peter-Post270]*** about any lack of experience on the other person's part and making social comments [***Peter-Post271***] to be friendly. Such posts highlight the ethos that makes AskNRICH 'a nice place to be' [see p173].

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[23] This example is incidentally also illustrative of the asynchronous aspect of different helpers finding a solution and pressing the send button later than others.

Peter's posts clearly show him being polite in welcoming newcomers to AskNRICH. For example a first time poster posting a question at three minutes past midnight and then three hours later making a further plea for help, is likely to be in a different time zone. Whilst the Moderator responds at 8.37am with a message suggesting patience, at 9.02am Peter provides help prefaced by a welcome:

> *first of all welcome to nrich. i am going to assume that you have done all of your other working* ***[Peter-Post333]***

A further examination of **H-P3** [Table 10.7 earlier] reveals Peter returning quickly to clarify meaning, appearing to write, post and then re-read, checking the help he has provided. Although this could be interpreted to imply some lack of confidence, it could equally imply conscientiousness on Peter's part to offer the most accurate help and advice he could. Peter, in still thinking about what he had written after he had posted, is perpetuating the ethos by example.

Peter appeared equally keen to ensure that other users of AskNRICH adhere to the protocols too. When a first time poster incorrectly started their thread in **PE**, Peter promptly 'reprimanded' them in a supportive manner:

> *can you post the question please. also bmo questions for future reference should be in onwards and upwards.* ***[Peter-Post288]***

The person responded by posting the question.

A further illustration can be seen in the following episode where Peter is 'defending' AskNRICH. A regular poster was trying to re-ignite the debate about the role of zero and was suggesting some fairly outlandish definitions that six other hardworking AskNRICHers were trying politely and using rigorous mathematics to refute. Eventually Peter joins in:

> *why do you insist in asking the same question in a different way when you have the AskNRICH team and other people have categorically told you that division by zero is undefined* ***[Peter-Post179]***

This did not exactly stop the debate immediately but it probably encapsulated what many were thinking.

The examples in this section are typical of the AskNRICHers' normal 'self-moderation' and their expectations of how AskNRICH should be used.

10.6 Features Summary 3

The Features Catalogue for this chapter, relating to People Characteristics, is presented in Figure 10.1.

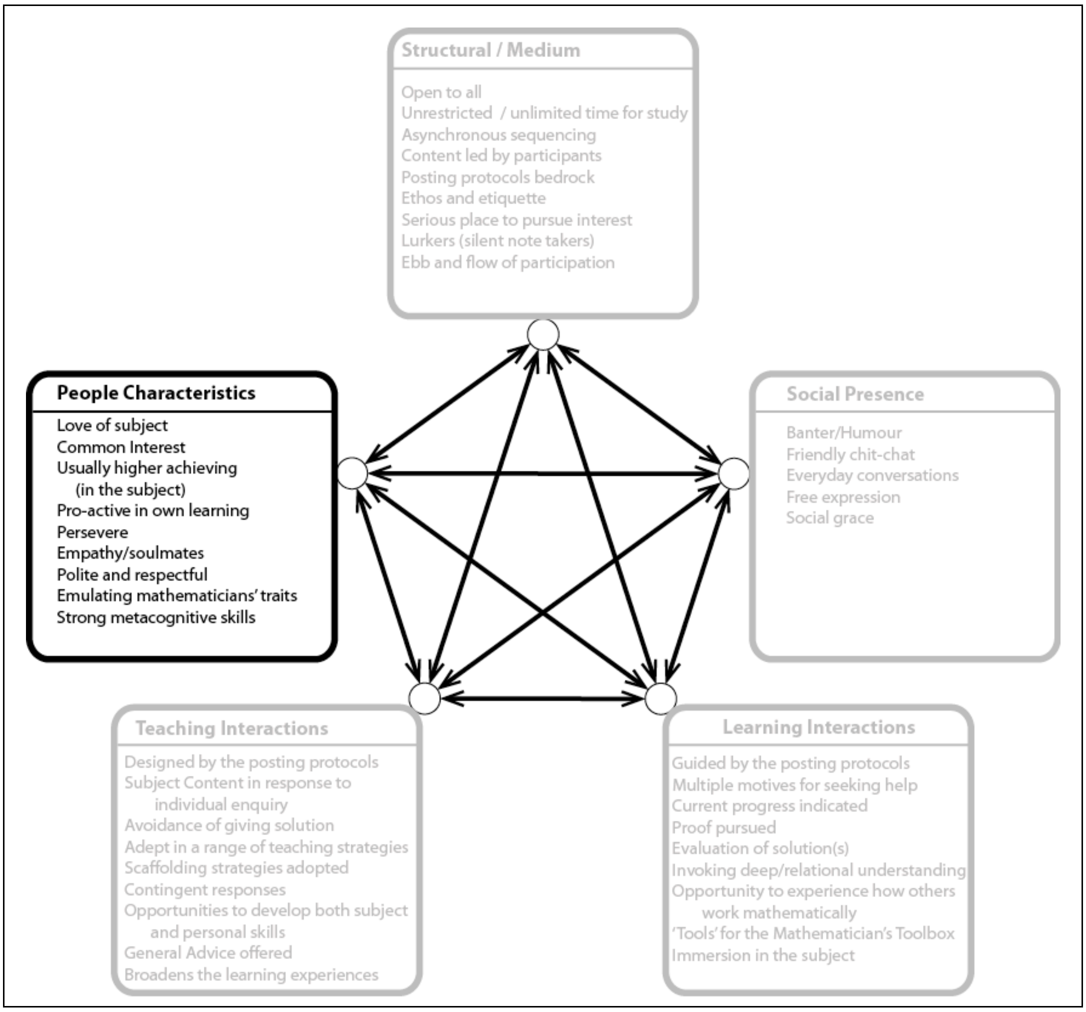

Figure 10.1 Features Catalogue: People Characteristics

10.7 Conclusions

This chapter has reported the second Perspective, an in-depth case study of one young mathematician, Peter, through his participation in AskNRICH and interaction with other AskNRICHers, analysing around 1900 posts in all. Peter used AskNRICH over an eighteen month period, at a time when he was much further advanced in his mathematics studies than other members of his school class and needed to work at challenging topics alongside others of comparable ability; a keen and enthusiastic mathematician pursuing independent study at a level above his current chronological age and beyond the school curriculum. AskNRICH provided the means for people working on their own, at home and alone, to (remotely) connect with like-minded others within this virtual environment [Sawyer 2006: 569] an opportunity rarely available anywhere else in the physical or virtual worlds.

From the threads used in the analysis throughout the case study a set of people characteristics are apparent that reinforce the picture of (school-aged) AskNRICHers engaged in mathematical study portrayed in the previous chapter. Peter perseveres to understand deeply the mathematics, seeking connections and relationships, pursuing proof and discussing aesthetic solutions. Peter is able to be open about his own achievements, thoughts and limitations. Peter is imaginative in his working, participating with good fun in his banter and display of humour. Peter is well-behaved in adhering to and maintaining the posting protocols. Peter shows and is shown politeness, respect, empathy, care and consideration to and by others.

Analysis of Peter's posting patterns confirmed that his participation was predominantly out of school hours and his posts were equally divided between asking for and offering help. In reporting the findings of this case study, the varying roles that Peter takes on at different times have been tested against van Lier's conceptualisation of an individual's four-part ZPD. The analysis starts with studying Peter in a learning role through two sample threads, and includes a discussion of learning opportunities through: the mathematics involved, experiencing habits of more proficient mathematicians and how his self-determined thinking and understanding allowed other AskNRICHers to exploit these qualities. Threads resulting from Peter's persistent interest in mathematical induction initiated by a 'have you heard of' remark are then used to track his transition from a learning role to a helping role. The interest led Peter over a period of three months or so to gain familiarity and mastery [Wenger 1998] of the topic that he could later share with others. An extensive analysis of threads with Peter in a helping role brought out Peter's engagement with other people's problems, involving him in: offering expert help on topics he had already mastered; at times offering help when he was himself unsure of the answer but could work with the person requesting help to find the solution, and joining in another's thread asking his own questions to further his own development and interest. These varied ways of *'teaching but learning'* allowed Peter to work with, and gain knowledge, from more experienced, equal experienced and less experienced others, whilst at the same time using his own internal processes.

In presenting the results of the various analyses of Peter in a Learning and/or Teaching role, instances of each of the four parts of the individual's ZPD were exemplified. Hence this chapter has demonstrated that van Lier's four-part ZPD can be adopted to model Peter's interactions in AskNRICH and hence those of AskNRICHers in general. Furthermore, given that this case study is based in a virtual environment, these findings also show that van Lier's model can be appropriated from the classroom to a web-board context. This is one of this study's claims to new knowledge [see Claim 2 Section 15.3 p308].

The focus of the next chapter, which concludes the three-way exploration of AskNRICH, is three distinct threads, all on the same mathematical question but posted at different times, incidentally all involving Peter. The exchanges in the threads are used to illustrate two subjects already touched on in this chapter: AskNRICHers' contingent conversations [van Lier1996] and behaviours demonstrating traits attributable to professional mathematicians' ways of working [Cuoco et al. 1996].

Postscript

Peter's use of AskNRICH is now only spasmodic. Even though after many months of regular posting the need to use AskNRICH decreased, it had helped him to become even more independent:

> *... From then* [the day I ventured in AskNRICH] *I began to use the site regularly to use askNRICH when I got stuck. ... I ask questions in askNRICH much less now as I do not use it so much anymore because I now have more of a determination to finish a problem than I used to and so spend more time on a single problem.*
>
> *[Peter email communication]*

Chapter Eleven

Playing the Game:
Mathematicians' Interactions and Conversations
Perspective Three: Three Threads on one problem

> *…[using AskRICH] it's great to be able to talk and discuss with other talented mathematicians - an opportunity which I don't really have at school.* *[Web-Survey Respondent Male Year 12]*

11.1 Introduction

This chapter is the third of three Perspectives reporting the findings of interpretive analyses of a selection of message threads. The Perspective for this chapter is the analysis of three separate threads [**3Thds**] spread over a four-month period, but all discussing the same mathematical question. The first of the two analyses reported in this chapter differs from those reported in the previous two in that it uses a novel, visual technique to *explicitly* represent the network of connections between the thread participants and posts that exists in such a complex melée of interactions.

The purpose of this chapter is to:

i. demonstrate how the process of analysis of threads is facilitated and augmented[1] by the newly designed visual technique, a connection diagram
ii. show, by using both the visual connection diagrams and text-based interpretative commentaries on the threads, how the AskNRICHers' exchanges can be considered as conversation-for-education
iii. present the results of further analysis of the threads that demonstrates that the AskNRICHers display traits that can be considered to be attributable to professional mathematicians' ways of working

The work reported in this chapter contributes to addressing all three questions of **RG3**, but particularly sub-questions of **RQ7**: *What types of interactions are shown between the participants as they engage with mathematics? In what ways does the behaviour of*

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[1] Or even '(e)nriched'!

AskNRICH participants emulate the working practices of professional mathematicians? [see Table 8.1 p165].

van Lier's contingent conversation-for-education [see LRIII pp144-145] forms the theoretical underpinning of the reporting in this chapter of the AskNRICHers' interactions and conversations. The literature on Mathematicians of LRI [Section 2.8 pp53-56] underpins the relation of traits occurring within AskNRICHers' exchanges to those of professional mathematicians in a social setting.

The next section of this chapter introduces the Three Threads. The remaining three sections follow the division used above in setting out the purpose of this chapter in discussing: diagrammatic representations of interactions, conversation-for-education and mathematicians: people-who-do-mathematics.

11.2 The Three Threads

This section starts with a short introduction to the ideas that initiated the work and the rationale for using the Three Threads. It continues by presenting the mathematical problem that was the subject of the threads, the participants involved and a brief account of how the threads were analysed.

11.2.1 Background and Rationale

AskNRICH's full title, the **Ask-a-Mathematician** service, reflects the intention that there would be 'mathematicians' available to provide help [see p168]. The web-survey response quoted at the opening of this chapter, together with my increasing awareness of the web-board environment and the mathematical culture in which I am situated, triggered the idea of considering the ways that AskNRICHers' behaviours emulate that of professional mathematicians.

When I came across the three threads, I was reminded of my own experience of talking with other colleagues around the table during 'coffee times'[2] discussing a current mathematical problem of interest. The inference made here is that my colleagues and I, in talking about and doing mathematics in this way, consider ourselves mathematicians. Clearly, as the chapter's opening quotation shows, in the same way (some) AskNRICHers, talk and discuss mathematics and also identify themselves as mathematicians. The contributions made in the three threads conjured up the vision of a set of people periodically dropping in and out during 'coffee breaks' to *talk about* and *do mathematics*, just as in the physical world people may gather round a table to do the same thing.

The three threads, posted under different titles though actually all on the same problem, were identified for detailed analysis when studying posts for the Case Study [see Section 6.3.3 p118]. These threads involved twelve AskNRICHers, contributing 38 posts. The number of posts and the number of participants were of a manageable size providing a bounded situation for analysis. The presence of the two participants across all three threads suggested a further potential dimension of analysing the interactions, both within a single thread and across threads. Thus the AskNRICHers' discussions have some inherent similarities to the intermittent coffee-table chats.

11.2.2 The Mathematical Problem

The mathematical problem [see Figure 11.1] comes from the 2005 British Mathematical Olympiad [BMO] paper and is in two parts: firstly proving a statement and secondly seeing if the statement would be true if considered 'the other way round'.

> Let n be an integer greater than 6.
> Prove that if n - 1 and n + 1 are both prime,
> then $n^2(n^2 +16)$ is divisible by 720.
> Is the converse true?

Figure 11.1 The 3Thds Question

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[2] See Appendix 11.7 for an email version of the 'coffee-table' exchanges between my colleagues that we continue to engage in periodically, even post-retirement.

The problem concerns a traditional piece of mathematics, but the work required is at a level of difficulty that it is most likely to be beyond that found in the classroom. That is, it is neither readily available nor common within the school environment although the AskNRICHers starting each thread are of school age as indeed are some of the helpers.

11.2.3 The Participants

The problem appeared in *Onwards and Upwards*, the appropriate section for the level of difficulty, each time initiated by younger[3] secondary aged pupils who were using the problem in preparation, and as practice for, a forthcoming BMO paper. School national examinations would not have questions that reach this level. Some respondents offering help had either used the problem as practice the previous year or indeed had taken the BMO paper in which it had first appeared. The first and third threads were started by participants aged 14 to 15; the second thread was started by someone a year younger. Twelve different AskNRICHers contributed, two across all three, all the others to only one of the threads.

11.2.4 Starting to Analyse the Threads

To start the analysis, two accompanying interpretative commentaries were made for each of the 38 posts, one on the mathematics and one on actions. These commentaries went through three iterations, as the depth of consideration increased. The final versions of the commentaries appear as Appendices 11.1 and 11.2 respectively. In order to gain further insight, a full prose narrative account was made of the three threads [see Appendix 11.3]. For all three threads both the two interpretative commentaries and the prose account were used to derive the response types for each post or part of post. The post, poster, response type and a synopsis and/or comment on the interaction were then tabulated. This table was then augmented with the information on which participants could be linked to each interaction. The full table provided the information from which connection diagrams of all three threads could be constructed [see Appendix 11.4].

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[3] Younger in the sense that the people who initiated the thread were in school years where the mathematics curriculum being studied would be at the level for asking questions in the (lower) *Please Explain* Section.

The preparatory analytical work just described provides the basis for detailed examination of the AskNRICHers' interactions within and across the three threads reported in the following sections.

11.3 Diagrammatic Representations of Interactions

This section explains the role of the diagrams and how they complement the text-based techniques to provide a clearer, more comprehensive and rich result of the analysis. It then presents in diagrammatic form, followed by a discussion, the interactions and response types of both the first thread and ***John***'s contributions to all three threads.

11.3.1 Purpose of the Connection Diagram

Text-based descriptions do not explicitly model the complex network of connections formed by posts that do not necessarily follow the same simple sequence as the chronological order of appearance on the web-board. It is, for example, quite common to see a new reaction resulting from a post further back in the sequence. Similarly, the types of interactions embodied in posts remain implicit within the text [See LRII p98 for discussion of methods for depicting interactivity]. Chapter Six [Section 6.3.4.5 pp129-131] described the connection diagrams and typology of response devised in order to explicitly portray the jumbled, interwoven network of posts. However, the response type and connection information in the diagrams cannot convey the quality of the message text and thus the diagrams are *complementary* to the textual analysis. Nevertheless, this pictorial representation greatly facilitates and enhances analysis that demonstrates the presence of conversation-for-education [van Lier 1996: 167] by being able to explicitly see and thus gain a better grasp of the network of interactions.

Moreover, the connection diagrams aid consideration of AskNRICHers' mathematical traits. They enhance the comparison of traits with those of professional mathematicians by providing a more vivid image of the AskNRICHers as 'people with personalities' and hence traits, even though the evidence for the traits actually comes from the textual analysis.

However, it is important to recognise that the design of the connection diagrams is a prototype[4] for the three threads, adequate for the present purpose, to improve the depth and quality of analysis leading to the characterisation of AskNRICH. The finalised diagrams were not intended for any other purpose such as, for example, finding patterns in threads. They are a representation of my interpretation, devised after careful deliberation and closure, and naturally will always be open to scrutiny and re-interpretation by others.

11.3.1.1 Connection Diagram for Thread One

Section 6.3.4.5 [pp129-131] gave an explanation of the connection diagrams that are used to portray the relationships between posters, posts and types of response, together with details of the notation used. Table 11.1 below lists the posts, posters, message texts, response types and synopsis for **3Thd1** alongside the connection diagram. The response type column lists each response by its type and the posters involved (their names are abbreviated by omitting "Help"). A fuller version of the thread with the addition of information on the interval between posts is presented in Appendix 11.5. Appendix 11.6 contains the connection diagrams for each of the **3Thds** separately and for all **3Thds** combined.

The diagram shows 22 entwined responses, including examples of all five types, within a thread that involved only five AskNRICHers and just ten posts. Thus the complex structure of the connection network representing a complex melée of interaction is immediately clear.

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[4] the prototype response types and diagrams proved appropriate and satisfactory for undertaking the follow-up funded research project referred to in Chapter Fifteen [p316].

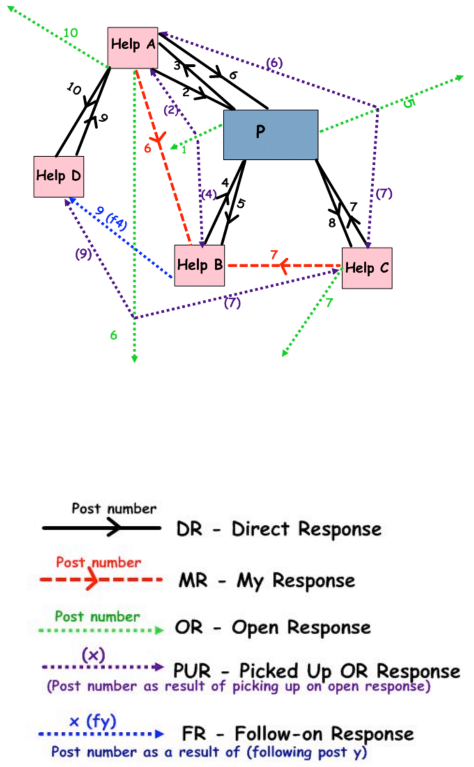
Post	Poster	Message text	Response Type	Synopsis of interaction / comments	Connection Diagram
P1	***Peter***	*Let n be an integer greater than 6. Prove that if n - 1 and n + 1 are both prime, then* $n^2(n^2+16)$ *is divisible by 720. Is the converse true?* *i have managed to prove the first part of the question using the fact that all primes are of the form 6n-1 and 6n+1. when i tried to prove the converse i cant do it.* *i know that 2 and 3 divide n and n is of the form 2mod5 3mod5 or mod5. from here where do i go? thanks*	OR PU by A & B	Completed first part of question but cannot do second part in finding if converse is true	 Figure 11.2 Connection Diagram for 3Thd1
P2	***HelpA***	*Do you think the converse is true?*	DR to Peter	Suggests starting with an intuitive approach – 'feeling' whether it is true or not true	
P3	***Peter***	*i presume that it isn't but im not very sure*	DR to A	Responds by saying that he assumes that it not true, but is not sure	
P4	***HelpB***	*If you look back over your proof, you used the fact that ALL primes are 6n-1 and 6n+1. However, is the converse of *this* true? Are all 6n-1 and 6n+1 prime? Using this, you can construct a counterexample.*	DR to Peter	Connects ***Peter***'s solution from the first part of the problem and suggests looking for a counterexample	
P5	***Peter***	*thanks i ve got it now,* *for anyone who's interested one counter example is 48.*	DR to B OR PU by A & C	Has found, and shares, 48 as a counterexample	
P6	***HelpA***	*or 24 ☺*	DR to Peter MR fr B4 OR PU by D	'Smugly' (via emoticon) suggests 24 would also do (in fact it does not)	
P7	***HelpC***	*Or if you really want to do no work whatsoever when it comes to multiplication just use 720*	DR to Peter MR fr B4 OR	Gives the 'blindingly-obvious-once-someone-has-pointed-it-out' solution of 720	
P8	***Peter***	*lol i totally missed that*	DR to C	Amused (lol - laughs out loud) at missing the obvious	
P9	***HelpD***	*Not to be a spoil sport, but I don't think 24 quite cuts it as a counterexample ☺*	FR to B4 DR to A	Politely suggests that 24 'does not quite cut' it as a counterexample	
P10	***HelpA***	*Sorry haha, I was thinking that all numbers 0 (mod 6) worked. Good job i didn't make that mistake when I took the paper last year!*	DR to D OR	Laughs at own error and shares mistaken thoughts	

Table 11.1 3Thd1: Posts with Response Type and Thread Connection Diagram

The interactions whose response type is easiest to categorise are those that are a **Direct Response** [**DR**] as they are always connect just two participants. Although, as signalled above, the diagram does not convey quality of the content, it is important to be mindful that many of these responses will contain the pedagogical exchanges that scaffold the learning [see Chapters Nine and Ten].

The nature of the web-board dictates that the first, trigger post, is open, sent out into the ether. However, the participants' common interest also engenders other posts addressing everyone in general. Both kinds of post are designated as **Open Response** [**OR**].

The other three response types are ways of recording instances of following/picking up on other posts. One of these three, **Picked Up Response** [**PUR**] records participants who pick up **OR**s, either from the trigger or other post(s). **My Response** [**MR**] has been used to record instances where a participant provides their own solution, to an idea (usually) presented to the person who has asked for help; for example in **3Thd1-P3** it is suggested to ***Peter*** that he may like to find a counterexample, which results in two other participants joining in, **3Thd1-P6&7**, with their own counterexamples. A **Follow On Response** [**FR**] records a participant who makes a direct reference to a post, having picked up on a suggestion from a different poster. For example, in **3Thd1-P9** ***HelpD*** joins in to correct ***HelpA***'s counterexample, **3Thd1-P6**, but this could only be done if ***HelpD*** was aware of the possibility of looking for a counterexample suggested by ***HelpB*** in **3Thd1-P4**.

The following sub-section illustrates a different use of a connection diagram in considering the participation of one poster, ***John***, in multiple threads.

11.3.1.2 Connection Diagram of One Participant’s Interactions Across Threads

Figure 11.3 [next page] illustrates the responses and interactions determined by the analysis of ***John***’s three contributions [**3Thds-P7**, **P21** and **P37** respectively]. ***John***’s three posts (in italicised text) are given in Table 11.2, together with a context and explanation of the mathematics or interaction.

Thread & Post Number & Date and Post Text		Comment
3Thd1-P7 Jan 2007	In reply to ***Peter***’s counterexample of 48 (strategy suggested by ***HelpB*** **3Thd1-P6**) and ***HelpA***’s (which turns out to be incorrect) 24 ☺ *Or if you really want to do no work whatsoever when it comes to multiplication just use 720*	When n=720, n-1=719 is prime but n+1=721 is not prime n^2 must automatically be divisible by 720 and thus the requirement that: $n^2(n^2 +16)$ is divisible by 720 is met ‘without doing any work’ This is a ‘blindingly-obvious-once-someone-has-pointed-it-out’ solution of 720 .
[3 minutes later ***Peter*** replied: *lol i totally missed that*: indicating his amusement at missing the obvious]		
3Thd2-P11 Mar 2007	Responding to request from ***R*** (the poster who has asked for help): I don't really understand this PETER. Sorry. Does anyone know a different way to give a hint/explain P's hint? (20 minutes later) *If n-1 and n+1 are prime, what are the possible remainders when you divide n by 5?*	[***R*** has returned the next evening to continue to try and solve the problem] This post offers a hint which connects with ***HelpE***’s [**3Thd2-P3**] questions from the previous evening to ***R***’s own reply [**3Thd2-P4**] to ***HelpE*** and ***Peter***’s [**3Thd2-P9**] afterthought of mod5
3Thd3-P11 Nov 2007	Responding to ***S*** (the poster who has asked for help) I've just realised that my counter example is exceedingly wrong as while 720 is divisible by 720, $720^2(720^2+16)$ isn't. (10 minutes later) quotes the above and replies: *Yes it is!* $720^2(720^2+16)/720 = 720(720^2+16)$	***HelpC*** should know that 720 is correct as it was they who suggest that it as a counterexample in thread one (which so amused ***Peter***)

Table 11.2 John’s (HelpC’s) Involvement across all Three Threads

John’s interventions are seemingly considered contributions responding to others’ posts and are potentially far reaching in their effect on others’ learning. In **3Thd1**, ***John*** picks up on a suggestion offered by ***HelpB*** of looking for a counterexample. Following ***Peter***’s offer of 48 as one possible counterexample and ***HelpA***’s seemingly rather smug, though actually incorrect, response of 24 being even better as it is smaller, ***John*** comes in with his own solution of 720 and ‘*no work*’ comment. This is delivered with a certain degree of humour and receives an equally humorous as well as admiring response from ***Peter***. Interestingly ***John*** did not correct ***HelpA***'s error of 24, even though the two posts were made five minutes apart. This is perhaps a fine illustration that not all can be ascertained from latent content!

In **3Thd2**, ***John***'s contribution, building on two earlier posts, attempts to move ***R*** on in their solution. Although ***John***'s post is in response to ***R*** asking if anyone else can help as they are not understanding ***Peter***, the remaining exchanges on the thread are all between ***R*** and ***Peter***. In **3Thd3**, ***John*** has taken no part until ***S*** suddenly returns to announce that their own solution of 720, which is of course precisely the same counterexample as ***John*** had so 'cleverly' made in the first thread, '*is exceedingly wrong*'. Again ***John***'s intervention is delivered with humour, reminiscent of a pantomime exchange. One can almost hear the laughter in ***S***'s reply, though of course this can only be a conjecture!

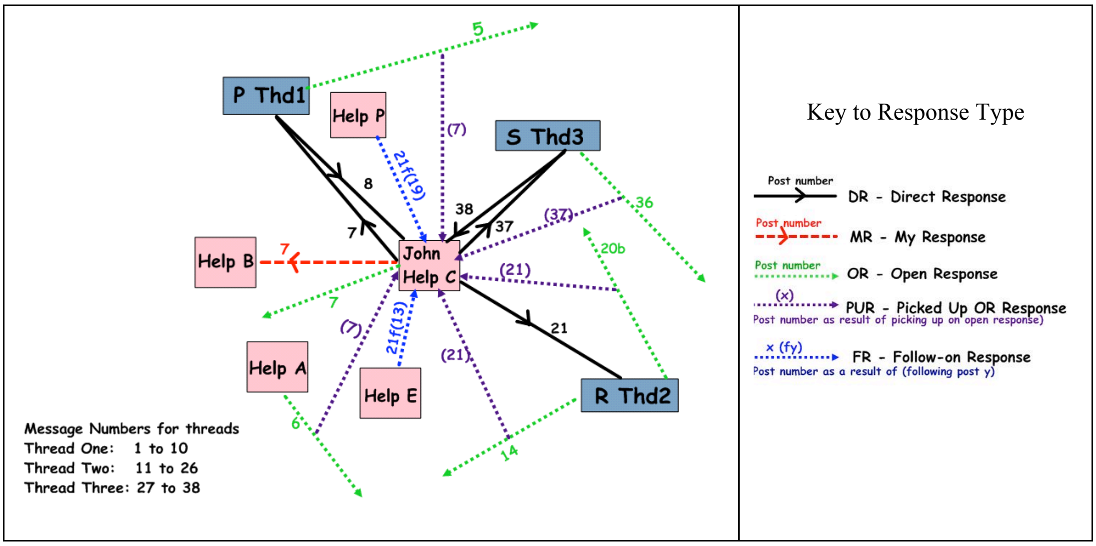


Figure 11.3 Responses and Interactions resulting from John's (HelpC's) contributions to the 3Thds

These excerpts involving ***John*** not only highlight the return in **3Thd3** of the 'clever' counterexample he offered in the first, but also convey the cyclical nature of the conversation that to me was 'typical' of how mathematicians might speak to each other 'round' a coffee-table. Co-incidentally and completely unrelatedly, the coffee-table is the same scenario that van Lier uses in his discussions on conversation types, stating that what others might perceive as idle chat needs to be defended for the advantages it brings [van Lier 1996: 168]. Indeed the exchanges within the three threads, whilst not idle, do indeed demonstrate the usefulness of seemingly light-hearted chat.

The next section now uses the connection diagrams for all **3Thds** [see Appendix 11.6] to analyse and relate the AskNRICHers' exchanges to Conversation-for-Education.

11.4 Conversation-for-Education

The 'lines' going in all directions, succinctly captured in the connection diagrams, shows visually the chaotic, in the sense of jumbled and interwoven, nature of the exchanges. Moreover, the need for five different response types and the requirement for a many-to-many, or multi-mapping between post and response type, is a further sign of complexity. In **3Thd1**, for example, seven out of the ten posts have been allocated to more than one response type [see Table 11.1 p249]. There are several possible multi-mapping scenarios: different response types may be allocated as a result of the entire content of the post or even just one fragment of it, or several different (discrete) fragments of the post each results in the allocation of one or more response types. Table 11.3 illustrates four allocations given to ***HelpA***'s brief post [**3Thd1-P6**]: **MR** as it was made because of ***HelpB***'s suggestion of looking for a counterexample; **DR** responding to directly ***Peter***'s solution, **OR** left for others to read and indeed **PUR** as it is later picked up by ***HelpD*** (to tell ***HelpA*** that the example was incorrect).

Thread & Post Number and Post Text		Comment
Thd1 P6	***HelpA***: or 24 ☺. Including ☺ gives the inference that 24 is a better answer than ***Peter***'s 48 as 24 is smaller solution	My Response from P4, ***HelpB***'s suggestion that ***Peter*** looks for a counterexample Direct Response to ***Peter*** (who had given 48 as a counterexample) Open Response Picked Up later by ***HelpD*** who has spotted that 24 as a counterexample is incorrect as it does not satisfy divisibility by 720

Table 11.3 Example with One Post Assigned More Than One Response Type

In Table 11.4 ***Peter***'s response [**3Thd1-P5**], has two fragments: he thanks ***HelpB*** [**DR**], and then shares his solution which in this case in turn results in having different types allocated [**OR**] that is also picked up by [**PUR**] ***HelpA*** & ***HelpC***.

Thread & Post Number and Post Text		Comment
Thd1 P5	***Peter***: *thanks i ve got it now* *for anyone who's interested one counter example is 48*	Direct Response to ***HelpB*** Open Response Picked Up by ***HelpA*** & ***HelpC***

Table 11.4 Example with Two-Part Post Assigned Multiple Response Types

Thus so far the connection diagrams provide evidence visually for the flexibility, unpredictability and symmetry of the exchanges that van Lier [1996: 175] states to be properties that define a conversation-for-education [see LRIII Section 7.3 pp144-145]. The information necessary to construct the connection diagrams is extracted from the interpretive commentaries [see Appendices 11.1&11.2 earlier] resulting from analysis of the three threads. This section continues to discuss conversation-for-education through a further iteration of analysing the AskNRICHers' exchanges using the connection diagrams and the interpretative commentaries together.

The conversations of two AskNRICHers (***Peter*** and ***John***) who contributed across all **3Thds** vary in format in the same way that 'ordinary' conversations may. ***John***'s three short responses are all in a helping role. In **3Thd1** ***Peter*** is in the role of learner, in **3Thds2&3** he is a helper. **3Thd2** ends up by ***Peter*** striking up an intense one-to-one conversation with ***R*** as ***Peter*** persists, even though ***R*** has asked generally if someone else can help as he apologetically explains that he is not understanding ***Peter***'s explanations. In addition to acknowledging ***R***'s predicament: *'since i don think that you understand modular arithmetic (dont worry about this) i shall write …'* [**3Thd2-P12**], ***Peter***'s subsequent teaching strategies [see Chapters Nine and Ten], respond to ***R***'s ideas and work, until ***R*** had '*Got it!'* [**3Thd2-P15**].

In **3Thd1-P5**, although ***Peter*** now has a solution to the problem, which could have ended the thread at **3Thd1-P4**, he openly shares it, thus continuing the conversation. Later, ***John***'s clever solution of 720 [**3Thd1-P7**] is instantly recognised in ***Peter***'s response: *'lol i totally missed that'* [**3Thd1-P8**]. This statement, although short, conveys laughter, humour, and admiration, qualities of a congenial atmosphere in which to engage with others.

Although ***John***'s responses might superficially appear minimal, suggesting 720 in **3Thd1-P7** and returning with it in **3Thd3-P11** provides insight for everyone as well as specifically contributing to ***Peter***'s and ***S***'s engagement with the problem. ***John***'s apparently sudden intervention at a crucial moment, picking up on the ideas and help of others, that he too could have contributed, conveys the sense of someone capable of giving cohesion to the conversations, further contributing to an amiable environment.

Chapter Seven [p145] relates that van Lier [1996] chose the word contingency to illustrate that, in free conversation, what is said is dependent upon what [others] have to say and what they chose to say is unpredictable; thus potentially there is an equal distribution of rights and duties. For the AskNRICHers, the content of the exchanges (the talk) is entirely in the hands of those AskNRICHers who choose to contribute *at that moment in time*. Thus the AskNRICHers' initiate and manage their own conversations; each chooses what they wish to write, although they should do so within the bounds of the Posting Protocols. Even if the protocols remove complete freedom, they nevertheless facilitate conversation-for-education and promote pedagogical interactions of the type that are positioned *towards* the outer reaches of the contingency line on van Lier's diagram [1996: 179] reproduced in this thesis [Figure 7.1 p146].

The analysis of the **3Thds** shows that the AskNRICHers' conversations are positioned towards the outer ends of the other six radii of van Lier's diagram. If not all talk is truly conversational, it is at least dialogic as exemplified by ***Peter***'s and ***R***'s exchanges in **3Thd2**, and certainly not monologic. There is always the potential for the talk to be substantially symmetrical rather than asymmetrical, in all three threads the person initially asking for help interacts as an equal with those offering the help. Although the problems posted on AskNRICH, as in the case of the three threads, are often based on mathematics examinations or tests, the underlying ethos of being involved with the problems is always one of process-orientation with the emphasis on engagement and academic and personal growth. The openness with which an AskNRICHer can show any difficulties and a helper checking whether an explanation is or is not being understood, ensure that the exchanges thrive on a proleptic understanding gap and an exploratory teacher role. Again ***Peter***'s and ***R***'s exchanges in **3Thd2** provide an example of this. Finally, by initiating and continue to pursue posts to their own satisfaction, AskNRICHers determine their own actions. Thus, taken together with the examination of contingency in the previous paragraph it can be seen that the pedagogical interactions in the **3Thds** tend towards the least restricted type of pedagogical interaction, designated as transformational. Furthermore, similar arguments may be applied to the **ExThds** and many of the **CSThds** and thus much of the activity in AskNRICH provides / promotes opportunities to engage in transformational pedagogy [see Claim 2 Section 15.3 p308].

From Figure 11.3 [p252] it can be seen that ***John***'s three contributions, all picking up on open responses, link with five different AskNRICHers. ***John***'s first response [**3Thd1-P7**] is unlikely to have been made if other AskNRICHers had not already been talking about counterexamples. The most serendipitous response of all was ***John*** in **3Thd3-P11** assuring ***S*** that they really were not 'exceedingly' wrong. It could not be predicted whether ***John*** would see ***S***'s remark or not, but that he did should have helped ***S***'s confidence. When ***S*** had realised their error, hopefully they did not mind others knowing that they had made a 'silly' mistake. They were after all in good company with ***HelpA*** of **3Thd1** who had originally thought that 24 was a counterexample. Such to and fro, 'free-fall', conversations constantly suggesting, correcting and debating are part of being a mathematician, the focus of the next section.

11.5 AskNRICHers Emulating Professional Mathematicians?

> When discussing the process of mathematical discovery, mathematicians now openly acknowledge making illogical leaps in arguments, wandering down blind alleys or around in circles and formulating guesses based on analogy or on examples that are hidden in the later, formalized exposition of their work. [Epp 1994: 257]

This section focuses on a part of **RQ7**, *examining ways in which the behaviour of the AskNRICHers emulates the working practices of professional mathematicians*. This is addressed by a revisiting and further analysis of the threads to draw out instances that demonstrate behaviours and activities, hereafter combined and labelled traits, attributable to the way that mathematicians work [see LRI Section 2.8 pp53-56] as exemplified in the quotation from Epp above.

Although AskNRICHers post from, and in, an isolated location they are also working together within a virtual environment. This 'working together' led to the vision described at the beginning of this chapter of people metaphorically sitting round[5] a coffee-table talking about and/or doing mathematics. The contention is that this is little different from the way that professional mathematicians work round a real table during coffee breaks, a well-recognised tradition as illustrated in Bollobás' [2006] book: *'The Art of Mathematics: Coffee Time in Memphis'*.

---

[5] 'Round' seemed appropriate as John's suggestion of 720 in **3Thd1** came 'full circle' in **3Thd3**.

The coffee-table scenario was examined using the content of the three threads.
Section 11.2.4 has already reported on how the initial intensive analysis of the three threads, to draw connection diagrams and to establish the presence of conversation-for-education [van Lier 1996: 167] was carried out. When undertaking the further analysis reported below, a reflective standpoint [Brown 2001] was adopted for revisiting the interpretations made of the posts, further informed both by the literature [LRI] and my “tacit knowledge … knowing-in-action” [Schön 1983: 49] of what it means (to me) to be a mathematician.

In order to report the findings I compiled a list of traits that, as Cuoco et al. [1996] stated about their collection of habits, is neither exhaustive (mathematicians may well do more than this) nor exclusive (other people may well do some or all of these). Nonetheless all the traits have been present during my own ‘doing’ and ‘talking about’ mathematics with colleagues. The traits presented below are partitioned into three sections to reflect those that can be considered as: (a) techniques for the mathematician’s toolbox, (b) customary occurrences when engaged in mathematics and (c) social graces embedded within the exchanges. The last of these adds more detail to the similar features first discussed in Chapter Nine under the Social and Personal theme. Some of the traits in the list and others may have already been observed in the work reported in the previous two chapters, but here the traits reported spring only from the content of the **3Thds**’ 38 posts now analysed for evidence of the coffee-table analogy.

For simplicity of presentation, the findings appear *solely* in a tabular form with the discussion element of this part of the chapter integrated into the table. Thus it is necessary for the reader to follow the content of the tables. In the first two tables each trait has an entry that consists of:

- illustrative example(s) extracted from the posts
- example(s) in the literature reporting the same trait
- discussion of the trait and/or the part it plays in emulating mathematicians ways of working

The final table builds on the work of Chapter Nine and thus contains only illustrative example(s) and discussion.

11.5.1 Techniques for the Mathematician's Toolbox

Findings: Techniques for the Mathematician's Toolbox
Trait 1: Follow Hunches/Intuition
Example The first response to ***Peter***'s request for help: *'Do you think the converse is true?'* [**3Thd1-P2**] is inviting ***Peter*** to follow a hunch/intuition. **Literature** *Develop your intuition – But don't trust it completely* [Houston 2009: x]. Intuition is one of five categories in the epistemological model for coming to knowing mathematics [Burton 1999, 2004]. 83% of Burton's study group supported the notion of intuition or insight. One of five common characteristics of Mathematical Creativity [Sriraman 2008a: 1]. **Discussion** Intuition is advantageous when the hunch is correct, less so when not. Nevertheless, there is often an inexplicable and unquantifiable feeling that one is on the right track whilst working on a mathematical problem. Even if it is difficult to quantify intuition the feeling for something to work mathematically comes from making an educated guess (with the presumption that it may be wrong). However ***Peter***'s reply *'i presume that it isn't but im not very sure'* [**3Thd1-P3**] indicates: (i) a still tentative feeling and not yet prepared to follow one's hunch and (ii) a possible interpretation by ***Peter*** that the wording of the problem suggests the converse not being true. If the converse were true it would be natural as a question setter perhaps to simply ask for a proof, or in fact most likely to set the whole question up as an 'if and only if'.
Trait 2: First search for Counterexamples
Example ***HelpB***: '*... you can construct a counterexample'* [**3Thd1-P4**] coincides well with a hunch that the converse is not true. **Literature** *...where the true mathematician has a chance to shine. ... given any statement try to find a counterexample* [Houston 2009: 93]. Promotion of using counterexamples in proof strategies [Stylianides nd; Stylianides & Stylianides 2009; Zazkis & Chernoff 2008]. Students' views on rigour of counterexamples [Simon & Blume 1996]. **Discussion** Looking for a counterexample is a common starting point in proving something is not true. If a counterexample can be found it is a quick and succinct way of completing a proof. If a counterexample cannot be quickly found, relatively speaking, generally there is a move to alternative, more complex methods but this decision is not taken lightly.
Trait 3: Always consider special cases (especially involving zero)
Example ***HelpG****: 'One small thing you've missed - n can be 0 mod 5, but that gives you* n^2 *is 0 mod 5 so you're still fine'* [**3Thd2-P6**]. **Literature** The translation to special cases is almost automatic (Talk Big & Think Small) [Cuoco et al. 1996: 384]. **Discussion** Part of a mathematician's toolbox is to ensure that any special case is carefully considered. The number zero can be problematic, for example division by zero is undefined. It either disappears for example when 'or mod5' should strictly be '0mod5' [**3Thd1-P1**] though admittedly no-one picked up on this, or is omitted, not especially considered in a proof [**3Thd3-P5**] which was picked up in ***HelpG***'s response above.

Table 11.5 Examples from the 3Thds of Strategies for the Mathematician's Toolbox

The metaphor of a *toolbox* containing mathematical tools, also sometimes labelled an arsenal of techniques [Houston 2009], is in widespread use [Black Douglas nd, Wolf 1998] and commonly understood. The AskNRICHers use this metaphor, for example [**ExThd2-P3** Chapter Nine]: *'always look to improve your problem solving 'toolkit' and to add more tools* *to it'*. Schoenfeld [2006: 500] draws a similar analogy between his passion for cooking, necessitating the gathering together of a range of implements, with his other passion of solving mathematical problems. Table 11.5 describes three strategy items that can be placed in the toolbox.

11.5.2 Customary Occurrences when Engaged in Mathematics

When mathematicians are working on problems, either individually or as a group, there are instances that can commonly occur attributable to either people and/or the situation. Opportunities to experience such traits arise through being active, observing and collaborating as advised by Houston [2009], taking part in the mathematical journey [Burton 2004]. Table 11.6 details seven such traits that mathematicians have or things that they do that can be seen in the three threads. Having these traits is not exclusive to mathematicians, as already stated, but mathematicians do have these traits and the AskNRICHers are working on mathematical problems.

Table 11.6 Examples from the 3Thds of Customary Occurrences when 'Doing Mathematics'

Findings: Customary Occurrences when Engaged in Mathematics
Trait 4: Thinking Out Loud
Examples ***R***: *'Is there a way to narrow it down further? Or can m have multiple values?* [**3Thd2-P13**] is followed nine minutes later by: *'Assuming m can have multiple values ... Yes, so n can be expressed as…'* [**3Thd2-P14**]. **Literature** The habit of noticing something and wondering why [Cuoco et al. 1996: 387]. *Talking is a good way of getting things done* [Burton 2004: 130]. **Discussion** ***R*** raises question(s) and before any reply arrived proceeded to answer them in the style of a rhetorical question. Just asking a question (saying it out loud) can provide the way forward, especially if it is likely that someone can answer it for you if necessary.
Trait 5: Amusing Howlers – Glaring Errors
Examples ***HelpA***: or 24 ☺ [**3Thd1-P6**]. ***S***: *'I've just realised that my counterexample is exceedingly wrong'* [**3Thd3-P10**]. **Literature** *Don't worry about being wrong* [Houston 2009: x]. **Discussion** On the contrary 720 is **not** exceedingly wrong [**3Thd3-P10**], neither is 24 a counterexample [**3Thd1-P6**], huge but simple mistakes to make (howlers). We all make them, and we neither mind making them nor mind seeing others do something silly. Maybe mathematicians have the confidence not to mind but life would be duller if we did not have howlers sometimes.

Findings: Customary Occurrences when Engaged in Mathematics

Trait 6: I wish I had thought of that!

Examples

HelpC: *'Or if you really want to do no work whatsoever when it comes to multiplication just use 720'* [**3Thd1-P7**].

Literature

Collaboration – work with others if you can [Houston 2009: x], *benefit from experience of others* [Burton 2004: 130].

Discussion

The obvious only becomes obvious when it has been made so by someone else or a subsequent thought. 720 is itself a comparatively high number but given that statement is already divisible by 720 then divisibility is assured. (n+1 = 721 is divisible by 3 so not prime). My reaction was similar to ***Peter***'s *'lol i totally missed that'* [**3Thd1-P8**] when one realises the 'of course' nature of the value given. There is always admiration and respect when a colleague comes up with the obvious when nobody else has considered it yet.

Simplicity is the ultimate sophistication [Leonardo da Vinci].

Trait 7: Scribbled working

Examples

See ***S***: *'This is what I've done …'* [**3Thd3-P4**].

Literature

… develop the habit of writing down thoughts … [Cuoco 1996: 379].

useful to formulate written and oral descriptions [Cuoco et al. 1996: 379].

See also quotation by Epp [1994] cited at the beginning of this section.

Discussion

To communicate in writing via the web-board, the level of detail in any explanation or working out will be relatively high when compared to how people might physically sit close together and scribble on the back of an envelope. Nevertheless informal working out definitely making sense to the writer will probably make sense to another mathematician. This is very different from the expectation that many teachers have in a classroom, often a consequence of the emphasis on method marks in an examination. ***S***'s explanation is succinct and clear, even if some working is lost between the lines e.g. ***S*** fails to include the step that as n^2 is divisible by 3^2 then $16n^2$ is divisible by 3^2 which is (strictly) needed to state therefore and indeed some parts omitted (the case of zero in Table 11.5).

Trait 8: Having Afterthoughts

Examples

Peter: *'sorry ive realised you can write this as mod5'* [**3Thd2-P9**] one minute after posting an explanation using multiples of 6 and mod 30 (probably connected/considered by 5x6=30).

S: *'I've just realised that my counterexample is exceedingly wrong'* [**3Thd3-P10**].

Literature

Any definition of a mathematician should probably include the attribute (or defect) of not being able to leave well alone [Brakes 1995: 388].

See account of working on a handshake problem [Rowland 2003].

Discussion

Problems can 'nag' away in one's mind even after proposing a solution, often subconsciously one is looking for a 'better' solution, whatever better might mean in this context. ***S***'s afterthought indicates that having one is not necessarily fortuitous (as 720 is actually correct)!

Findings: Customary Occurrences when Engaged in Mathematics
Trait 9: A Moveable End Point to any Finished Solution
Examples ***Peter***: *'thanks i ve got it now, for anyone who's interested one counter example is 48'* [**3Thd1-P5**] is not the end of the thread with other offers arriving. In **3Thd3** two participants (***ANP1*** & ***ANP2***) who add points beyond the immediate problem (that has been solved). This opens up conversation [see LRIII p147] and has the potential to add further knowledge: *'If the converse were true, then it would be a really, really fast way to find big prime numbers!' 'Not to mention being a proof of the twin prime conjecture!'*[**3Thd3-P8&9**]. **Literature** This is a variation on 'not leaving alone' [Brakes 1995] above, but the distinction between this and 'afterthoughts' is that here other people are the ones who are returning to the problem. This is similar to some of the advantages of collaborating given by Burton's [2004] mathematicians: e.g. increase in quality and quantity of ideas, get into areas that one may not have thought of. **Discussion** ***Peter***'s post [**3Thd1-P5**] suggests the problem is concluded but others remain fixed on the problem finding their own examples. (Even though ***Peter***'s post above indicates that he is satisfied he still looks back at the thread and comments back to ***HelpC***). Fortuitously in minimising incorrect solutions remaining as errors, ***HelpD***'s posts some six hours later pointing out that 24 is not a counterexample and again ***HelpA*** responds. In **3Thd3 *S***, having found the required solution with satisfaction, like ***Peter*** returns an hour later with the comment: *'By the way, for it to be a proof of the twin prime conjecture…'* [**3Thd3-P10**] although whether this had had any contribution to the howler mentioned above remains open to question. When a solution is found to a problem it is not necessarily the end point. Someone might at any time (immediately or much later) return to it and start another conversation. This might be even more prevalent within a virtual environment as new posts are flagged as such, and thus potential intrigue can draw the correspondents (and lurkers) back. Using the two examples that illustrate the on-going nature of a solved problem, the three participants (***ANP1***, ***ANP2*** & ***HelpD***) who extend the conversation, though regular posters, had not contributed to the thread before the solution had been shared. In this sense they 'lurked' within these threads.
Trait10: Tutoring on Unfamiliar Territory
Examples ***Peter***: *'since i don think that you understand modular arithmetic (dont worry about this) i shall write in a basic algerbraic form'* [**3Thd2-P12**]. **Discussion** There can be situations where the mathematics needed to solve a problem is new or unfamiliar to a participant in the discussion when someone in the group may resort to a detailed explanation or even some direct teaching. ***Peter***, after trying with some minimal hints, decided to introduce ***R*** to modulo arithmetic using a fairly didactic manner. The explanation given eventually became sufficient for ***R*** to complete the solution successfully. Some 'coffee-table' discussions between professional mathematicians would include explanations, presented to colleagues/peers in a similar way as ***Peter*** did here, seeking to explain in a way that ***R*** would find understanding, rather than just telling.

11.5.3 Social Graces Embedded within Exchanges

A collegiate group like that proposed of mathematics 'people' sitting round a table during a coffee break is likely to include some social and personal exchanges that contribute to a friendly atmosphere. Similar exchanges can be seen within the **3Thds**, see Table 11.7 below: a sense of camaraderie appears, evident through a combination of banter, humour, admiration, praise, politeness, success accompanied by exhilaration and personal asides. The headings in Table 11.7 evolved from the intensive analysis described earlier in this chapter and provides further findings that relate to the social graces of the delivery of the responses.

Findings: Social Graces Embedded within Exchanges
Trait 11: Banter/Humour
Examples ***HelpA***: or 24 ☺ (seemingly smug?). ***HelpC***: Or if you really want to do no work whatsoever when it comes to multiplication just use 720 ***Peter***: lol i totally missed that [**3Thd1-P5-7**]. 'Pantomime' like response by ***HelpC***: *'Yes it is!* to ***S***'s *'while 720 is divisible by 720,* $720^2(720^2+16)$ *isn't'* [**3Thd3-P10&11**]. **Discussion** See Section 9.3.3 p200 for distinction made between banter, light-hearted teasing as in ***HelpA***'s comment and humour, the genuine neutral witty remark, interpreted as present in the other examples above.
Trait 12: Politeness
Example **R***: 'I don't really understand this PETER. Sorry. Does anyone know a different way to give a hint/explain Peter's hint?'* [**3Thd2-P10**]. **Discussion** Although the posting protocols ask for politeness, the comment above is indicative of the sensitivity that people show to each other when realising that someone is genuinely trying to help but is not succeeding.
Trait 13: Admiration
Example [See 'blindingly-obvious' remark in Table 11.1 above]. ***Peter***: *'lol i totally missed that'* [**3Thd1-P7**] in response to the simplicity of choosing 720. **Discussion** Suggestion of admiration made more explicit by using the shorthand text for laughing-out-loud
Trait 14: Pleasure at Success
Example ***R***: *'Got it!'* [**3Thd2-P15**]. **Discussion** Image of ***R*** jumping off their chair, punching the air and so pleased that at last and after a struggle the problem was solved.
Trait 15: Praise
Example ***Peter***: *'yes well done this completes the proof. ... yes anyway. well done'* [**3Thd2-P16**]. **Discussion** A justly apt congratulatory post, implicitly recognising the work that ***R*** has put in. (It is always good to have one's endeavours praised and the additional recognition of some hard work finely accomplished).
Trait 16: Personal Comments
Examples ***Peter***: *'i remember fondly this question. this was my first bmo question i completed. arrr memories ...'* [**3Thd2-P16**]. ***Peter*** (at 7.38pm) *'I think that there is a nicer way but this is still nice and simple and im tired at the moment'* [**3Thd2-P8**]. **Discussion** The first is a personal reminiscence of fond memories that offers a sociable 'joining-the-club' feeling. In keeping with many of the other threads analysed, there is a personal comment at the end of each of these three threads, though in the case of the first and the last it is in response to having made a 'howler'.

Table 11.7 Examples from the 3Thds of Social Graces Embedded within Exchanges

Although Houston [2009: x] makes it clear in his list of advice that there is no competition in collaborating, Burton's [2004] research suggests that professional mathematicians can still experience competition even within a collaboration and/or co-operation situation [pp131-134]. Any competitive element or one-up-man-ship in the posts has, after some consideration, not been included as a feature since it is to some extent problematical to determine where it appears. Having read a very large number of posts, including the web-board's private area, I have concluded that the care that participants show for each other is much more explicit than any attempt to be competitive. Moreover, there appears a respect for those who are the most talented (or rather, score highly in any competitions) and a great deal of compassion for the many who actually score few marks[6].

This section has examined ways in which the AskNRICHers working together could be considered to emulate the working practices of professional mathematicians in a social setting. The findings have supported the argument, alluded to in Section 10.3.2.2 [p223], that because the AskNRICHers are immersed in a rich mathematical environment, engaging with others who are enthusiastic about the subject and experiencing their mathematics, they will themselves engage in activities in ways similar to professionals in the field.

11.6 Features Summary 4

The Features Catalogue for this chapter, relating to Social Presence, is presented in Figure 11.4 below.

The term 'Social Presence' was used as the identifier for this Catalogue as the Features listed in Figure 11.4 are strongly similar to some that Garrison and Anderson [2003: 51] classified[7] as social presence. The term Social Grace was adopted to portray the camaraderie in a social setting conveyed in the proposal of mathematicians sitting round a coffee-table. The banter, humour, friendly 'chit-chat', often accompanied with a peppering of emoticons and texting abbreviations are liberally sprinkled throughout the threads is part and parcel of everyday conversations taking place within an environment that allows free expression. These qualities are pervasive throughout AskNRICH and provide a major part of the cohesion that binds the AskNRICHers together.

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[6] In the 1989 Putman competition in the USA the median score was 0 out of a possible 120 which was not unprecedented [Larson 1994: 33].

[7] A classification that used the indicators of expression of emotions, use of humour and self-disclosure for the Affective category and vocatives, inclusive pronouns and phatics and salutations for the Cohesive category. The collaborative model underpinning of the Open Communication category is incompatible for AskNRICH.

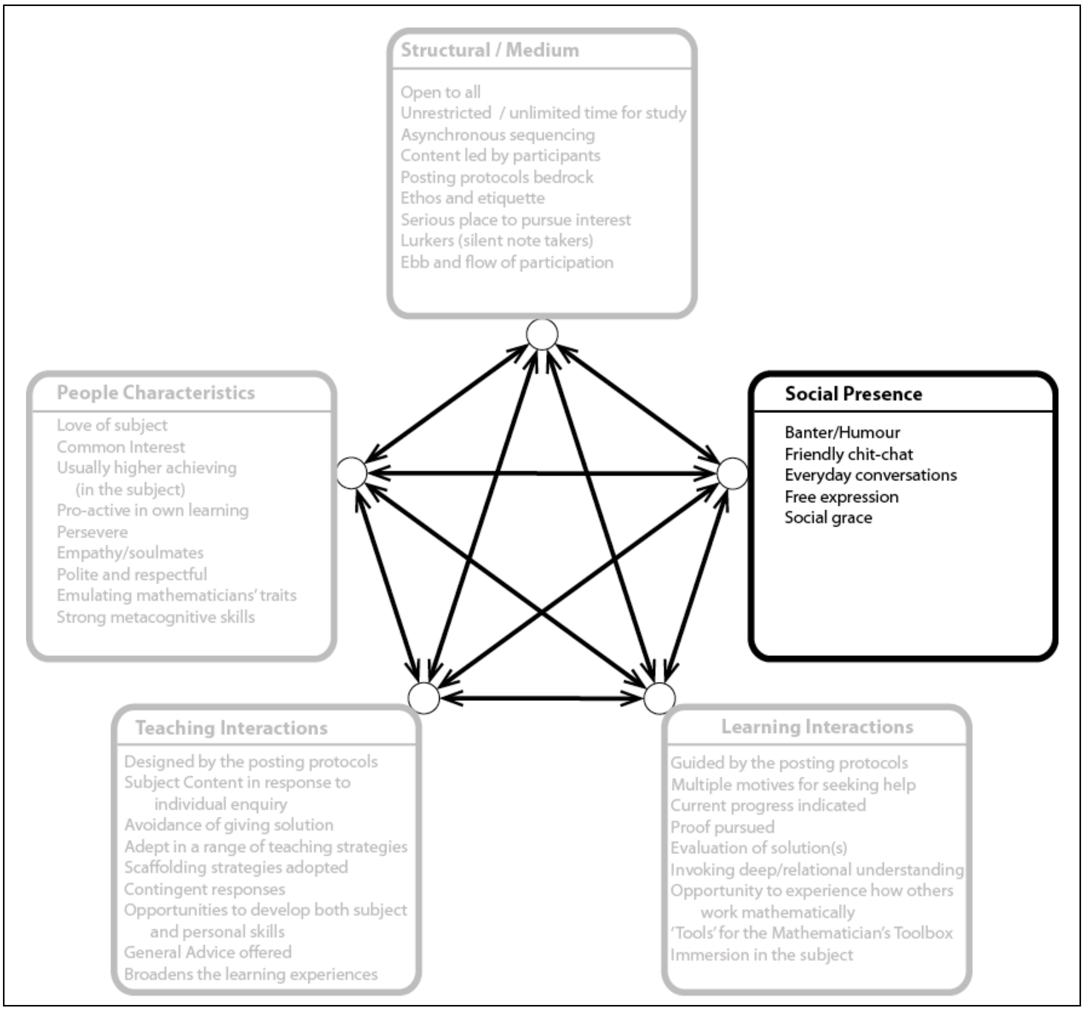

Figure 11.4 Features Catalogue: Social Presence

11.7 Conclusions

This focus of this chapter has been on the mathematical activities, interactions and exchanges within three specific threads all on the same problem that had appeared over a three-month period on AskNRICH.

The visual mapping of the Three Threads confirmed that the complexity of the network of interactions within the AskNRICHers' conversations was well beyond that of simple turn-taking. The jumbled and interwoven nature of the exchanges made clear by the visual mapping is used to argue that these are contingent conversations which thus met the criteria to be considered 'conversation-for-education' [van Lier 1996: 175]. Further consideration of the threads in relation to individual components of contingency leads to the claim that the type of exchanges also tend towards the most free of pedagogical interaction that is called transformational [van Lier 1996: 180].

Further analysis of the Three Threads made against perceived ideas of what it might mean to be a mathematician resulted in establishing a number of ‘Traits’, a collective label for behaviours and activities. These traits each belonged to one of three groupings. One group brings together the techniques that an individual can add to their own (mathematician’s) tool-box, comprising of a set of traits made possible where there is a mixture of the experienced and less experienced, expert and (relative) novice. The other two groups emerged as more focused on the person: customary occurrences when engaged in mathematics and social graces embedded within exchanges that help to maintain the desire to meet. The findings show that the AskNRICHers’ work, activities, interactions and exchanges emulate those of professional mathematicians in the coffee-table analogy. Moreover, the camaraderie revealed by the investigation of that analogy combined with the to-and-fro, free-fall conversations highlights the Social Presence amongst the AskNRICHers.

This chapter has reported the third and final Perspective used to explore AskNRICH. Its findings are combined with those from the other two Perspectives in a wholistic view presented in the next ‘Interlude’ chapter.

Postscript

> I became a mathematician by falling in love with mathematics
> [Papert 2006: 581]

And as another very famous mathematician, Erdös, was apparently fond of saying:

> A mathematician is a machine for turning coffee into theorems
> [Hoffmann 1998: 7]

Chapter Twelve

Taking Stock - A Reprise of Findings

> *I feel that my mathematics improves by learning from other people's solutions and methods and being able to organise my thought into a method/proof that is comprehensible by other people.*
>
> *[AskNRICHer aged 14, 2010]*

12.1 Introduction

The previous four chapters have reported on the analysis and findings of an in-depth exploration of AskNRICH using the three Perspectives. The purpose of this brief interlude chapter is to:

i. take stock of the many findings reported separately in the four previous chapters through three diagrams illustrating a wholistic overview of the AskNRICH artefact
ii. indicate the directions taken in the remaining chapters that lead to defining a characterisation of AskNRICH, building on the findings already established

The chapter begins with the three diagrams [Figures 12.1, 12.2 & 12.3] presenting different aspects or levels of detail of the wholistic view: methodological, identified features and theoretical underpinnings respectively. The second part outlines the steps taken to produce a final characterisation of AskNRICH.

12.2 Diagrammatic Overview of Findings

The diagram in Figure 12.1 [shown earlier as Figure 6.4 p132] below depicts the interconnections (blue double-headed arrows between ovals) between the particular sets of threads chosen for the three Perspectives, in essence portraying their synergies and complementarities. The diagram also shows the five identifiers of the summary tables of features (Feature Catalogues) provided at the end of each of Chapters Eight to Eleven. Black double-headed arrows are used in order to emphasise the interwoven nature of inter-connections between the features in the five catalogues. The red double-headed arrows depict the further inter-connections between the three Perspectives and the five Catalogues.

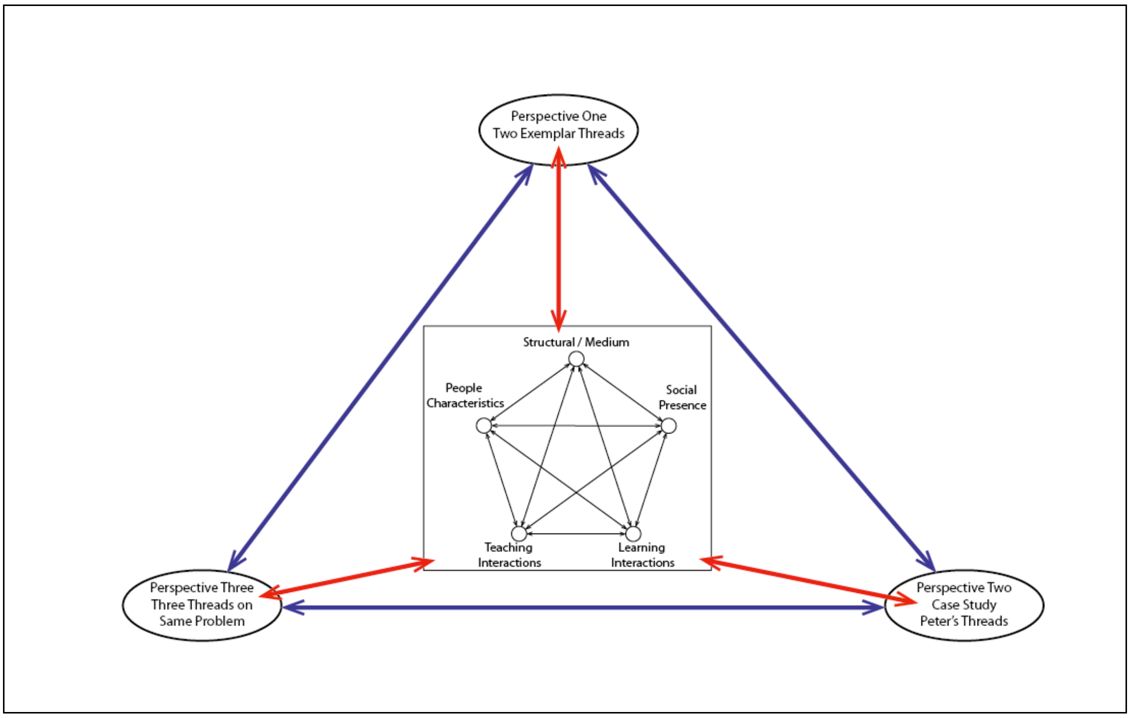

Figure 12.1 Diagrammatic Representation of Interconnections between Perspectives and Features
[Figure 6.4 of Chapter Six]

Figure 12.2 [next page] is a copy of the completed version of the diagram built up through Chapters Eight to Eleven setting out the features and illustrating their interwoven interrelationships. It shows each table with its identifier with a listing of all Features within and thus can be seen as an expanded view of the inner box of Figure 12.1.

As in Figure 12.1, in Figure 12.2 the double-headed arrows on the pentagon indicate the inter-dependence between the Features in the Catalogues. Explanations and discussions around each of the features shown in Figure 12.2 are contained in the individual Chapters [Eight to Eleven].

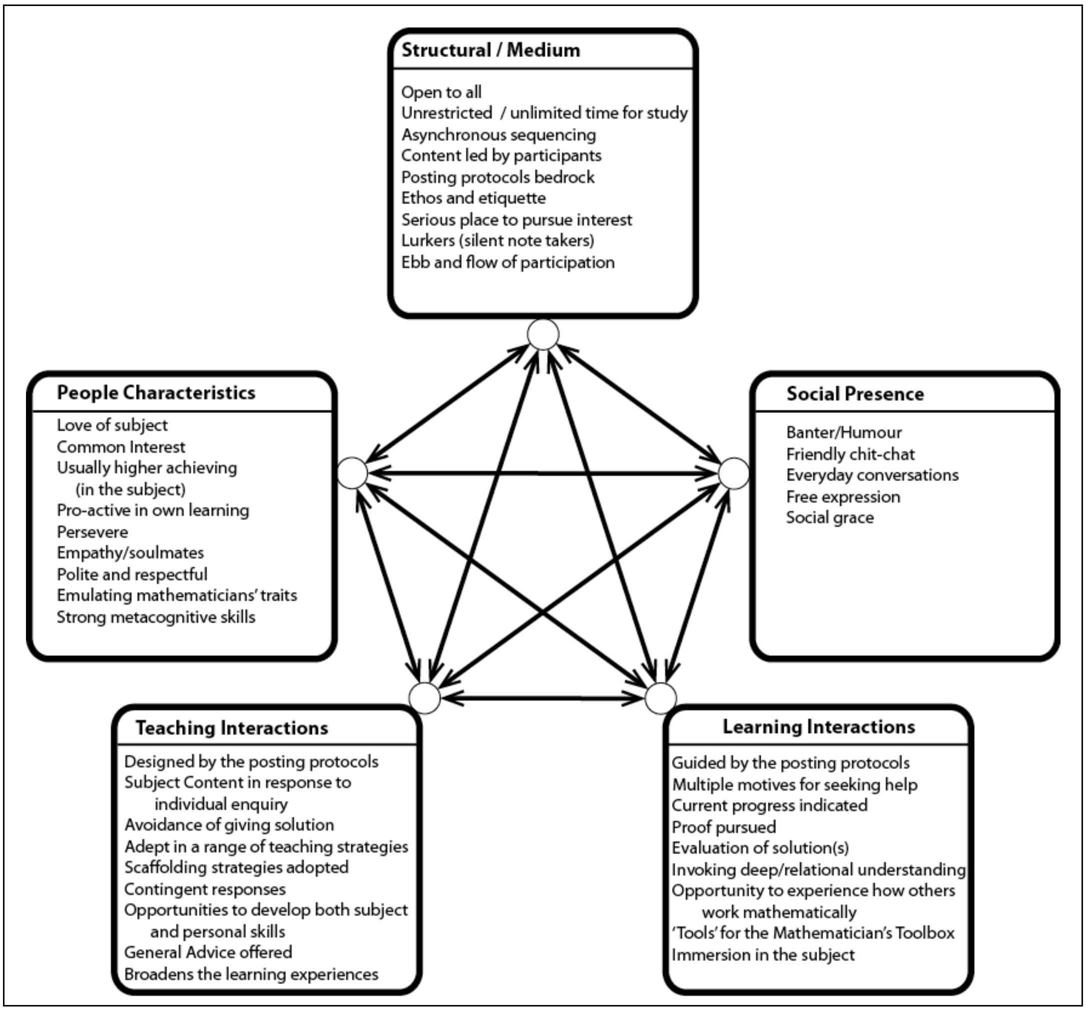

Figure 12.2 Detail of Figure 12.1 listing Features in each Summary Table

Figure 12.3 [next page] shows, visually, the five theoretical underpinnings produced in the iterative process within Exploring and Defining the Characterisation of AskNRICH [see Figure 6.5 p134] that were also used in reporting the three Perspectives [see Figure 7.4 p164]. The relationship between these underpinnings and all the interconnected Features depicted in Figure 12.2 is portrayed by superimposing a transparent layer listing those underpinnings over the depiction of Perspectives and Feature Catalogues of Figure 12.1.

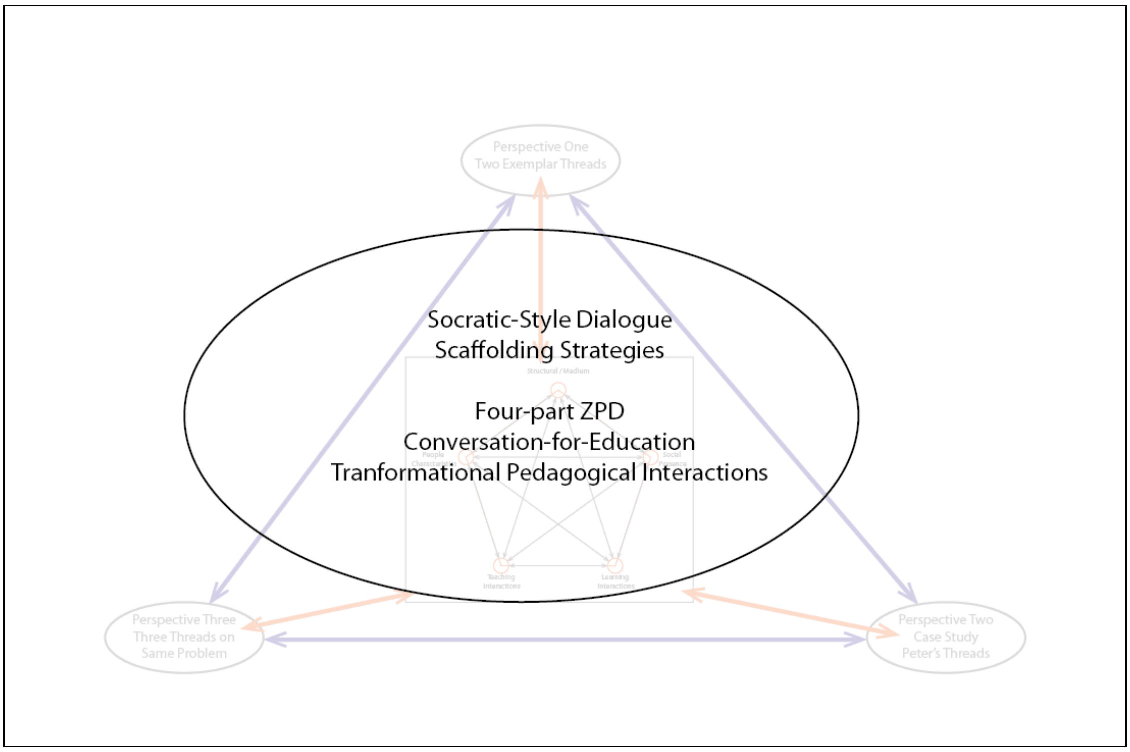

Figure 12.3 Theoretical Underpinnings used in Reporting the Exploratory Examination

Reflection focusing on the five underpinnings depicted in the topmost layer of the diagram in Figure 12.3 leads to a realisation and affirmation of the pivotal role of the Posting Protocols in shaping the nature of AskNRICH. Two Protocols, in particular, are revealed as crucial: firstly, the entreaty to the helper i.e. 'teacher' to provide hints and guidance not a solution, and, secondly, the onus put on the 'learner' to share current thoughts and ideas. These two Protocols have the immediate consequence of establishing a Socratic-Style of Dialogue [see LRIII pp151-152] that permeates the threads, leading to the adoption of scaffolding strategies [Chapter Nine] and providing the opportunity for conversation-for-education [van Lier 1996: 167] [Chapter Eleven]. The opportunity for this type of conversation is further enhanced by the freedom with which the AskNRICHers can participate in contingent conversations [van Lier 1966: 175] in both a teaching and learning role. In turn, when this conversational style exists it clearly matches the upper levels of van Lier's transformational pedagogical interactions [van Lier 1996: 179]; all content on AskNRICH is determined by the AskNRICHers themselves. Chapter Ten revealed that '*teaching but learning*' was an important aspect of Peter's activities in a helping role. He sometimes entered a thread in a helping role, but subsequent interactions provided him with the opportunity to also increase his own learning. John [p174] provides a complementary example of teaching but learning in which the helper knows the topic but finds out that they have gained a greater understanding of it through helping the learner. ‘*Teaching but learning*’ interactions such as these ensure the opportunity for intermental activities to forge intramental capabilities [Vygotsky 1978: 57]. Thus, whatever their experience, AskNRICHers may, at different times, be working within different parts of van Lier’s [1996: 194] proposed four-part ZPD model.

12.3 Outline of the Remaining Stages: Defining the Characterisation

At this point in the thesis narrative the Exploratory Examination of the AskNRICH Artefact and the part of Exploring and Defining the Characterisation of AskNRICH [see Figure 6.5 p134] have been presented. Building on these foundations, in the remaining chapters the final characterisation is defined through a consideration of the arguments, touched on in Chapter One, for viewing AskNRICH in terms of a (virtual) space in which to meet and collaborate. The next chapter is a literature review, beginning with the topic of collaboration and cooperation. Consideration of this topic has been delayed until this point because awareness of the findings of the Exploratory Examination of the AskNRICH Artefact is necessary in order to relate the reported definitions of concepts to AskNRICH. The literature review continues by analysing Gee’s concept of an Affinity Space identified through the iterative consultation of the literature portrayed in Figure 6.5. The context in which the identification was made is also described. The final definition of the Characterisation of AskNRICH is developed in stages during the subsequent chapter through further detailed consideration of the collaboration and cooperation between the AskNRICHers combined with the appropriation and modification of the Affinity Space concept.

Part Four

Chapter Thirteen

Literature Review IV
Collaboration & Cooperation; Affinity Spaces

> *The most significant lesson, derived from both initiatives, however, concerns shared goals.[…]. The lesson, perhaps, is that in order to achieve the levels and quality of interaction anticipated, it is important first to make the goals clear and second to ensure that the participants subscribe to the philosophical underpinnings of the initiative.*
>
> *[Joubert & Wishart 2012: 118]*

13.1 Introduction

This chapter presents a review of literature in two key areas necessary in order to make the final characterisation of AskNRICH. Each of these areas is reviewed with particular reference to, and consideration of, the findings of the earlier explorations. The first topic area is the definitions of collaboration and cooperation, the second is Gee's concept of an Affinity Space [AS].

13.2 AskNRICHers' Collaboration and Cooperation in Context

This section reviews the literature in order to both discuss the meanings given to the terms collaboration and cooperation and, based on the findings of the study so far, to relate the meanings to AskNRICH. This allows for a definition of the terms, with qualifications appropriate to the AskNRICH context, to be formulated and deployed in the final Characterisation of AskNRICH.

The special nature of AskNRICH has had a profound influence throughout this study. As alluded to in previous chapters [see specifically LRII], it is difficult to reconcile the meanings that are commonly assigned to the terms collaboration and cooperation in the literature with the nature of the AskNRICHers' interactions. A situation in which participants are together seeking to solve a problem, whose solution they do not know, would be called collaborative, cooperative learning in the 'traditional' sense used in the literature [see for example Littleton and Häkkinen 1999: 21; Stahl et al. 2006: 411]. The quintessential difference remains that, in the sections of AskNRICH under review in this study, an individual will be helped, through interacting with like-minded others, to find a solution to their own problem *because others participating will already know the solution* and in this sense may or may not be *learning the content*. Nevertheless, a brief review of some of the arguments presented in the literature provides a useful background and context to how the terms will be employed in later characterising AskNRICH [Chapter Fourteen].

Although everyday usage provides a loose and general meaning of collaboration as “people are working together to get something done”, there has been considerable debate in education as to how to define collaboration and collaborative learning [Littleton and Mercer 2010: 272]. The terms collaboration and cooperation are sometimes used synonymously [Dillenbourg 1999: 11] or interchangeably [Littleton & Häkkinen 1999: 21]. However, a distinction is made by Burton [2004] in her study of professional mathematicians. She used the term collaboration for a team or group working together, in for example publishing joint papers, and writes about the mathematicians working co-operatively across different disciplines or areas and later bringing separate tasks together.

Dillenbourg [1999: 9] uses his own framework to present four aspects of learning that can be described by the adjective collaborative: *situations, interactions, learning mechanisms and effects of collaborative learning*. The first of these, *situations*, is now discussed at some length to show the difficulty of considering what happens between the AskNRICHers in terms of what Dillenbourg proposed as collaboration. For Dillenbourg a situation can be termed collaborative if peers are approximately at the *same level*, able to *perform the same actions*, have a *common goal* and *work together.* All of these criteria can be viewed as being met in AskNRICH, but examining the detail of Dillenbourg’s explanation exposes a degree of mismatch.

Firstly, consider the *same level peers* aspect of Dillenbourg’s definition. It is difficult not to see the ‘mixed bunch’ of AskNRICHers as peers at the same level able to *perform the same actions*. Their common interest provides homogeneity to their actions. As is evident from their posts, the majority of AskNRICHers have a talent for mathematics and thus, in this respect, might be viewed as being at the same level. Furthermore, the conceptualisation of the four-part, AskNRICHer’s ZPD [see Figure 7.3 p159] where both the helper and the learner ‘learn’, could be viewed as supporting this argument. Moreover, Dillenbourg’s emphasis on symmetry of action, knowledge, and status that he affords to same level peers [1999: 9] has a strong correspondence with van Lier’s symmetrical conversations [see LRIII Section 7.3 p144] and the equality of status afforded by Gee’s ASs [see below]. Nevertheless, the AskNRICHers certainly cannot be considered as equal (same level peers) in terms of mathematics level or expertise. Thus in AskNRICH, because *someone will know the answer* there is *asymmetry* of knowledge. Furthermore, as the findings of this study have shown, this asymmetry is *not* played out in any power or competitive relationship but is a key aspect of the nature of AskNRICH that is used to great advantage.

In relation to *common goals,* the AskNRICHers are again in a different position from Dillenbourg’s collaborators as AskNRICHers are not solving a shared problem. Furthermore, Dillenbourg [1999] argues that shared goals are shaped through the collaboration and that the collaborators become mutually aware of these shared goals [p11]. Joubert and Wishart [2012: 118] [see chapter’s opening quotation] state that more than awareness is required in that the participants must “subscribe to the philosophical underpinnings of the initiative”. In AskNRICH, due to its raison d’etre, just by participating there is already collaboration and an awareness of the goals of AskNRICH. The AskNRICHers’ common interest in mathematics provides them with an *inherent* common goal of enabling like-minded peers to engage in doing mathematics. It is thus the AskNRICHers’ interest that results in their subscribing to a common goal rather than any explicit awareness of the goal itself.

As far as Dillenbourg’s situation criteria is concerned, *working together*, it is the degree of division of labour that is significant in distinguishing the words collaboration and cooperation, as earlier exemplified by Burton’s use of the terms. Burton’s use of a hyphen in co-operation emphasises the individual tasks undertaken before they are combined together to make a whole. Dillenbourg suggests that in collaboration the division of labour is *low,* but not necessarily absent. With the common purpose inherent within AskNRICH and the willingness of any AskNRICHer to provide help when needed, it seems inappropriate to consider *any* division of labour or a *sharing out* of tasks. I will later explain the different sense of the word cooperation that I will use in the characterisation of AskNRICH.

Similar comparisons can be made of AskNRICH against the other aspects [listed above] of Dillenbourg's framework: *interactions, learning mechanisms and effects*. So, for example, Dillenbourg assigns three criteria to collaborative interactions: interactivities, synchronicity and negotiability [ibid: 11]. For the first of these criteria, the stress he gives to the quality of the interaction assessed by the influence of the peers' cognitive processes presents a strong match with the way that the AskNRICHers work [see Chapters Nine to Eleven]. However, Dillenbourg's exposition of his second criteria leads to specifying synchronous communications as necessary for collaborative interactions and asynchronous communications for cooperative interactions. Moreover, in the third criteria he describes negotiation more as argumentation in a similar format to that of critical thinking [see LRII pp92-93]. Thus neither the second or third criteria match AskNRICH, which is neither synchronous nor fundamentally a forum for critical thinking. Nevertheless in stating that "at the most fundamental level the learning mechanisms involved must operate in the case of the individual cognition, since there are still individual agents involved in group interactions" [ibid: 14], Dillenbourg succinctly supports the arguments made for conceptualising a multi-component and multi-participant ZPD appropriate for the AskNRICHers. Dillenbourg lists cognitive load, self-explanation and conflict, as three mechanisms central to individual cognition. Findings of previous chapters have shown that each of these can be present in AskNRICH, along with the *internalisation* process that Dillenbourg adds as a learning process specific to social interactions. Furthermore, his description of the internalisation process as the transfer of tools from interaction with others in a social plane to the inner plane of reasoning parallels the Vygotskian conceptualisation of inter- and intra-psychological functions [Vygotsky 1978: 57].

Littleton and Häkkinen [1999: 21] contributing a chapter to Dillenbourg's edited book, reference a range of alternative, nuanced descriptions for collaborative learning suggested by others e.g. *collective learning* [Pea 1994: 286] and, in response to Pea's work, *coordinated learning* [Koschmann 1994: 220] both of which emerged from 'computer support' environments. Nonetheless, the situations they were describing remain different from AskNRICH, both then and now. A later description by Stahl et al. [2006] recognises the tension between views of computer collaborative learning for a 'group' and for an individual within it, that I will later show parallels my own analysis ('community' versus 'space') of AskNRICH.

However, ultimately the foundation of all of the cooperative and collaborative learning situations described in the literature remain essentially confined to somehow solving a problem together that could not be done by any of the individuals alone. In the classroom context, Littleton and Mercer [2010] state: "it is usually agreed that collaboration means something *more than*[1] children working together in a tolerant and compatible manner" [p272]. In the special context of AskNRICH, what is considered necessary but not sufficient in the classroom is '*enough*'[2] for AskNRICH, a view accepted in private communication [priv. comm. Mercer 2011].

Hence it is proposed that for AskNRICH, collaboration means the AskNRICHers ensuring a problem is solved, and cooperation means accepting the good behaviour required by the posting protocols. *Working together in a tolerant and compatible manner* maintains AskNRICH as 'that friendly place' [p173]. The harmonious environment provides opportunities for symmetries that enable interactions that foster individual cognitive development and for a seeming obliviousness of common goals and negotiation as described in earlier chapters. Thus the levels and quality of interaction in AskNRICH are achieved, as envisaged in the chapter's opening quotation, through a sharing of goals and subscription to philosophical underpinnings that is *inherent* in participation.

The next section reviews the second key area of literature necessary for completion of the Characterisation of AskNRICH: Gee's concept of Affinity Spaces, where again sharing of goals and philosophy is inherent in participation.

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[1] My italicization.

[2] In Mathematics: here 'more than' would mean necessary and not sufficient, 'enough' is necessary and sufficient!

13.3 Affinity Spaces

In this section reporting of the literature begins by recounting the context in which Affinity Spaces emerged as a key area influencing the final Characterisation of AskNRICH and continues by presenting a detailed exposition of their definition.

13.3.1 Background to Affinity Spaces

As discussed in the introductory chapter [see Section 1.5.2 p28], finding the ‘X’ in characterising AskNRICH as a Community of X (an original thought/intention of the study) was proving to be an inadequate approach. The turning point was in discovering a recent (at that time) edited book: ‘*Beyond Communities of Practice: Language, Power, and Social Context*’ [Barton and Tusting 2005]. The word ‘Beyond’ was used in two senses. Firstly, as more people had *developed* the ideas of a Community of Practice proposed by Lave and Wenger [1991] and Wenger [1998] and reported on their own Communities of Practice, ‘beyond’ is being used in the sense of ‘broader’. Secondly, since several years had passed since the original concept was proposed, new concepts had been invented that ‘went beyond’ the original. However, the two senses are not equally represented, the second sense is addressed only in the last of ten chapters. Nonetheless, it was this particular chapter by Gee on Semiotic Social Spaces [SSS] and Affinity Spaces [Gee 2005a: 214-232] that proved to be so instrumental (and inspirational) to my characterisation of AskNRICH. Gee ‘simply’ suggested with an example, an alternative view of focusing on the space in which people interacted rather than fixing them with a community label. This is similar to the group versus individual tension described by Stahl et al. [2006] referred to earlier. Gee adapted a well known saying by stating that “we could be missing the trees for the forest” [p215] to argue that the seemingly ubiquitous practice of defining any group of people working with a common aim as a Community of Practice has rendered the term almost commonplace so that the true meaning as defined by Wenger may have been lost [see support for this claim from Bruckman 2006b: 617; Kling & Courtright 2003: 224]. Incidentally, Wubbels [2007] makes a similar point in article entitled *‘Do we know a community of practice when we see one?’* published, perhaps ironically, as an end-piece in a special journal issue on *‘Online Communities of practice in education’*.

A trawl of Gee's previous work was to prove even more illuminating. In my mind, not only is Gee's work substantial, but realising the trajectory it has taken, shows how selecting it fits with my own explorations. In introducing himself, Gee [2004] pronounces:

> I am a linguist whose interests have changed over the years … I don't do theoretical linguistics any more … [but] for last number of years been an educational linguist interested in how language and learning work at school and in society at large. [pp1-2]

So although I would introduce myself as a 'mathematician', it was actually the work of van Lier [used extensively in earlier chapters] and Gee, both rooted in the study of language, that I found invaluable in arriving at my characterisation of AskNRICH.

Two of Gee's earlier books, each of which was later updated, though first published dates are useful in showing the 'history', have the word 'Discourse' in the title. To paraphrase, one concerns Ideology [Gee 1990, 3rd edition 2008], the other theory and method [Gee 1999, 2nd edition 2005b]. In both books, Gee carefully distinguishes between discourse with a little 'd' (language-in-use, on-site to enact activities and identities), and Discourse with a big 'D' (the non-language aspects, signs and symbols, of those activities and identities) [2005b: 7; 2008: 155]. I did not use the terms discourse or Discourse in my analysis of AskNRICH; instead I use the word 'exchange' to cover both, but my analysis of the exchanges implicitly recognised the difference.

I find it slightly strange that Gee's work, as far as I can ascertain, does not appear to be referenced in the CMC literature and thus that meant that I did not come across it during the literature survey for LRII. In a completely different context, I had in fact encountered Gee and Green's [1998] account of a methodological study on research undertaken in a science classroom and, although I thought the ideas presented could shape my analysis of AskNRICH, I did not pursue it at the time. This was due to the lack of reference, and thus a seeming lack of connection, with the methods of CMC analysis reported in the literature.

Although Gee's chapter in the '*Beyond Communities of Practice'* book had alerted me to the concept of an AS [Gee 2005a], this turned out to be an edited version of a chapter that had first appeared in Gee's [2004] book '*Situated Language and Learning: A critique of traditional schooling*'. The second part of the title does much to convey Gee's stance. Gee indeed criticises the type of learning that he considers is rife in (US) schools that bears little resemblance to how people learn in real-life. He brings together his life's professional work and an obvious deep interest in (or passion for?) video and computer games to unequivocally argue how the learning experience might be vastly improved in schools. He does this through a detailed account of how he learnt to play '*Rise of Nations*', a real time strategy computer game in a virtual world, having not successfully mastered other games in this genre, even though his twin brother and seven year old son were "impressive" [p59]. At the end of this specific chapter he lists twenty-five learning principles that he considered had been built into the game and would, he believed, be "efficacious" in areas outside games (i.e. in school for example) though admits he needs to leave the argument for another time [pp73-75]. He uses these principles to introduce his conception of an AS [pp77-89], a description and explanation that is easier to understand than the single 'cut-down' chapter appearing in the edited book [Gee 2005a]. For example, technical details such as "grammars" [Gee 2005a: 226] are replaced with more everyday language of content and organisation [Gee 2004: 85]. In retrospect, finding the original (whole book) before attempting to interpret the edited version would have involved less effort in understanding the whole concept, though it is at least reassuring that the result of the 'struggle with terms' was a correct interpretation. In order to provide theoretical underpinning to the Characterisation that I give to AskNRICH [see next chapter] the following section describes Gee's concept of an AS.

13.3.2 Defining Affinity Spaces

Gee makes it quite clear that "the notion of a community of practice has been a useful one" [Gee 2004: 77]; indeed he had previously worked with it [Gee & Green 1998: 146] and thought originally that he would define an Affinity Group (not space) [Gee 2004: 73]. Nevertheless by attaching a community label, Gee argues that the focus is on people within that community, quickly leading to defining the community in terms of its membership. This can be problematical as it immediately leads to making decisions as to who is and who is not in the community and indeed how far they are in (or out) [Gee 2005a: 215]. This is true for AskNRICH as 'membership' is, as already demonstrated in previous chapters, extremely fluid, some AskNRICHers remaining for many years, some for just one week, some returning after a gap of a year or more. In addition, AskNRICH is borderless in that it is open to anyone who happens to come across it. Now for Gee, place (space) and what goes on within it becomes the important feature. In this alternative view he emphasises that "is a particularly important contemporary social configuration with implications for the future of schools and schooling" [Gee 2005a: 214], an important statement in the context of the AskNRICH environment that spans home and school.

Gee has developed his arguments to define the space that a group works within rather than defining the group, labelling this a Semiotic Social Space. When necessary he refers to the real time computer game '*Age of Mythology*' by way of example, before detailing one special type of SSS which he terms an Affinity Space [AS]. I see this as a development from his earlier work with Green. In this Gee and Green [1998] took an ethnographic perspective [p126], to present a framework to "understand the relationships among discourse, social practices and learning" [p134]. They describe this framework through two sets of elements. The first they call the MASS system (material, activity, semiotic and socio-cultural aspects of discourse), the second the "building tasks (i.e. what is accomplished through discourse that simultaneously shapes the discourse and social practices)" [ibid].

Gee starts to define a SSS by its content, as the space needs to be 'about' something. Whatever gives space content, he terms **generator(s)** [Gee 2004: 80]**.** The space can then be looked at in two different ways: (i) directly in what signs (the semiotic part of the SSS) it has and how they are organised (content (including design) organisation) (ii) how people *interact* with that content or with each other over that content (interactional organisation) [ibid][3]. To me, this is analogous to the separating out of the mathematics (content) from the actions of, and interactions between, the AskNRICHers to gain a better insight into AskNRICH [see Section 6.3.4.1 pp121-122].

In addition, there is a need for **portal(s)** to allow people to enter the space and then "gives access to the content of space and to ways of interacting with that content, by oneself or with other people" [Gee 2004: 81]. Gee provides explanations and examples (using a science

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[3] It is this that is referred to as "internal and external grammars" [Gee 2005a: 219] though a table listing [ibid: p221] describes these under the definitions above, which considerably aids understanding.

classroom) to further illustrate that portals can be or become generators and that generators can also be portals. This completes the definition of a Semiotic Social Space.

An AS is defined by proposing eleven features that it will have to greater or lesser degrees; any SSS is more of an AS if it possesses more of these features than another SSS. Table 13.1 lists the eleven features that Gee [2004] defines to be within an AS [pp85-87] each with an explanatory summary.

	Feature
1	**Common Endeavour is Primary**. People relate to each other in terms of common interests, endeavours, goals or practices rather than identity by gender, social class, or race. Thus there are essentially no power relations.
2	**Newbies and Masters** (and everyone else, including 'lurkers') share **Common Space**. This is not to imply that all space is common but that there is at least some space that is open to all.
3	**Some Portals are strong Generators.** The space itself allows people to create new material that then generates new activities.
4	**Content Organisation is Transformed by Interactional Organisation**[4]**.** The activities of the people within the space can change what is on offer within the space.
5	**Encourages Intensive and Extensive Knowledge**. There are opportunities both for people to develop and display specialised (intensive) knowledge and through the general 'melee', broaden (extend) their more general (less specialised) knowledge.
6	**Encourages Individual and Distributed Knowledge.** Individual knowledge is that which is stored in one's own head. Distributed knowledge is that which belongs to other people or which is in the materials available within the space that the individual can connect to and thus "allows people to know more and do more than they could on their own" [p86].
7	**Encourages Dispersed Knowledge.** The Affinity Space encourages and enables users to pursue further (related) sites. No constraints are imposed on users accessing external knowledge and skills.
8	**Uses and Honours Tacit Knowledge.** The Affinity Space allows users to both pass on their tacit knowledge by simply participating in the activities but also provides the opportunity when appropriate to articulate that knowledge as others want to know how or what the user is doing.
9	**Many Different Forms and Routes to Participation.** Not only can participation be peripheral or central in different parts of the space, but also the pattern of participation can be equally fluid.
10	**Lots of Different Routes to Status.** Users are able to achieve status (determined by other users) through a range of activities and not through a set of pre-ordained tasks.
11	**Leadership is Porous and Leaders are Resources.** Within the space there are no 'bosses' and although there may be some people who have a leader role in the background, the boundaries between these and other users is "vague and porous" [p87] as in essence everyone contributes, thus providing the resources whilst being in this leadership role.

Table 13.1 Gee's Description of Eleven Features of an Affinity Space

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[4] This is labelled "Internal Grammar is transformed by External Grammar" in Gee 2005: 225-228.

Gee argues that “Affinity Spaces are a particularly common and important form today in our high-tech new capitalist world” [Gee 2004: 83]. He suggests that, out of school, young people of ‘*The Internet Generation*’ are constantly ‘living’ in such spaces. Gee points out that they will compare and contrast such virtual spaces with their classroom experiences and strongly implies that the latter will be found sadly wanting. However, he does go on to suggest that classrooms, a physical space, could embrace the features of an AS and gives an example of a science class. In later work [Gee & Hayes 2011] these ideas are repeated but only in the out-of-school and non-school curriculum context.

13.4 Conclusions

The review of literature presented in this chapter has been in the two key areas: collaboration and cooperation, and the concept of Affinity Spaces [Gee 2004, 2005a]. Both are instrumental in making the final synthesis that completes the Characterisation of AskNRICH. The literature definitions for collaboration and cooperation have been shown to be a weak match to what the findings from the previous chapters demonstrate AskNRICHers’ collaboration and cooperation to be. In showing this weak match the uncomplicated definition “working together in a tolerant and compatible manner” [Littleton & Mercer 2010: 272] has been shown to be entirely appropriate to be used in the characterisation of AskNRICH in the next chapter.

This chapter has also recounted how Affinity Spaces emerged as a critical topic to enable a robust, defensible characterisation of AskNRICH that fits with the findings of this study. The importance of the Affinity Space concept derives from the way in which it forces the focus to be on space/location/place instead of on community/membership. By comparing the features of AskNRICH, drawn from earlier analysis with those listed in Table 13.1, I will also show in the next chapter that AskNRICH has a high degree of similarity with an Affinity Space. However, I will go on to demonstrate that AskNRICH has some quintessential features that make it worthy of its own distinct definition and devise a model to show the relationship between the two.

Chapter Fourteen

The AskNRICHers' Virtual World
A Second Learning Place

> *Without a doubt, the physical locations we call schools and classrooms will have to change; and they will become less exclusively the spaces where certain kinds of learning are possible.*
>
> *[Burbules & Callister nd]*

14.1 Introduction

The previous chapter reviewed the literature, with particular reference to, and consideration of the findings so far of this study, in two key areas necessary for the final characterisation of AskNRICH. It firstly set down and clarified definitions of collaboration and cooperation appropriate for use in the context of AskNRICH. Secondly it considered concepts relating to a type of 'space' that will be used in this chapter to create a robust, defensible characterisation consistent with all the preceding findings of this study.

The purpose of this chapter is firstly to present the four stages that lead to the final characterisation of AskNRICH:

i. establishing AskNRICH as a Place of Nurture and a Safe Haven Commune in which to learn
ii. considering the implications of the nature of AskNRICH for relating it to the concept of Gee's [2004, 2005a] concept of an Affinity Space [AS]
iii. examining AskNRICH as revealed by the exploration against the features of an AS
iv. extending the concept of AS to define two new concepts of a Pupil Learning Place [PLP] and a Second Learning Place [SLP]

The chapter then concludes having characterised AskNRICH as an SLP by continuing with stage (iv) to explore the nature of an SLP through a detailed discussion comparing Affinity Spaces, PLPs and SLPs.

14.2 Stage One:
Establishing AskNRICH as a Safe Haven Commune

Much of the content of the threads examined in this study, and reflected initially in the features within the Social and Personal theme [Table 9.6 p200] and later in the Social Presence Features Catalogue [Figure 11.4 p264], conveys the apparent friendliness between the AskNRICHers and provides the evidence for the ‘good nature’ of the postings [see Appendix 14.1 for illustrative extracts from **ExThd1** and **ExThd2**]. This is perhaps an even greater achievement given that this is an open access and not pre-posting moderated board. An apparent and important factor in making AskNRICH a place where young people enjoy doing mathematics, is the respect and politeness, i.e. consideration and the care for others that emanates from them. In many of the threads, including those in the private posting area, the desire of one student to keep another on board, to *continue to study mathematics*, by taking time to write a sympathetic and encouraging post provides further evidence for the existence of ‘kindred spirits’. In other words, the AskNRICHers’ general practices can be interpreted as AskNRICH providing a pleasant environment, *a Place of Nurture*. Furthermore, LRIV established that the appropriate definition of cooperation and collaboration for AskNRICH was the uncomplicated “working together in a tolerant and compatible manner” [Littleton and Mercer 2010: 272]. The collection of the four qualities: cooperation, collaboration, consideration and care will hereafter be referred to as the 4Cs. Since these four qualities are diffused through the features reported in the findings of previous chapters, they thus may be viewed as lying at the heart of AskNRICH, a situation conveyed by placing them at the centre of the pentagonal diagram of Figure 12.2 [p268] as shown in Figure 14.1 below. In summary, AskNRICHers are *cooperating* in acceptance of, and keeping to, the posting protocols, showing *consideration* and *care* towards all others as they *collaborate* on aiding like-minded peers to pursue their interest in mathematics.

The harmonious world where the 4Cs predominate, automatically engenders a feeling of ‘pleasantness’ as exemplified by the quotation first presented in Section 8.4.2 [p174]:

> *I love the way everybody [in AskNRICH] is so friendly, and not obnoxious when you post the most obvious of questions, which you see the answer to the minute you post it.* *[Web-Survey Female Upper School]*

AskNRICH was described above as providing *a Place of Nurture* and by assimilating the 4Cs, this can be further developed to describe AskNRICH as:

> *a safe haven commune in which to learn*

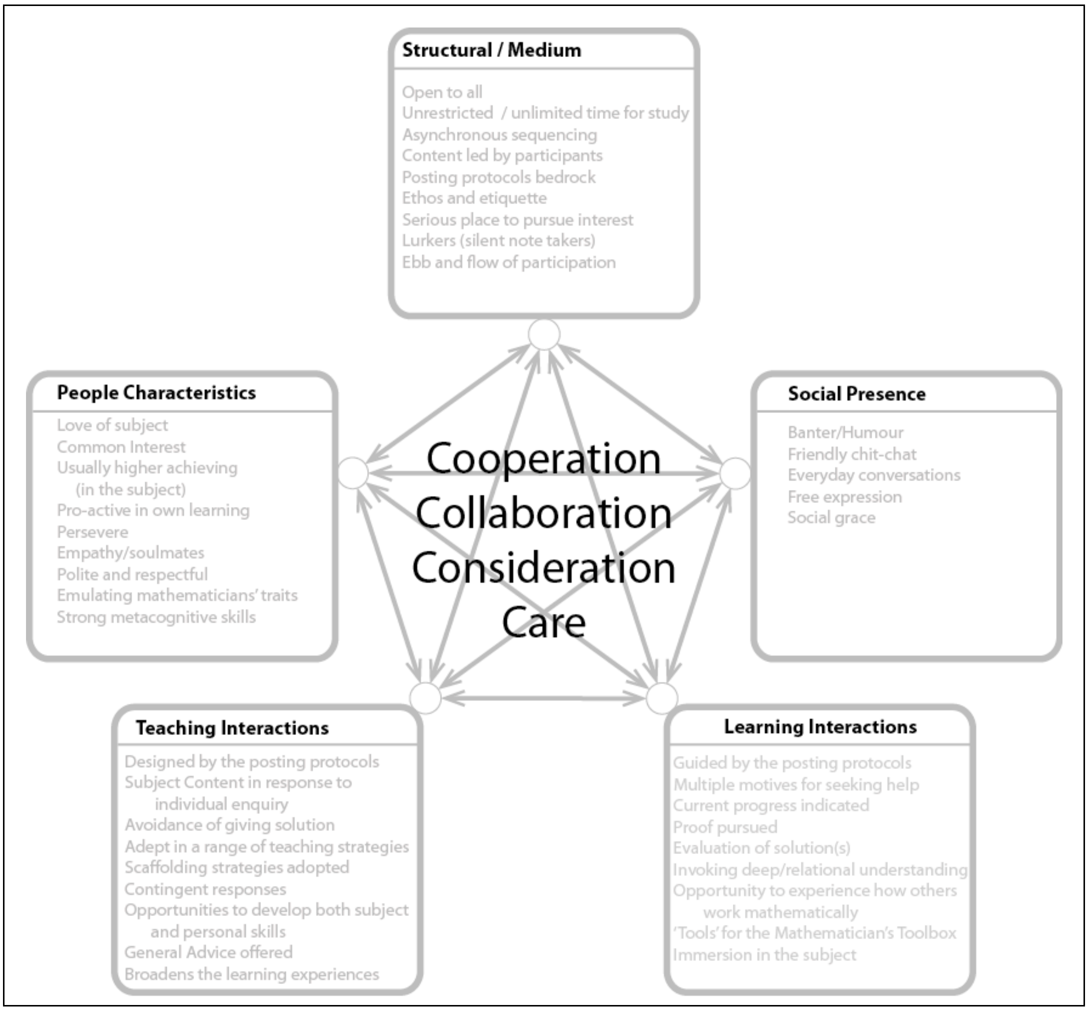

Figure 14.1 4Cs at the heart of AskNRICH

In order to ensure the correct meaning of this is accurately conveyed a short explanation of the choice of words/terminology is included here. In earlier chapters I have stated why it is inappropriate to label AskNRICH as a Community of X. However I have chosen to use the word 'commune'. The meanings of commune and community overlap, but they are not synonymous. One reason for using commune is that it does not bring unwanted connotations, especially 'permanent membership', whereas community does. Furthermore, commune has aspects of its meaning that I do particularly want to emphasise in relation to

AskNRICH: shared, communal and location (space). Moreover, the dictionary definition[1] contains the word ‘settlement’. An associated word could be ‘encampment’ as it would also bring nuances of a transitory nature, which would be appropriate for the fluctuating and dynamic participation in AskNRICH referred to in Chapter Eight [p172]. The addition of the words ‘safe haven’ brings associations with the idea of a friendly and safe location[2].

The most prolific/sustained users of AskNRICH are often those who are high, or in some cases exceptionally high, attainers in the subject. The stereotypical view of such people, reinforced somewhat by the media, is usually of being ‘odd’ or ‘nerdish’, finding it difficult to make social conversation and be ‘normal’. Some can find it hard to feel accepted by other members of their school class which exacerbates the situation [Beardon, Jared & Way 1999; Frieman & Sriraman 2008: 115; Freeman 2001].

Indeed, it is hard to imagine a stronger evidential example for the idea of a safe haven than the following comment:

> *I certainly find in an offline setting (i.e. school) it is harder to ask a question than to answer one, as I get mocked and jeered at for asking questions. But online, I feel more confident in asking a question, as usually, I won't be victimised because of it.* *[anonymous AskNRICHer]*

A web-board set up such as AskNRICH allows each individual to contribute anonymously, without being subjected to either any prejudice or any of the indications, either to or from the person posting the message, that a social face-to-face meeting could convey. This parallels Gee’s first facet of an Affinity Space where common purpose is primary [Gee 2005a: 225], but it is of extreme importance here for those young people who can feel vulnerable, ‘sticking out’ from the norm. This anonymity helps to create the empathetic atmosphere that allows AskNRICHers to flourish, and in turn AskNRICH itself.

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[1] As in Concise Oxford Dictionary meaning1 (b): a communal settlement especially for the pursuit of shared interests [Eighth edition 1991:229].

[2] The word Oasis also came to mind as in meaning 2: an area or period of calm in the midst of turbulence [ibid: 816], although any reference to turbulence is probably inappropriate!

14. 3 Stage Two:
Implications of the Nature of AskNRICH

The purpose of this intermediary stage is to set down aspects of the nature of AskNRICH that have implications for the following stage that will relate AskNRICH to the concept of ASs.

Special Nature of AskNRICH	Consequences that have implications for relating AskNRICH to the concept of Affinity Spaces
participants belong voluntarily	(i) there is fluid participation – a choice to come & stay or go; some who go return later (ii) there are complex reasons for posting: from hope for a quick fix for homework (that is quickly dispelled when it is clear this will not be met) through to a more common and predominant sustained self-learning of a subject that holds interests; common interests provide some participants with an altruism in supporting others of kindred spirit Hence there is a possible common purpose, but definitely an individual need and each participant decides on their own role(s) (asking and/or giving help)
it is not part of any fixed syllabus, curriculum or subject course, primarily used only at home	(iii) although content may be driven by need to perform well in terms of public examination systems or competition style questions (such questions can but not exclusively be directly related) this is at one level removed (at least) from tying the content to any examination assessment (i.e. AskNRICH is insulated from delivering to assessment systems) (iv) quality/quantity of postings have no assessment purpose attached to them, as participation is predominantly an out-of-school activity pursued for pleasure (even though such 'pleasure' can be relevant to 'school based' examination type questions)
topics are only raised if they are of importance to the individual making the initial post	(v) content totally determined by participants and thus there is no teaching course design (vi) there is little variation in threads' patterns – essentially the start is by someone asking for help, receiving help by one or more others, and messages continue until originator successfully solves their problem to bring conclusion
there is no teacher/lecturer led element	(vii) content totally controlled by participants (viii) no student/teacher power relationship (ix) each participant has essentially equal status in choosing to post
In addition, there are three further implications (though these could be present in other CMCs)	
Medium of AskNRICH	(x) posting protocols very strictly adhered to, more often peer moderated than by 'official' moderator (xi) the anonymity of the web-board is treated with respect by participants who cannot make direct personal contacts via AskNRICH that other social networking sites might allow (xii) the web-board is the object that enables the activities of participants. The activities of participants then themselves become enablers for other participants

Table 14.1 The Special Nature of AskNRICH and its Implications in the Further Stages of Characterisation

Table 14.1 lists four of the five properties of AskNRICH [Table 1.1 p26] and summarises alongside the consequences of each that have implications for relating AskNRICH to the concept of ASs. The fifth property, that AskNRICH is used by many who are of school age, has not been included here since age is not a factor that needs to be considered in this aspect of the study [see p294 later]. The final row of Table 14.1 lists three further consequences that relate to the medium itself and thus are not necessarily exclusive to AskNRICH. The twelve stated individual consequences are based on either recognised facts about AskNRICH or findings already reported in previous chapters.

14.4 Stage Three:
Examining AskNRICH in relation to the Concept of an Affinity Space

There are easily discerned similarities between AskNRICH and Gee's only exemplar of an AS, the real-time computer game '*Age of Mythology*', in that, for example, participants are voluntary and feel passionate about the subject[3]. Nonetheless the roots and purposes of AskNRICH and the game are substantially different and thus Gee's AS model could not be adopted unquestioningly in its entirety as a characterisation of AskNRICH. However, undertaking a comparison of the two provided a valid and viable starting point for further conceptual development.

In order to compare AskNRICH with Gee's construct of an AS, Gee's eleven constituent features [Table 13.1 of LRIV] are considered in the context of the findings of the earlier explorations of AskNRICH. This comparative exercise is presented in Table 14.2 [pp290-293] in which Gee's eleven AS features are also categorised in terms of what I consider to be a space or person focus. I have re-ordered Gee's listing of features in order to group them into three categories: technical (two features), participatory, individual and communal (five features) and differing forms of knowledge (four features). Each feature's entry includes my interpretation of it, how it is exhibited with reference to Table 14.1 and earlier chapters, and a ranking according to the degree I perceive it to be matched within AskNRICH.

In the technical category there is a strong match on feature [4], content organisation, but, given AskNRICH has only one portal and must be its own generator, ranking that feature [3] is problematical. There are extremely strong matches in two participatory features [1 & 2] relating to common endeavor and interests, with less strong matches in parts of two others [9 & 10] concerning forms, routes and status. Defining leadership in AskNRICH is not

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[3] Indeed Gee's most recent work [2011] uses the term Passionate Affinity Spaces.

simple and thus matching for that feature [11] is also ranked as problematical. Most features in the knowledge category match strongly with the exception of [7] since there is limited opportunity for Dispersed Knowledge given the single subject focus and AskNRICH's inherent self-sufficiency.

Gee's example is based on knowledge gained in pursuit of what would be generally thought of as a 'pure' hobby. AskNRICH was set up to allow school pupils the means to develop their own mathematical knowledge through independent, personal learning. Mathematics is a traditional education subject, taught at school and university, even if the AskNRICHers, as indicated in Chapter One, view doing it as a hobby! As argued earlier in this chapter, in AskNRICH learning takes place with, and is delivered by, the 4Cs: cooperation, collaboration, consideration and care. Equally the 4Cs give, and also result from, a feeling of affinity/empathy with other participants; there is thus a sense of harmony. Indeed, whilst AskNRICH is a space where knowledge can be created, to different degrees for different forms [see Table 14.2], the more self-sufficient, self-confining aspects locate it as a place that hosts a nebulous group of people that has learning, aided by good teaching, at its core. Such learning occurs as a result of other participants' pedagogical skills that they offer in the pursuit of an existing mutual and individual interest. Thus AskNRICHers not only partake of their learning within the virtual world but also in their own physical location, generally at home, alone. These points together with the mixture of matchings demonstrated in Table 14.2, leads me to contend in the section that follows that AskNRICH should be characterised by a construct, worthy of its own identity, that has a relationship of difference from, but some overlap with, an AS as is illustrated later in Figures 14.2 [p295] and 14.3 [p296] and discussed subsequently.

Table 14.2 Correspondence between AskNRICH and Affinity Space Features

[Please note: The eleven features of Gee's Affinity Space are presented in a different order from Gee's listing]

AS No	Space/People Focus	Affinity Space Feature [Gee 2004: 85-87]	How the feature is exhibited within AskNRICH	Perceived degree of matching
Features 3 & 4 focus on 'technical' aspects of any Semiotic Social Space (SSS) and of which AS is a special type				
3	Space	Some portals are strong generators of 'content'	As access to AskNRICH is achieved through logging onto AskNRICH, then AskNRICH is itself a portal [First entry level]. Given that anyone is free to post [second entry level] new material which in itself shapes the content then AskNRICH is also a generator. Moreover as all content is shaped by the postings the generator is strong. [See (vii) & (xii) in Table 14.1 above].	Problematical to interpret though some degree of matching
4	Space (in terms of what people can do)	Content organisation is transformed by interactional organisation	The content organisation of AskNRICH is essentially the threads and the mathematics (and other content) that is contained therein. Thus the content organisation appears through and is shaped by (i.e. transformed by) the actions and interactions, the interactional organisation, of the people who post to AskNRICH. [See (v) & (vi) in Table 14.1].	Strong
Features 1 & 2 recognise that people exist within the space but the individual's characteristics (e.g. gender, race etc.) is irrelevant to any action				
1 & 2	Space	Common endeavour is primary	In AskNRICH, all have the common aim to do mathematics (for a variety of reasons), enthusiasts in their own interest of the subject. [See (i) & (ii) in Table 14.1] Exemplified throughout the exploration but see in particular when working as mathematicians [Chapter Eleven]. AskNRICH allows each individual to contribute anonymously, and thus diminishes the impact of any victimization. [See section 14.2].	Extremely strong
	People	People relate to each other in terms of common interests, endeavours, goals or practices rather than identity by gender, social class, or race	Being able to choose a posting name, (e.g. 'Fluffy Slippers' 'Weathergirl' 'Mathsandme'), although some do use their own, allows for identities to be hidden. [See Chapter Eight].	

AS No	Space/People Focus	Affinity Space Feature [Gee 2004: 85-87]	How the feature is exhibited within AskNRICH	Perceived degree of matching
Features 9 to 11 focus on the individual (people) in term of possible participatory actions rather than establishing the extent of membership				
9	People acting within space	Many different forms of participation	Participants range from lurkers of whom no-one is aware, but who may look at and work on the mathematics by themselves [in ***Julia***’s case this has been for years]; to those who only ask for or only offer help, through to those who take on both roles all participate at different rates and at different levels [see Chapter Eight]. Likewise some people participate for long periods of time, some intensively, some sporadically. [See ***Peter***’s posting patterns in Chapter Ten].	Strong
	Space	and routes to participation	However in AskNRICH there is essentially only one route to participation via posting a message though and any distinction could only emanate from whether a participant decided to ask a question or offer help when beginning to post. [See ***John***’s interview Chapter Eight].	Partial
10	Space	Lots of different routes to status	Not only do newbies and masters share the same space (feature 2) there is essentially equal status for anyone who wishes to post [See (ix) in Table 14.1].	Strong
	People		However, consideration of status in AskNRICH is problematic/complex and hierarchy is not to the fore, but admiration (if this is assumed to confer status) in recognition of various actions (e.g. success in competitions or a ‘clever’ answer [as ***HelpC’***s 720 was to **Peter** in **3Thd1** Chapter Eleven]).	Partial

AS No	Space/People Focus	Affinity Space Feature [Gee 2004: 85-87]	How the feature is exhibited within AskNRICH	Perceived degree of matching
11	People	Leadership is porous and leaders are resources	AskNRICH has a professional leader (moderator) who is always there and must take responsibility for the ‘well-being’ of the site. Participants, who willingly adhere strictly to the posting protocols [see (x) in Table 14.1 concerning self-moderation] will if appropriate, defer to the moderator for points of order. Many of the moderators and deputy moderator posts are no different from posts of other people. Whilst posts that appear in the colour cyan indicate members of the AskNRICH team there is no hierarchical structure for posting conduct or who can respond to whom [see (viii) in Table 14.1]. Gee describes leaders (amongst other things) as ‘enablers (teachers)’ that therefore implies that all of those who offer help are leaders for that moment in time and who are additionally, in that situation, resources within AskNRICH. [See (viii) in Table 14.1]	Problematical to interpret though some degree of matching
The remaining Four Features (5 to 8) focus on what the space allows people to do in terms of different types of knowledge				
5	Space (content)	Encourages intensive and extensive knowledge	AskNRICH allows people to develop and display specialised (intensive) knowledge (e.g ***Peter*** developed his knowledge of MI [see Chapter Ten] at the same time as displaying his knowledge of modular arithmetic to ***R*** in the **3Thds** [see Chapter Eleven]. Extensive (broader, less specialised) knowledge is shared within general advice messages [see **ExThd2** Chapter Nine] – though by AskNRICH’s ‘specialised’ content extensive is less prominent in these terms than intensive.	Strong
6	People	Encourages individual and distributed knowledge	Individual knowledge is that which is stored in one’s head – e.g hence ***Peter*** stored his knowledge of MI (and later shared the knowledge gained with others) [see Chapter Ten]. By being exposed to all the resources and items on offer within AskNRICH, knowledge is spread around. Distributed knowledge “allows people to know more and do more than they could on their own” [Gee 2005a: 227] which in terms of the original poster asking for help is illustrated by virtually any AskNRICH thread but e.g see **ExThds** in particular [Chapter Nine]; also adding to ‘toolbox’, seeing other people’s mathematics, enquiring mind and immersion in the subject all evident in **CSThds** [Chapter Ten].	Strong

AS No	Space/People Focus	Affinity Space Feature [Gee 2004: 85-87]	How the feature is exhibited within AskNRICH	Perceived degree of matching
7	Space (other content)	Encourages dispersed knowledge	Due to the single subject (specialist) focus this is somewhat limited in AskNRICH though would never be discouraged. The basic content core is 'just' mathematics and AskNRICH is self-sufficient in providing resolution to a mathematical query i.e. it is a self-containing network. However by e.g. recommendations to other sites [evidenced in threads] and bringing in experiences from other 'courses' (including work at school) and networks, resources extend the boundaries to appropriate knowledge and skills.	Problematical to interpret though some degree of matching
8	People	Encourages, enables and honours tacit knowledge	Working on a problem together (originator of plea and helper) sometimes involves just passing on (implicitly) processes and skills. Given the message medium of text, tacit knowledge is often articulated in words (passing on) …. … highly regarded and remarked upon by others The respect that participants have for each other ensures that any tacit knowledge that is displayed is honoured. [See **ExThd2** Chapter Nine and **3Thds** Chapter Eleven].	Strong

14.5 Stage Four:
Defining a Pupil Learning Place and a Second Learning Place

Thus the final stage has now been reached where I can make a characterisation of AskNRICH as being a special example of a **Pupil Learning Place [PLP]**. In order to maintain the idea that AskNRICH is an example, i.e. an example of one of the different types of Pupil Learning Place, I term it a **Second Learning Place [SLP]**. In the following sub-sections the first defines the more generic construct of a Pupil Learning Place, the second gives a definition of a Second Learning Place and compares ASs, PLPs and SLPs. Finally, it explains the choice of the word **Second** for the specialisation of PLP as SLP.

14.5.1 Pupil Learning Place

I first coined the term "*Pupil Learning Place*" [Jared 2004: 66] as a result of conducting the first two evaluation studies of the NRICH website in the late 1990s [Jared 1997, 1998] and considering the potential of being able to undertake NRICH type problems at home and/or at school. The concept of different "*sites of learning*" [Jared 2005: 135] has now evolved in this thesis into two different *learning places*. Others had clearly envisioned something similar as exemplified in this chapter's opening quotation from Burbules and Callister [nd]. Such a phenomenon had only recently been made possible by the development of the Internet. Originally I viewed 'pupils' as young people who attended school aged 5 to 18 being taught a curriculum subject. Access to the Internet not only removes restrictions of a physical classroom [Bentley 1998, Furlong et al. 2000] it also removes the age limit. 'Pupils' can be of all ages. When considering the findings from the exploration of AskNRICH, the word 'pupil' becomes even more appropriate to encompass people of all ages for not only does the dictionary definition of pupil as a "person who is taught by another" [Hawkins 1989] fit well, but the word 'pupil' has been used for centuries for 'learned' apprenticeship schemes e.g. barristers in law firms 'take on' pupils. Hence I propose Pupil Learning Place as a generic term, for *any* place where *learning* occurs with the *aid of teaching* amongst people of *any* age. So for example schools, universities, training placements, 'evening' classes would all come under this umbrella term.

14.5.2 Second Learning Place

The diagram presented in Figure 14.2 represents the relative positioning of the Affinity Space, Pupil Learning Place and AskNRICH (termed a Second Learning Place). The circles are used simply as a visual means and the areas are not indicative of any measures. It is important to realise that although it resembles a Venn diagram it is not one since the underlying characteristics (in Table 14.3 [next page]) are not defined in terms of Boolean Logic, but in terms of a three valued system: ‘always’, ‘not always’ and ‘not/never’. This diagram is developed further in Figure 14.3 later to illustrate the key characteristics of AS, PLP, SLP and their similarities and differences.

It can be seen from Figure 14.2 (and later Table 14.3 and Figure 14.3) that all of the features possessed by an SLP are also possessed by a PLP (though a PLP can contain more features that an SLP). An SLP draws on the features of an AS as described in Table 14.2 and in the subsequent discussion. Only some ASs and some PLPs will have a school curriculum subject as a base, but an SLP is defined always to do so. The 4Cs of cooperation, collaboration [delivered with] consideration and care, may or may not be present in any particular AS or PLP but definitely are present in an SLP; making it the safe haven commune in which to learn.

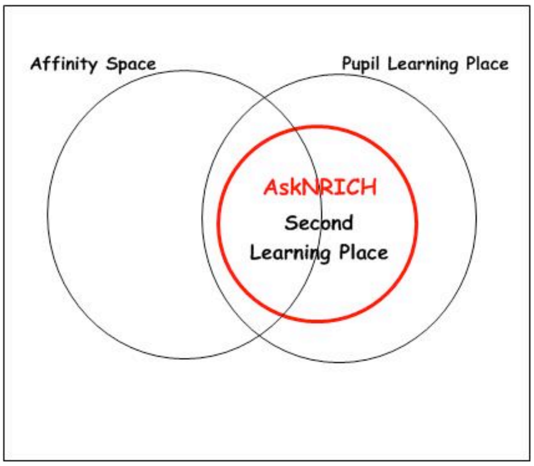

Figure 14.2 Relative Positioning of Affinity Space, Pupil and Second Learning Places

Table 14.3 clarifies this by listing five general characteristics emerging from the discussion above to be always, sometimes and never present within the three types of Place/Space under consideration. For both the PLP and SLP columns the entries are definitive. The third column is harder to be absolute in choice as the only practical example of an AS provided by Gee is based within the gaming domain, although he adds a theoretical hypothesis that an AS could be present in a traditional classroom. Whether an AS is always the Safe Haven that I am attributing to the SLP remains open to debate.

Characteristics	Pupil Learning Place	Second Learning Place (AskNRICH)	Affinity Space (Gee 2004, 2005a)
Curriculum Subject	Not Always	Always	Not Always
Physical Classroom	Not Always	Never	Not Always
Peer Led	Not Always	Always	Always
Affinity between members	Not Always	Always	Always
Presence of 4Cs (Safe Haven)	Not Always	Always	Not Always (?)

Table 14.3 Presence of Characteristics in Affinity Spaces and Learning Places

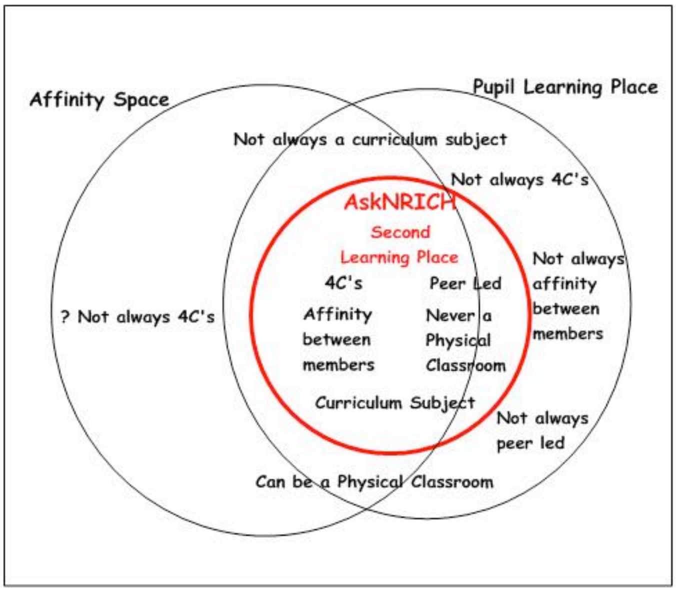

Figure 14.3 Relative Positioning of Characteristics in Affinity Spaces and Learning Places

The choice of the word Second in SLP was made for a variety of reasons. As discussed above, AskNRICH participants not only partake of their learning within the virtual world where others are ‘present’ to help but also on their own in a physical location alone i.e. to paraphrase Zuckerman [2003: 186] “together and alone”. Participants in this study are doing mathematics (as the thesis title suggests) in (two) different places, in reality that of school and home. Posting times reveal that participation in AskNRICH is mainly out of school-time and thus it seems fair to assume that such participation generally takes place at home. Both locations are thus, from the definition above, pupil learning places; each one neither first nor second (main or subsidiary). However in terms of compulsory schooling where the vast majority of the UK population of school age attends a school, culturally AskNRICH can be considered as the second location. Furthermore AskNRICH is an additional (second) mathematical resource to the texts used and teaching help available in school mathematics. In the light of evidence [including e.g. ***Peter*** in Chapter Ten] that the AskNRICHers are frequently working on mathematics by themselves as they have already completed the syllabus level considered appropriate for their school year, whether AskNRICH is actually their second or first source of learning could be debatable. Nonetheless the term 'second' is used to encompass the meanings of additional and/or alternative.

Moreover, as Figure 14.1 depicts, inside the pentagon are the 4C words, collaborative, cooperation [delivered with] consideration and care. These 'warm and friendly' words are part of a list that, in an ideal world, one would work hard to have inside a loving family home. So, to push a metaphor to extremes, calling AskNRICH a safe haven makes it become a 'second' home for many of its users.

A Second Learning Place is therefore being defined as a specific pupil learning place, different to the cultural norm, where people pursue a common interest *in an academic curriculum subject* in a **c**ollaborative, **c**ooperative environment showing **c**onsideration and **c**are for others with participants either aiding the pursuit, being aided in the pursuit or doing both.

14.6 Conclusions

This chapter has presented two new concepts developed from the findings from the preceding and earlier chapters in four stages, finally resulting in the achievement of the overarching research goal of this thesis of characterising AskNRICH. The first is that of a Pupil Learning Place, a generic term to refer to *any* place where learning occurs, not necessarily in a school curriculum subject, with the aid of teaching amongst people who are not necessarily of a specific age. By considering the activities and actions within AskNRICH that are a result of, and result in, being delivered with cooperation, collaboration, consideration and care, the definition of Second Learning Place (a subset of a Pupil Learning Place) has been given to embrace the empathy and harmonisation that these key qualities present; a safe haven commune in which to learn.

This chapter concludes the reporting of the exploration of AskNRICH and findings that emerged from it. The final chapter of this thesis will provide a reflection on the thesis findings and characterisation of AskNRICH, the extent to which the research goals were achieved, the claims made to new knowledge, the limitations of the research, and implications of, and for future, research.

Conclusion

Chapter Fifteen

Conclusions and Reflections

15.1 Introduction

The research undertaken explored the activities of young people as they made independent use of AskNRICH, a mathematics web-based discussion board that allows people to meet in a virtual world; a classroom-in-the-air. An exploration of AskNRICH within an interpretative paradigm led ultimately to defining the characterisation of the AskNRICH environment as a Second Learning Place. Three specific Research Goals [**RG**] with a total of seven associated Research Questions [**RQ**] were developed in order to reach the ultimate, overarching researching goal of characterising AskNRICH. As a preliminary to investigating the AskNRICHers' world, the research began with an investigation into pupils' general perceptions of doing mathematics in school and using NRICH type problems in home/school settings (**RG1**). **RG2** was to develop an analytical approach appropriate to the nature of AskNRICH that could then enable **RG3** the exploration of AskNRICH.

This thesis claims a contribution to knowledge in five parts summarised as follows:

Claim 1: A set of techniques, that includes some new elements, has been formed that manage the complexities, size and nature of the task of analysing the AskNRICH web-board

Claim 2: Analysis of teaching and learning aspects of exchanges within AskNRICH has demonstrated that the virtual world of AskNRICH and the behaviours of the AskNRICHers strongly promote opportunities to engage in a transformational pedagogy

Claim 3: The AskNRICH environment (i) engenders a harmonious mathematical learning experience and (ii) provides an example of positive, Internet-based, learning benefits

Claim 4: AskNRICH can be successfully characterised using a concept of 'place', based on a modification of Gee's model of an Affinity Space, through the introduction and definition of two new concepts, Pupil Learning Place [PLP] and Second Learning Place [SLP]

Claim 5: The nature of AskNRICH as a learning place embodies qualities having the potential to complement learning in schools

Appendix 15.1 details the claims against the appropriate **RG** and associated **RQ**s.

This chapter briefly summarises findings associated with each **RG**, commenting on the extent to which each has been addressed, before turning to explain and justify the claims made. The chapter concludes with a discussion on the limitations of the study and the implications of the findings for further research.

15.2 Summary of Findings

This summary is presented taking each of the **RG**s and associated **RQ**s in turn. Each section concludes with a brief comment on the extent to which the **RG** has been achieved.

15.2.1 Background Study

RG1: To investigate pupils' general perceptions of doing mathematics in school and of using NRICH type problems in home/school settings

RQ1: What are the common practices of using NRICH problems in the home context?

RQ2: What views do students using an on-line mathematics resource (NRICH) have concerning their experience of school mathematics?

The supporting evidence and analysis for this and the following summary paragraphs is contained in Chapter Four. Findings in relation to **RQ1** confirmed those of an earlier study of some years ago [Jared 1998] and (re)-established that there were indeed some pupils working independently away from the school classroom, pursuing their interest in mathematics on their own and without their teacher knowing. Students made reference to new opportunities that the Internet provides for being in touch with like-minded others who live far away and cited AskNRICH as a useful means both for doing this and for pursuing their interest in mathematics.

In relation to **RQ2**, student views on the 'benefits' of doing puzzles and problems in mathematics lessons are consistent with other studies [e.g. Boaler 1997; Brown et al. 2007; Miller et al. 1999; Nardi & Steward 2003; Ofsted 2006, 2008; Schoenfeld 1989] with an emphasis, balanced by discernment, placed on the 'fun' nature that doing such work can bring. Students generally believed that it was more important to understand the mathematics than to merely remember a set of rules. This was especially true of the respondents who stated that they only used NRICH at home and not at school. Open comments on the web-survey provide support to the educationists' views of the restrictive nature of school mathematics that is focused on examinations [Boaler 1997; Hatch 2002; Schoenfeld 1989; Watson 2006] and contrasted that to the stimulation of challenging mathematics that is intrinsic to NRICH type problems.

Thus **RG1** was met. The research elicited ways that young people work on mathematics in both home and school settings, and their general perceptions of doing mathematics in school. There is a further discussion in Section 15.4 outlining the limitations to this part of the study.

15.2.2 Method Shaping Perspective

RG2: To formulate an analytical approach appropriate to the nature of AskNRICH

RQ3: Can existing methods / frameworks for analysing Computer Mediated Communication forums be employed in analysing AskNRICH?

RQ4: How should the exploration of AskNRICH be conducted (planned, structured and executed)?

In practice, there was a strong interaction between the tasks of addressing each of these two research questions. The increasing depth of knowledge of AskNRICH not only enabled further and more meaningful comparisons with frameworks reported in the CMC literature, but also shaped the design of the exploration process. Furthermore, the manner of addressing **RQ4** also continued to evolve iteratively as the practical exploration of AskNRICH proceeded.

By necessity, given the reliance on analysis of web-board postings, the methodology developed for this study [see Chapter Six] was based on the hermeneutical, interpretative paradigm [Brown 2001; Crotty 1998; Denzin 2001; Heywood & Stronach 2005], common to some other CMC studies [Potter & Levine-Donnerstein 1999]. There is a substantial literature reporting on the use of CMC Technologies and the varied methods of analysis relating to web-board systems [see LRII]. The key methodological considerations highlighted in the literature were pertinent to the exploration of AskNRICH. However a systematic assessment of frameworks and methods found no off-the-shelf solution, but revealed some elements that could be adapted or adopted, in devising an analytical approach in which to explore AskNRICH. Thus the development of an approach required techniques incorporating new elements that could deal with the special nature of AskNRICH and impose order on, and make sense of, the complexities and the vast data source available. **RQ3** has thus been answered.

RQ4 was addressed through the two sub-questions: *Which threads should be selected for analysis? How should individual threads be analysed?* The selection of threads for detailed analysis from the 6000 threads available was based on the idea of having three Perspectives that had emerged during the reading of threads: (1) the general practices and use made by participants would be elicited by studying a number of general threads [see Chapter Nine]; (2) tracking through postings would provide a case study [see Chapter Ten] of one representative participant and, (3) how the AskNRICHers emulated traits of professional mathematicians would be investigated by examining three separate threads on the same mathematical problem [see Chapter Eleven].

In order to analyse an individual thread it was necessary to understand the mathematics contained therein by working through it. This considerably improved the quality of the subsequent characterisation of the activities and nature of the work within the thread. The activities and the mathematics were then articulated in two separate prose interpretive commentaries to be included in coding for emerging themes [Strauss & Corbin 1998]. In addition, a typology of responses was derived and used in a connection diagram which provided an accessible visual form that further facilitates the understanding of the complexities of interactions within the threads [see Chapter Eleven]. The development of techniques for analysis of threads was moulded throughout by the continual evolution of the organisation of the Exploratory Examination of the AskNRICH Artefact.

In developing an appropriate approach to analysing the web-board content, **RG2** has been achieved in full. The analytical approach and techniques, combined with the organisation of thread selection and reporting, present a principled way of managing and making sense of the complexities of AskNRICH. This leads to enabling Claim 1 of this thesis to be made [see Section 15.3 below].

15.2.3 The Exploratory Examination of AskNRICH

RG3: To undertake the exploration of AskNRICH (that will subsequently lead to the characterisation of this virtual world)

RQ5: What does AskNRICH offer to participants to enable them to pursue their mathematical practices?

RQ6: What are participants' common practices when using the AskNRICH web-board?

RQ7: What results from participants' practices when using the AskNRICH web-board?

The Exploratory Examination was undertaken whilst focussing on these three connected, fundamental questions. The comprehensive reporting on AskNRICH in Chapters Eight to

Eleven and summarised below illustrates the extensive analysis carried out in fully addressing these questions.

AskNRICH provides an environment in which young people can be enthusiastic about mathematics, find help that may not be available elsewhere, enabling them to develop their mathematical skills, and build their confidence among like-minded peers who, in that moment at least, live and breathe the subject.

In the AskNRICH environment the help offered scaffolds learning [Wood et al. 1976] enabling the learner to seek relational understanding rather than a simple rule-based instrumental understanding [Skemp 1976]. A Socratic-Style of Dialogue [see LRIII Section 7.5.1] provides the learner with the means to move forward in, in Vygotskian terms, their Zone of Proximal Development [Vygotsky 1978]. In addition, the banter, humour, friendly chit-chat and peppering of emoticons are representative of everyday conversations. The Posting Protocols are a primary factor in ensuring that the exchanges/interactions are ‘contingent conversations’ for both ‘teacher’ and ‘learner’ matching the transformational pedagogical interaction, the upper, most free, level of interaction for rich learning [van Lier 1996]. Such an environment enables a wealth of generally accepted good classroom practices [Hodgen & Wiliam 2005; Mercer 2000; Mercer & Littleton 2007; Tanner & Jones 2000a] to emerge.

The types of problems posed and the mathematics discussed are commonly above the level expected for the particular age, indicating that participants are generally higher-attaining, in mathematics at least, often displaying flair, imagination and creativity [Sriraman 2008b: 97], and an awareness of both metacognitive knowledge and skills [Hacker 1998]. AskNRICH also provides the opportunity to experience how others are working mathematically, and a means by which ‘tools’ can be collected for the mathematician’s toolbox. Chapter Eleven exposes the extraordinary/special practices of young people emulating professional mathematicians’ whilst using the Internet in their proactive, independent pursuit of mathematical studies beyond the confines of the classroom.

Although the Posting Protocols form the foundation for well-mannered conduct, it is, in the end, the actions of the participants in a variety of conversational exchanges that make AskNRICH the valued place that it clearly is. The consistently high-quality exchanges between equal peers, with evident absence of power relationships, are characterised by respect and consideration, interspersed with a scattering of witty remarks. The findings of the in-depth analysis of posts confirm the perceptions expressed by the AskNRICHers that the participants show care, empathy and a voluntary determination to nurture one another in a common pursuit.

15.2.4 Characterising AskNRICH

Overarching Research Goal: To characterise the network that constitutes AskNRICH, a virtual world that allows young people to meet within it and engage in doing mathematics

The 4Cs [see Chapter Fourteen] provided the basis for the final characterisation of AskNRICH that took place in four stages, starting with establishing AskNRICH as a Safe Haven Commune (in which to learn). The next two stages draw on Gee's concept of an Affinity Space [2004, 2005a], considering the extent to which it can match with AskNRICH. In the final stage AskNRICH is characterised as a Second Learning Place [SLP], a special case of a Pupil Learning Place [PLP], two new concepts generated through the work of this thesis. In so doing the overarching research objective of this thesis has been addressed.

This thesis, through exploring young people's activities (doing mathematics in different places), as they make independent use of a mathematics web-board identifies AskNRICH as an SLP. It presents the summarised characteristics of AskNRICH that so define, and are defined by, an SLP.

As a result of the findings just presented and consequent conclusions, this thesis makes claims to new knowledge summarised under five headings set out and discussed in the following section.

15.3 Thesis Claims and Contribution to Knowledge

This thesis makes five claims, the first is methodological in nature, the remaining four result from the process leading to the characterisation of AskNRICH as an SLP.

Claim 1: A set of techniques, that includes some new elements, has been formed that manage the complexities, size and nature of the task of analysing the AskNRICH web-board

The work undertaken to address **RQ3** highlighted the differences between AskNRICH and other web-boards, both in the unique nature of AskNRICH and the size of datasets studied [see LRII]. Furthermore, the approach needed to address the jumbled, interwoven nature of the exchanges, an aspect not specific to AskNRICH but still adding to the complexity of the task of analysis. The novel research approach devised, and reported on at length in Chapter Six, adds to the other approaches and methods currently available. The advances include:

i. employing the combination of multiple Perspectives to allow systematic and comprehensive coverage thus producing a more wholistic view
ii. the construction of interpretative commentaries on threads
iii. the separation of mathematical content from actions in the threads and their encapsulation into two separate commentaries [exemplified in Chapters Nine and Eleven]
iv. the derivation of a set of response types used to classify posts and fragments of posts
v. a diagrammatic representation of the linkage of interactions, posts and participants [exemplified in Chapter Eleven]

These last two advances also permitted the visualisation of the intricate relationship structure of the posts that allowed an additional demonstration of the presence of transformative pedagogical interactions and contingent discourse/conversations [see Claim 2 below].

Overall the whole approach derived allowed a rigorous and valuable exploration to address both the complexity and the vast amount of data studied. The claim is in effect that a methodology including several new elements has been assembled and its use resulted in successful completion of the task, a task that had not been done before on a web-board of this size and unique nature.

Claim 2: Analysis of teaching and learning aspects of exchanges within AskNRICH has demonstrated that the virtual world of AskNRICH and the behaviours of the AskNRICHers strongly promote opportunities to engage in a transformational pedagogy

The work of van Lier [1996], developed in a classroom situation, pre-dates the widespread use of the Internet. The theoretical frameworks encompassed within his work all play a part in making sense of and demonstrating the rich learning experience provided by AskNRICH as presented in Section 15.2.3 above. van Lier's theories on the categorisation of types of pedagogical interactions and his conceptualisation of multiple Zones of Proximal Development were used widely in the analysis of teaching and learning in AskNRICH in all three Perspectives.

The Exemplar Threads Perspective [Chapter Nine] shows that the effect two of the Posting Protocols on the AskNRICHers' actions/behaviours has the immediate consequence of establishing a Socratic-Style of Dialogue [see LRIII Section 7.5.1] that permeates the threads, leading to the adoption of scaffolding strategies. This Perspective also shows that the teaching and learning interactions taking place within exchanges have a spontaneity and conversational tone that leads to a high degree of contingency, a key concept within van Lier's categorisation of pedagogical interactions.

The analysis of the threads within the Three Threads Perspective [Chapter Eleven] presented a more detailed consideration of contingency levels by examination of the extent to which constituents of contingency as represented diagrammatically by van Lier [1996: 179] were present in the threads. Findings showed that the teaching and learning interactions present within AskNRICH generally would fit more towards the outer limits of each of the constituents of contingency and thus the AskNRICHers have the opportunity to engage more freely in a transformational pedagogy.

The Case Study Perspective [Chapter Ten] demonstrated that Peter's multiple teaching and learning roles were consistent with van Lier's four-part ZPD model. The opportunities afforded within AskNRICH to (i) engage in a variety of roles, and with a variety of peers who maybe more, equally or less capable and (ii) initiate and manage the conversations, provide a potentially transformational effect both in its environment and on the mathematical development of an AskNRICHer.

Thus this study has shown how van Lier's series of ideas/concepts concerning transformational pedagogical interactions can be appropriated into the virtual world, at least as represented by AskNRICH.

Claim 3: The AskNRICH environment (i) engenders a harmonious mathematical learning experience and (ii) provides an example of positive, Internet-based, learning benefits

This is a two-part claim, the first of which is that AskNRICH provides a harmonious mathematical experience and this harmony produces learning benefits. The theoretical frameworks selected to underpin the analysis reported in Chapters Nine to Eleven directly relate to teaching and learning concepts. However, as the Figures in Chapter Twelve portray, these concepts provide only a partial view of the AskNRICH environment. The complete, wholistic picture can only be formed by simultaneously considering the five interdependent groupings and their interplay together. Bringing in the People Characteristics, the Structural/Medium and the Social Presence groupings leads to the inference, in Chapter Fourteen, of the 4Cs cooperation, collaboration, consideration and care being at the heart of AskNRICH [Figure 14.1 p285], producing a self-sustaining and self-perpetuating environment with each of the 4Cs resulting from and contributing to the other three. These four qualities give the sense of harmony that abides within the AskNRICH environment: a place available for anyone who wishes to engage in doing mathematics; a place for doing mathematics in a way that is analogous to 'playing the game', as professional mathematicians might be perceived as doing, rather than practising the 'dribbling' [see LRI p54]; an inclusive, open and pleasant place of nurture in which to learn mathematics.

The second part of this claim relates to use of the Internet by AskNRICH, which would not and could not exist without it[1]. The activities within AskNRICH and the outcomes as a consequence of these activities have been shown as purposeful in supporting learning, thus demonstrating an exemplary, simple, worthwhile and beneficial use of Internet-mediated communication. Just by providing the opportunity to communicate with like-minded people, the Internet here has made a tangible and real difference [Abbott 2001: 88] for a group of young people [see quotations from AskNRICHers across Chapters Eight to Twelve]. Given the 'bad press' that much of the Internet receives, the AskNRICH narrative is, I would argue, undoubtedly an example worthy of wide dissemination.

Claim 4: AskNRICH can be successfully characterised using a concept of 'place' based on a modification of Gee's model of an Affinity Space through the introduction and definition of two new concepts, Pupil Learning Place [PLP] and Second Learning Place [SLP]

As presented in Chapter Fourteen: the voluntary and fluid nature of participation, with some participants only being in either the teaching or learning role; the need of the individual to find the solution to their (for, and at, the moment) problem, and the fact that someone will always know the answer, all ultimately provide strong arguments to suggest that what goes on within the AskNRICH virtual world is of greater importance than the individuals themselves, i.e. 'placeness' is paramount.

In defining the final characterisation of AskNRICH, this study introduces and defines two new concepts [see Chapter Fourteen] that can be added to the collection of definitions describing virtual learning worlds: Pupil Learning Place [PLP] and, as one type of PLP, Second Learning Place [SLP]. PLP is a generic term for *any* place where *learning* occurs *with the aid of teaching* amongst people who are not necessarily of *any* specific age. An SLP is characterised, in general and specific terms respectively, by the 'safe haven commune' in which to learn, and by the virtual world that is AskNRICH. AskNRICH, and by equivalence, an SLP, is where people pursue a common interest (in an academic curriculum subject) in a

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[1] Incidentally, it is worth noting that AskNRICH is not reliant on any of the recent, more sophisticated innovations in the technology (Web 2.0), having used the same unpretentious interface and thrived for more than a decade.

collaborative, **c**ooperative environment showing **c**onsideration and **c**are for others with participants either aiding the pursuit or being aided in the pursuit, or doing both.

Although the concept of an SLP derives from Gee's [2004, 2005a] work on Affinity Spaces and AskNRICH demonstrates, to varying extents, all the features that Gee suggests characterise such spaces, there are important differences that make SLPs worthy of having a separate identity [see Chapter Fourteen]. AskNRICH in essence exists to provide the means for students of mathematics, predominantly at school or undergraduate level, to develop their own mathematical knowledge. That is participants of AskNRICH are undertaking independent, personal learning in a curriculum based subject at an alternative (second) location to that of formal education. The fundamental nature of an SLP springs from this definition of location, purpose and activity together with the qualities of a safe haven commune underpinned by the 4Cs as referred to above. In other words these aspects are required for the existence of an SLP and are key to producing the benefits of the harmonious learning experience referred to in Claim 3.

Claim 5: The nature of AskNRICH as a learning place embodies qualities having the potential to complement learning in schools

This claim results from reflection on elements that had been 'discovered' during the characterisation of AskNRICH and on the potential they might have to influence practice in school-based teaching and learning. This claim rests on the fact that the teaching and learning strategies employed by the AskNRICHers, underpinned by the Posting Protocols, have been demonstrated to be successful within the AskNRICH environment [see Chapters Nine to Twelve]. LRI portrays a generally 'poor' tone of mathematics teaching and consequent 'unhappy' learning experiences. Thus it may be argued that the successes achieved through the nature of AskNRICH as a learning place are worthy of wider consideration and possible experimentation within the school environment.

In particular, one lesson drawn is the effectiveness of the Posting Protocols (within AskNRICH as an SLP) imploring the helper not to give the answer and the learner to share current progress. As stated in Claim 2, these two Protocols encourage a conversational tone beyond IRF and provide *natural* opportunities to become involved in types of talk that are less reliant on the teacher taking the lead [see LRIII]. Furthermore, the elements of the environment present within an SLP (as detailed above in Claim 4) enable a 'happier' learning experience, that safe haven in which to commune [see Chapter Fourteen] providing a further quality with the potential to complement school-based learning. Finally, the learning experience in AskNRICH results from the successful strategies of peers who are of mixed age and mixed experience. In AskNRICH there is no apparent power relationship between helper and learner, with each appearing to embrace the asymmetry of knowledge and both gaining from the interactions [see for example Chapters Eight and Ten]. Creating opportunities for such 'mixed peers' to interact is again a quality whose benefits are worthy of consideration in a school context.

Having presented and discussed the claims proposed as contribution to knowledge, the remaining two sections of the chapter focus on the limitations to the study and the implications of the research, also including suggestions for future work.

15.4 Limitations to the Study

In addition to the inherent limitations of the Initial Study deriving from those associated with web-surveys and questionnaires in general [see Chapter Three], the investigation of pupils' perceptions of school and home mathematics (**RG1**) was only taken up to the point necessary to support the other goals. Although ethically it was not possible to consider pursuing 'stranger contact' with web-survey respondents, using NRICH staff contacts findings from the web-survey were strengthened by additional data obtained from an in-depth face-to-face interview with a keen user of NRICH from abroad and three email correspondents. Even so, the fact that many web respondents moved to AskNRICH led to only a minimal necessary investigation being made into perceptions of NRICH problems. Balancing competing time restrictions also led to a greater concentration on the central core of the research on AskNRICH and necessary curtailment of the analysis of all of the data obtained from the web-survey.

There are some limitations to be discussed concerning the methodological approach and the organisation of the exploration of AskNRICH. Any chosen methodology is likely to bring with it intrinsic limitations, and those associated with the qualitative methods used here are already well known and documented in the literature. In this study the impact of, for example, subjectivity in interpreting text with latent content and reliability and validity of coding by a lone researcher is considered and steps taken to minimise such methodological limitations are reported at length [see Chapter Six]. Chapter Six also reports the constraints on choosing a method appropriate to characterising AskNRICH with all its complexities and special nature together with the vast amount of data available for study. There is scope for other researchers to choose different options in determining their approach, but throughout the thesis care has been taken to report and explain the choices and decisions and resulting paths through an unknown territory. Thus I am content that the coverage of AskNRICH achieved can support the claims presented and stands up to scrutiny.

This research has limited its focus to just one mathematics web-board, AskNRICH, albeit a well-established one now in its second decade of use and unchanged since its inception. In addition to the longevity and stability of AskNRICH, its roots remain firmly placed within the mathematics education department that conceived it and thus ensures that it is a web-board worthy of serious educational research study. However, findings from the research conducted here cannot start to be transferred to a general population perspective without accounting for two potential limiting factors relating to the participants. Firstly, the number of active participants i.e. AskNRICHers at any one time remains relatively small [within the low hundreds]. However this figure does not include lurkers, who the technical team believe, account for around 90% of all AskNRICH traffic. A second, previously acknowledged, limiting factor is that AskNRICHers tend to be extremely high-attainers in mathematics – and it may be speculated probably in other subjects too, though this remains indeterminable. Nevertheless, AskNRICH was established because the founders felt that these high-attainers' special needs had been neglected. As the findings show AskNRICH has provided the opportunity for such students to be in 'virtual' touch with like-minded peers and thus it serves its purpose.

On further reflection, my declaration and confession at the very beginning of this thesis [p22] signals the clear view in my own mind of what constitutes good/effective teaching and learning of mathematics and consequently the values that I hold in relation to them. Thus the vision of what is to me good teaching and learning has clearly governed the judgements made in the analyses in this thesis. The excitement, enthusiasm for, and enjoyment of doing mathematics shown by the AskNRICHers in the threads in the three perspectives, and frequently in the many other threads considered in the process of analysis, vividly reflects my own pleasure in learning. Similarly, the AskNRICHers' pedagogy parallels my own and the associated values.

Thus it might be argued that it is hardly surprising that AskNRICH emerges from this Study as a stimulating environment and, to me, exemplifies my values. Indeed, my values and personal experience contributed to shaping the Posting Protocols. Furthermore, NRICH and AskNRICH were set up to provide a resource for (high-attaining) people who were keen to pursue their study beyond that of 'normal' school mathematics. Hence it might be presupposed that the 'discovered' characteristics would be inherently present given the web-board design and the high-attaining nature of the participants [see p208]. Nonetheless, I was genuinely surprised and gratified by the tangible and pervasive effect of the Posting Protocols [see p269] exemplified by the degree of self-moderation apparent on the web-board [see for example p211 and p239]; there is no built-in guarantee, beyond a light-touch moderation, that the AskNRICHers will inevitably and routinely conform. Thus, although the designers of AskNRICH had set out to create an environment based on their imaginings of the needs of hypothetical AskNRICHers, the existence of the characteristics that have been drawn out or 'discovered' is firmly rooted in the methodology and findings of the analysis of the AskNRICHers' work.

15.5 Research Implications and Further Work

The work undertaken and the claims made potentially contribute to a variety of fields: firstly to use of the Internet for learning and secondly in defining communities and virtual spaces. The work also adds a new, potentially highly influential perspective to the field of Mathematics Education that emanates from AskNRICH's innovative use of a web-board technology and the implications for transformation of school-based practices.

The new techniques developed in this study were designed specifically to meet the challenges posed by the complexities, size and nature of the task of analysing the

AskNRICH web-board. However, there is potential for future researchers analysing web-boards, which may or may not be restricted to those similar in nature to AskNRICH, to adopt or adapt these techniques or elements of them. Thus there is also a possible contribution to research methods associated with CMCs.

This study has demonstrated the far-reaching effects of the Posting Protocols on the nature of and behaviours within the AskNRICH environment, a lesson with potentially widespread applications in using Internet-mediated communication for peer teaching and learning. The final characterisation of AskNRICH in terms of a space rather than a community widens a debate [see Chapter Fourteen] already begun by e.g. Gee [2005a] and Wubbels [2007].

The findings of this study demonstrate that the AskNRICH environment successfully supports learning amongst the AskNRICHers whose participation shapes interactions to offer a rich mathematical experience to all. The environment is neither a classroom nor constrained by any assessment or examination systems [Hatch 2002; Schoenfeld 1994; Watson 2006]. In addition the AskNRICHers show themselves to be high-attainers in the subject (if not others). Nevertheless such differences should not preclude considering placing similar strategies within the school-based environment. After all AskNRICH has been made sense of here through using theories developed for the classroom and, as the literature shows, adopting the findings from these theories has long been advocated by educationists, although implementation may not have been as full as desired [Boaler 1997; Watson 2006]. This study has provided the evidence that what takes place within AskNRICH and how it takes place leads to successful learning within an Internet environment. As Claim 5 argues the successes achieved through the nature of AskNRICH as a learning place have the potential to be applied more widely back within the school environment. Thus this could imply, for example, some adoption/adaptions of the Posting Protocols that should ensure a quantity and quality of more symmetrical talk and the creation of vertical aged groupings. Taken together these measures have the potential to begin to invoke an atmosphere resembling that of AskNRICH. Whether schools retain their current format or not and whatever impact Internet environments make on education, the lessons from this study should still apply.

The work reported in this thesis will be extended, expanded and tested as NRICH has secured funding for a further two-year research project involving AskNRICH. Although the research reported in this thesis indicates that AskNRICH is a valued asset by a number of long-term users [see Chapter Eight] there are others who come to AskNRICH and make a few posts and then disappear. A longitudinal study is planned to target particular posters for views staged after making a specified number of posts. Moreover, each post as it arrives at AskNRICH will be entered into a database that can systematically record details; a task that I could only do by hand and retrospectively, from the thread to which the message had been posted, using what was available on the public part of the board. Furthermore, the work from this thesis study will itself be critiqued. Posts will be recorded, indexed, read, interpreted, analysed and tested against the coding system and response typology devised in this thesis. In addition to the testing of the existing systems devised, threads will be coded to typify the 'study area' of each new thread and all posts within the threads individually allocated the researcher's perceived reason for the poster making a posting.

Laughter and Tears were sometimes not far away as I read many of the AskNRICH posts. The *joie de vivre* shown by the AskNRICHers as they pursued their mathematics study exemplified all that I have held dear to my beliefs of what true education should mean.

Their Life and their Work is a Tale Worth Telling

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Appendices

List of Appendices

RG1 To investigate pupils' general perceptions of doing mathematics in school and of using NRICH type problems in home/school settings	
Research Questions	**RQ1: What are the common practices of using NRICH problems in the home context?** *Why and with whom do these students do NRICH problems at home? To what extent do these students perceive their teacher knowing that they do mathematics problems at home? Why do these students not tell their teacher?* **RQ2: What views do students using an on-line mathematics resource (NRICH) have concerning their experience of school mathematics?** *What are students' perceptions about doing mathematics puzzles and problems in lessons? What are students' perceptions about the relative merits of rules, methods and understanding? How do these students seek help with school mathematics?*
Research Undertaken to Collect Relevant Data	Questionnaires: Different pilots in five schools* preceding final publication on NRICH website. [c 220 and 117 responses respectively] Tape Recorded Group Interviews [2 at Key Stage 3 groups* (8 participants), 3 with A level groups (11 participants)]. Tape Recorded Individual interviews: 2 teacher interviews*, 2 individuals – one an active user of NRICH problems. (*data not explicitly used in thesis)
Location in Thesis	**Chapter Two:** Related Literature Review I **Chapter Three:** Methodology **Chapter Four**: Findings from Preliminary Studies [relating to Preliminary Investigations into current situation as perceived by school pupils]
Further comment	The work undertaken for this research goal provided background information to the exploration and characterisation of AskNRICH. As such no claims to new knowledge are being made though confirmation of the finding from earlier evaluative studies [Jared 1998]. This current study establishes the home and alone school student (predominantly not telling their teacher) interested in (independently) pursuing their mathematical studies
RG2 To formulate an analytical approach appropriate to the nature of AskNRICH	
Research Questions	**RQ3: Can existing methods / frameworks for analysing Computer Mediated Communication forums be employed in analysing AskNRICH?** *What different types of frameworks have been reported? What different methods/approaches already exist? What key methodological issues are reported?* **RQ4: How should the exploration of AskNRICH be organized (planned, structured and executed)?** *Which threads should be selected for analysis? How should individual threads be analysed?*
Research Undertaken to Collect Relevant Data	Extensive Literature Review of a range of reported studies. Derivation of an analytical approach and techniques needed to explore the complexities of the web-board content that contained approximately 50 000 messages in 6 000 threads
Location in Thesis	**Chapters Five:** Related Literature Review II **Chapter Six:** Derivation of Methodology and Methods for Exploration of AskNRICH
Contribution to Claims (New Knowledge)	**Claim 1**: A set of techniques, that includes some new elements, has been formed that manage the complexities, size and nature of the task of analysing the AskNRICH web-board

RG3: To undertake the exploration of the AskNRICH artefact	
Research Questions	**RQ5*:* What does AskNRICH offer to participants to enable them to pursue their mathematical practices?** *Rudimentary information about AskNRICH: What is it? What are the different sections of the conference board? What posting protocols are used on AskNRICH? Who are the core participants? Why do they belong? How is AskNRICH typically used?* **RQ6: What are participants' common practices when using the AskNRICH web-board?** *What characteristics do participants of AskNRICH exhibit as they pursue their interest in mathematics? What mathematics teaching and mathematics learning roles are manifested within AskNRICH?* **RQ7: What results from participants' practices when using the AskNRICH web-board?** *What types of interactions are shown between the participants as they engage with mathematics? In what ways does the behaviour of AskNRICH participants emulate the working practices of professional mathematicians?*
Research Undertaken to Collect Relevant Data	(i) Literature Review on peer interactions (ii) Exploration of the AskNRICH site (artifact) to document evidence, plus e-mail correspondence with three school-aged participants and three tape recorded interviews [two team members and one 'lurker'] (iii) Reading in excess of four hundred threads involving four thousand messages, annotation of text of 150 threads [events, key words/actions] (iv) Detailed interpretative commentaries made alongside individual posts within two threads. Open coding to propose key characteristics first from these two threads, and through comparison with other threads, additions made (v) Detailed (chronological) study of 484 retrievable posts made by 'Peter' (96.6% of Peter's total postings) (vi) Interpretation of three threads, at a micro-level, on the same mathematical problem appearing over time. Interpretation then analysed for collective interactions between the participants. Diagrammatic representation devised to demonstrate a derived typology of interactions
Location in Thesis	**Chapter Seven:** Related Literature Review III Primarily Using (ii) **Chapter Eight:** General Description of AskNRICH Primarily Using (iii) and (iv) **Chapter Nine:** Analysis of general postings to AskNRICH Primarily Using (v) **Chapter Ten:** A case study of one AskNRICH participant Primarily Using (vi) **Chapter Eleven:** Three Threads on the same mathematical problem **Chapter Twelve:** Summary of findings from Chapters Eight to Eleven
Contribution to Claims (New Knowledge)	**Claim 2:** Analysis of teaching and learning aspects of exchanges within AskNRICH has demonstrated that the virtual world of AskNRICH and the behaviours of the AskNRICHers strongly promote opportunities to engage in a transformational pedagogy **Claim 3:** The AskNRICH environment (i) engenders a harmonious mathematical learning experience and (ii) provides an example of positive, Internet-based, learning benefits

Overarching RO: To characterise the network that constitutes AskNRICH, a virtual world that allows people to meet within it and engage in doing mathematics	
Research Undertaken	Related Literature Review and findings from RG3 further synthesised to lead to a characterisation of AskNRICH
Location in Thesis	**Chapter Thirteen:** Related Literature Review IV **Chapter Fourteen:** Characterisation of AskNRICH as A Second Learning Place
Contribution to Claims (New Knowledge)	**Claim 4:** AskNRICH can be successfully characterised using a concept of 'place', based on a modification of Gee's model of an Affinity Space, through the introduction and definition of two new concepts, Pupil Learning Place (PLP) and Second Learning Place (SLP) **Claim 5:** The nature of AskNRICH as a learning place embodies qualities having the potential to complement learning in schools

Date	Item	Participants	Rationale/Use
Initial Study Stage One: School-based Investigations			
1 July 2004	Questionnaire PQ1	31 pupils across ability range in years 7 to 9 in local comprehensive school	Pilot Study
2 July 2005	Questionnaire PQ2	124 pupils in years 7 (63) and 8 (61) across ability range in two local schools	Adaptation of pilot study and used as basis for interviews
3 July 2005	Tape recorded Group Interviews	Two groups of four pupils, two girls and two boys, one group from each of year 7 and year 8 in one of the schools used in 2 above	Gaining additional responses to open questions on questionnaire
4 July 2005	Tape recorded Interviews	Two teachers, one from of each of the schools used in 2 above	Gaining background information from teacher perspective of problem-solving opportunities within the curriculum
5 July 2006	Part of Questionnaire PQ3	51 pupils in years 7 (27) and 10 (year 10 top set) in local comprehensive school	Pilot re-design prior to completing web survey design
Data analysis of above is not extensively reported within this thesis. Evaluation undertaken of questionnaire design and quality of responses used to inform next stage of research is reported within the methodology section [Chapter Three].			
Initial Study Stage Two: Primarily Online Investigations			
6 January to May 2007	Web Survey WQ	Public access – open to respondents (with Internet Access) of any age from any country. 117 replies used	Obtain general perceptions of school mathematics across a wide range of schools
7 May 2007	Tape recorded Individual Interview	17 year old male participant in NRICH for five years (from EU country)	Long time user of NRICH monthly problems
8 September 2007	Tape recorded Individual Interview	11 year old male	Known to do problem solving at home without teacher knowing
9 October 2007	Questionnaire PQ4	15 A level Mathematics students (final year) in Boys Comprehensive	Additional information from high achievers at school level
10 October 2007	Tape recorded Group Interviews	Three groups of 3/4 students from school used in 9 above	Gaining information from known users of Mathematics Internet Site
Initial study reported in Chapter Four			
Main Study [Exploration of AskNRICH]			
11 August 2007	Tape recorded Individual Interviews	Director of NRICH Deputy Moderator of AskNRICH	Gain increased knowledge from two key personnel involved with NRICH
12 October 2007	E-mail correspondence	One female user of NRICH site One male regular participant in AskNRICH	As result of web survey analysis, investigation into NRICH and AskNRICH users' activities
13 August to December 2007	AskNRICH Sampling live threads and trawling the Archive	AskNRICH participants Considering some 250 threads involving some 2000 posts	Scoping the Research and Reconnaissance of artefact, recognising the means of establishing order on the vast amount of material available, leading to finalizing sub-questions of RQ3-7
14 January to May 2008 (and at times until study's end)	AskNRICH website (daily visiting late January to mid-April)	AskNRICH participants During the period reading in excess of 500 threads involving 4000 posts	In-depth study of the workings of AskNRICH, consider and evaluate possible analytical approaches and techniques, selecting threads
15 March 2008	E-mail correspondence Tape recorded Individual Interviews	One male member of AskNRICH answering team One male regular participant in AskNRICH One female 'lurker'	To add to information being gained from AskNRICH explorations
Analysis of AskNRICH data used for Chapters Eight to Eleven and to inform final arguments presented in Chapter Fourteen. The development of the technique which allowed this analysis is reported in the second methodology chapter [Chapter Six]			

1.3A: Introductory interview invitation letter to individuals

UNIVERSITY OF CAMBRIDGE
Faculty of Education

From Mrs Libby Jared
Lecturer in Education (Mathematics)
Tel 01223 – xxxxxx
E-mail: ecj20@cam.ac.uk

Research into Mathematics and the Internet: School and/or Home

I am in the process of collecting data as part of my PhD studies at King's College, London.

In 1996 I was one of a 'gang of four' who helped to set up the NRICH website with the primary aim to have a monthly set of challenging mathematics problems accessible on-line for anyone to try (if they wanted to). At that time, the 'new' Internet seemed a good way for making mathematics become more accessible to lots of people – the technology had some natural advantages for what it could offer in the way of open problems and other resources. The problems could be taken off the Internet and used by schools in lessons and mathematics clubs and/or by people at home.

Now in 2007, some (not all) students (from under 11s to sixth formers and beyond) appear keen (though some prefer to keep quiet about it) to spend time at home (or away from the classroom) doing some of the problems. My research includes looking at reasons for making that choice and in investigating the role that Internet resources might or might not play in establishing 'learning places' in the future.

Your participation in interview is purely voluntary - even though I am hoping that you will help me out! I need to give you the assurance that anything you say or write will be kept confidential and used only for the purposes of my research. In any publication relating to this research all names will be referred to by a pseudonym.

Thank you for the time you have given in participating in this interview.

Libby Jared [Date]

1.3B: Introductory letter to School Class Teacher

UNIVERSITY OF CAMBRIDGE
Faculty of Education

From Mrs Libby Jared
Lecturer in Education (Mathematics)

Date

Dear

I am someone who has been involved in teaching for some thirty years and for the second half of this time I have been training. I contacted [name] School seeking help with a mathematics education research project that I am currently conducting into the role played by the Internet on young people learning mathematics.

I would like to conduct some student interviews which would be tape recorded and so I am now writing to ask whether you feel that you give me permission to interview (name) on (date).

Thank you for your speedy reply suggesting that your school may be able to help me with my research. I am hoping that the research will in part inform my current PhD study at King's College London as well as being a basis for further dissemination to the mathematics community. I am hoping (though as a researcher I need to remain neutral) to show the added value that the Internet can provide in helping with extra-curricular activities and additional materials to foster and stimulate students who like and enjoy doing mathematics. The pressures on working with the ever-present examination syllabus might make it hard for some to find time in lessons to 'relax' and have 'fun' with problems.

First, though, maybe we need to see if, everything else below being acceptable to you, we can find a date that can I visit. I am not sure precisely when you break up for the Summer Holidays but I could come on any day between Wed 4th until Tuesday 10th July inclusive and then any other days which remain from Monday 16th July onwards. I hope that one of these will be convenient for you. As I will be traveling from Cambridge, it may be safer to say that I could arrive at anytime from 11am onwards. Please feel free to have timings during the day that suit you best.

I am attaching a draft interview schedule (semi-structured as they say!). If there is anything on this schedule that you feel uneasy about then please feel free to say so. I would really need to tape record all the interviews so I can undertake a transcription afterwards.

I am happy to interview any students you feel you can choose – and who, of course, agree to be interviewed. I am not sure whether your students work on the problems in school or at home or in both places but if there is any variety then I would really appreciate interviewing all ‘types’ of location working. I am particularly seeking any home workers - they can be hard to find, as I am not in their network! I think the students would feel more at ease if they were in a group situation, though numbers between 2 and 4 can flexibly form a group. An individual interview can be possible if you feel that would be OK for the person concerned. I would ‘only’ want to conduct four interviews (group or otherwise) as a maximum and would try hard to keep each interview to be no more than thirty minutes. I am happy to ‘hang around’ over the day if this would disrupt lessons the least.

I am not sure what ages of students you can give me to interview. It may depend upon their ages (especially for those not in the sixth form) as to whether your school feels that I need to gain permission from the parents before proceeding. I am attaching some draft paperwork if this would help in any way.

I wonder whether you and a colleague would mind being interviewed for about 20 to 30 minutes at some stage. If you agree I would like to keep that schedule secret for a while!

I should also tell you that I have been CRB checked and would always bring it with me for your school to see.

I am sure that I have left some information out. Do please ask for any clarifications you would like, but as I mentioned in my initial letter I will be out of contact for the end of today until Monday July 2nd.

With kind regards
Libby.

1.3C: Information for Sixth form students questionnaire to be given out by class teacher

Research into Mathematics and the Internet: School and/or Home

Background Information

My name is Libby Jared and I am a lecturer in Mathematics Education at the Faculty of Education, University of Cambridge.

In 1996 - around the time I think that you were moving from KS1 to KS2 - I was one of a 'gang of four' who helped to set up the NRICH website with the primary aim to have a monthly set of challenging mathematics problems accessible on-line for anyone to try (if they wanted to). At that time, the 'new' Internet seemed a good way for making mathematics become more accessible to lots of people – the technology had some natural advantages for what it could offer in the way of open problems and other resources. The problems could be taken off the Internet and used by schools in lessons and mathematics clubs and/or by people at home.

Now in 2007, some (not all) students (from under 11s to sixth formers) appear keen (though some prefer to keep quiet about it) to spend time at home doing some of the problems. Others would prefer to do other things and never visit the website out of school. Either of these two scenarios are equally interesting to me – NRICH is not there to force anyone to do anything!

However I would like to find out a little more about the reasons for choosing between 'to do or not to do' and in investigating the role that Internet resources might or might not play in establishing 'learning places' in the future. Looking at the NRICH site your school gets a proud mention several times because some members are doing the problems somewhere (school? home?). Your teacher has kindly allowed me to visit and have a chat with you - if you are willing - by way of a questionnaire and through some group interviews.

Your participation in both the questionnaire and interview is purely voluntary - even though I am hoping that you will help me out! I need to give you the assurance that anything you say or write will be kept confidential and used only for the purposes of my research. In any publication relating to this research all names will be referred to by a pseudonym. If you are willing to be interviewed it is best for you to sign what is known as a consent form. (You show consent for the questionnaire by filling it in).

The questionnaire appears overleaf. The interview consent form and a draft* of the interview questions (you can think of replies in advance, if you wish to) are also included. *During the interview, depending upon your replies I might ask some supplementary, follow-up questions.

Libby Jared. 08.10.07.

1.3D: Interview Consent Form

PARTICIPANT CONSENT FORM

Title of Project:

Mathematics/Internet research project

Please sign below if you are willing to be interviewed for this project. Before you do, please note that:

The interview will be tape recorded.

During the interview you need not answer any question you would prefer not to.

You are free to ask me any questions before, during and at the end of the interview.

Anything you say will be kept confidential and used only for the purposes of the study. In any publication relating to the project, all names will be referred to by a pseudonym.

____________________	_______	____________________
Name of Student	Date	Signature

1.3E: Example of email contacting potential interviewee

Dear […],
Emma [AskNRICH Moderator] has passed on your e-mail message to her in response for me trying to find some people to 'talk' to as it were as part of my research.

From looking within AskNRICH, I've calculated that you are now in Year 12 and over 16. So I think that as long as you are willing to talk to me via e-mail, I can talk to you. However you say that your Mum also dips into AskNRICH sometimes to see what is going on, so please feel free to invite Mum to add any two pennyworth as well.

It doesn't matter a jot whether you post on nrich now, as I am looking more for people who like you obviously find maths interesting (I note your ambition, which is great and good luck) and become proactive in using the Internet in some way to, dare I say it, enrich the subject. I am thinking (or hypothesising as they say) that, now we have the Internet available, there are more opportunities for people like you to find extra people to communicate with than just teachers.

I should say at this point that if I write about you in any of my research you would have a new, anonymous identity - either no name or a completely different one!

So if you are happy with all the above, can I ask you a few questions to start with and then if I think of others from your replies, maybe I could maintain a conversation for a short while. (You are free to choose not to answer particular questions if you don't want to). If you cover a later question in an earlier response it doesn't matter and please don't waste time repeating it in a different place. I will get the overall picture regardless as to where you reply.

Code				

CONFIDENTIAL

Questionnaire - Maths and the Internet at school and at home

Everything you write on this questionnaire (including your name) will be kept confidential and only read by researchers

The purpose of this questionnaire is to ask you about the way you do maths related work, with or without the Internet – at school and home.

The questionnaire has been written in five sections - A, B C, D and E. Depending on your answers you may not have to answer all five sections.

Please start by answering all of Sections A, B and C.

Whenever you are asked to write in a box, please try to write down something.

Section A Individual Details

This section is just to get some general information about you. If you would like to give your name you can do so just under this writing, but if you would prefer not to, then that is OK.

(Your name, if you would like to give it: ________________________________ *)*

1. How old are you? __________ years

2. What year are you in? Year __________

3. Are you *(please tick)* Male ☐ Female ☐

Section B School Information

This section is asking for some general information about your school and how the maths classes are arranged.

4. For any part of this school year, are you being taught maths in sets?

   (please tick) yes ☐ no ☐

5. If yes, is your maths set a:

   (please tick) upper set ☐ middle set ☐ lower set ☐

In the table below, please tick either yes, no, or not sure, to answer the following:

	Question	yes	no	not sure
6	In lessons, do you ever do maths games and puzzles that are not part of the maths topic you are doing?			
7	In lessons, do you ever do any maths problems which have been taken from the Internet?			
8	Is there a maths club in your school?			

*Answer the next question if you answered **yes** to question 6.*

9. When you do maths games and puzzles in lessons, do you feel that it helps you to learn maths in a different way?

   (please tick) yes ☐ no ☐

10. *If you answered question 9 and ticked **yes,*** write a few words in the box below to say why you think it is different.

    *If you answered question 9 and ticked **no,*** write a few words in the box below to say why you think it is not different.

11. If you think that there are other subjects you do on your timetable which help you with your maths, then please write them in the box. below.

Section C Maths Feelings

Not everyone in this world likes maths or finds it easy or interesting. This section asks you to think about how you do in maths, what you feel about the subject and the ways you understand it.

It is impossible to give a wrong answer here as you are just giving your own thoughts.

How you do in maths and what you think about it as a subject

Some statements are given in the table below. For each one put a tick in the box which you feel most comfortable with.

	Question	Definitely	I mainly think so	Not sure	No, not really	Certainly not
12	I feel I am quite good at all subjects					
13	I feel I am better at maths than I am at most other subjects					
14	I feel that I am worse at maths than I am at most other subjects					
15	I generally enjoy doing maths					

How you understand Maths

This part is about maths rules and methods e.g. adding a zero when multiplying by 10.

16. In maths lessons do you think that your teacher usually gives you maths rules and methods for doing your work

(please tick) quite often ☐ sometimes ☐ never ☐

Some statements are given in the table below. For each one put a tick in the box which you feel most comfortable with.

	Question	Definitely	I mainly think so	Not sure	No, not really	Certainly not
17	In the maths lessons I understand how the rules and methods work					
18	I like understanding how the rules and methods work					
19	I want to understand how the rules and methods work					
20	If I am given a rule or method in a maths lesson, I can remember it for at least the next few days					
21	If I am given a rule or method in a maths lesson I can generally remember it forever					
22	I think I am quite good at applying rules and methods in maths					
23	I like to make up my own rules and methods					
24	I think it is more important to understand a maths topic than to try to remember how the rules and methods work					

25. In this box please describe how, in school, you feel you learn the maths you do.

26. In this box, write down a favourite topic that you have done in any of your maths lessons? Try also to write why it is your favourite.

If you **DO NOT use the Internet at home**, there are no more questions for you to answer.

THANK YOU for filling in this questionnaire.

If you **DO use the Internet at home**, then it would be helpful if you continued with the next section.

Section D Using the Internet at Home

For this section I am interested in seeing how you use the Internet at home. This section is not very much to do with maths or school work but how and for what purposes you are using modern day technology to live in a modern world and what you feel about having the Internet to use for either homework or free time.

27. Please list up to four different things that you use the Internet at home for and say roughly how much time (either in hours or minutes) each week you spend on each one.

	I am using the Internet to:	Time spent per week
1.		
2.		
3.		
4.		

28. Please name up to four different maths websites that you have visited whilst using the Internet at home and indicate roughly how often this is - e.g. just once, a few times, once a month, every week, very often. You do not need to give the web address. If you never visit any maths website write NONE on the first line.

	I visit the following maths websites:	How often?
1.		
2.		
3.		
4.		

Some statements are given in the table below. For each one put a tick in the box which you feel most comfortable with.

	Question	Definitely	I mainly think so	Not sure	No, not really	Certainly not
29	I think that the Internet was a good invention					
30	I think that the Internet can help me with some of my homework					
31	Even when I am using the Internet for some reason other than homework, I can still sometimes learn about a subject or topic I am doing at school					
32	I think that people who do not have access to the Internet at home are at a disadvantage					
33	Within the next five years, the Internet will change the way our teachers teach us					

If you **DO NOT use the NRICH website at home**,

there are no more questions for you to answer.

THANK YOU for filling in this questionnaire.

If you **DO use the NRICH website at home,**

then it would be helpful if you continued with the next section.

Section E Using the NRICH Website at Home

This final section is concerned with how and why you use the NRICH site. It also asks you a little about what you think about the problems which you can find on the NRICH site.

34. Do you use the website:
(please tick) only at home ☐ both at home and at school ☐

35. Do you think that your maths teacher knows that you use the website at home?
(please tick) yes ☐ no ☐ not sure ☐

36. If the answer is yes, is it because *(please tick)*:
you have told your teacher ☐
your teacher suggested that you use the site ☐

37. or is there some other reason, which is:

____

38. Please write in this box some reasons why you choose to do maths work from the NRICH website at home.

39. When you are doing maths from the NRICH website at home, do you try the problems: *(you may tick more than one)*
alone ☐ with a group of friends ☐
with a friend ☐ with brothers/sisters ☐
with adults in your family ☐ ☐

Other – please describe who with ____________________________

40. Please give the name of the last three problems on the NRICH site you have tried. (If you cannot remember the name then just write a few words about what it was about.)

1. ____

2. ____

3. ____

41. When you choose a problem to do, do you generally find it:

(please tick) fairly easy ☐ just right ☐

a little difficult ☐ very difficult ☐

42. For each of the two statements below put a tick in the box which you feel most comfortable with.

	Question	Definitely	I mainly think so	Not sure	No, not really	Certainly not
a.	Learning maths through NRICH is more enjoyable than learning maths at school					
b.	Using the NRICH website has made you more interested in maths at school					

43. In this box, please give some reasons for your answer in (a.) above.	44. In this box, please give some reasons for your answer in (b.) above.

THANK YOU for filling in this questionnaire.

Code				

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Questionnaire
Maths and the Internet at school and at home

The purpose of this questionnaire is to ask you about the way you do maths related work, with or without the Internet – at school and home.

The questionnaire has been written in six sections - A, B C, D, E, F and G. Depending on your answers you may not have to answer all seven sections. Please start by answering all the questions in Sections A, B and C.

When you think you have completed your questionnaire, please turn to the very last page (the blue sheet) to fill in a few details about yourself. (Section G).

It might be quite hard, but whenever you are asked to write in a box, please try to write down as much as you can.

Please note:

Everything you write on this questionnaire (including your name) will be kept confidential and only read by researchers.

THANK YOU

Section A General Information about your School

1. If you are in a maths set, is it a:
   (please tick) upper set ☐ middle set ☐ lower set ☐

2. In maths lessons, do you sometimes work on questions taken from the Internet?
   (please tick) yes ☐ no ☐ not sure ☐

3. Do you belong to a maths club in your school?
   (please tick) there is no maths club ☐ yes ☐ no ☐

4. Does your school use e-mail or the Internet to send information home?

   (please tick) yes ☐ no ☐ not sure ☐

Section B Doing Puzzles and Games in Maths

This means work you do in maths lessons that asks you, for example, to solve a puzzle, or to do a logic problem, or to play a game that needs some maths or logic to find out how to win.

	Statement	Definitely	I mainly think so	No, not really	Certainly not
5	Doing puzzles and games in maths makes me feel I am doing maths in a different way to my other maths lessons				
6	Doing puzzles and games in maths is more interesting than my other maths work				
7	Doing puzzles and games in maths is less fun than other maths				
8	Doing puzzles and games in maths makes me think for myself more than when I am doing other maths work				
9	It is harder to learn maths by doing puzzles and games				
10	When doing puzzles and games, more of us work together than with other maths work				
11	When doing puzzles and games in maths, I feel I am working in a more problem solving way than at other times				
12	Puzzles and games are generally more challenging than other maths questions				
13	When doing puzzles and games in maths, I try harder to solve the problems than with other maths questions				

14. *Use the box below if you have anything more you would like to say about doing maths puzzles and games.*

Section C Maths Feelings

Not everyone in this world likes maths or finds it easy or interesting. Here you are giving your own thoughts and feelings so it is impossible to give a wrong answer.

Some statements are given in the table below. For each one put a tick in the box that you feel most comfortable with.

	Statement	Definitely	I mainly think so	No, not really	Certainly not
15	I feel I am quite good at all subjects				
16	I feel I am better at maths than I am at most other subjects				
17	I feel that I am worse at maths than I am at most other subjects				

How you understand Maths

This part is about maths rules and methods e.g. adding a zero when multiplying by 10 or being told that two minuses make a plus

Some statements are given in the table below. For each one put a tick in the box that you feel most comfortable with.

	Statement	Definitely	I mainly think so	No, not really	Certainly not
18	If I am given a rule or method I can generally remember it forever				
19	I like to be told the rule and method to be used, without any explanation, so I can simply use it				
20	I am interested in knowing why the rule or method works				
21	If the rule or method is explained I generally understand how the rule and method works				
22	Making up my own rules and methods helps me to understand the work				

23. Put a circle round the letter A or B below, for the statement that you think is the more important of the two:

A. Understanding the mathematics so you have a way for doing it for yourself

B. Remembering the rule or method

How you help yourself to learn your maths

This part is asking you to describe how you feel you help yourself to learn your maths. Below there are several statements that different people have given before.

For each of the statements please tick one of the boxes which best matches how much you use each way to help you learn your maths.

		Often	Quite often	A bit	Never
24	I ask my teacher to explain it again to me				
25	I ask a friend to show me what they are doing				
26	I talk about it with other people in my class				
27	I ask someone at home to help me				
28	I try and quietly work it out for myself				
29	I explain the work to others in my class to help them				

30. If there is some other way you use, please write it down here

If you **DO NOT use the Internet at home**, there are no more questions for you to answer but **Please go to the last (blue) page** to fill in a few details about yourself.

If you **DO use the Internet at home**, then it would be helpful if you continued with the next section.

Section D Using the Internet at Home

The table below lists a range of activities which people use the Internet at home for. If you do any of these activities at least once a week, choose a box to show roughly how much time each week you spend doing the activity.

	Time each week in ho	Over 7 hours	From 3 to 7 hours	1 to 3 hours	Less than 1 hour
31	Sending and receiving messages from friends and family				
32	Finding out about things for topics set as homework				
33	Choosing to use websites which have school related work but are not directly related to any homework task				
34	Searching for general information, e.g. train times, shopping/booking tickets etc.				
35	Downloading music, or graphics				
36	Anything else – please write down what this is and tick a time as with the others above				

37. On average how many hours each week in total do you use the Internet at home: ____ hours

Some statements are given in the table below. For each one put a tick in the box that you feel most comfortable with.

	Statement	Definitely	I mainly think so	No, not really	Certainly not
38	I think that the Internet can help me with some of my homework				
39	Even when I am using the Internet for some reason other than homework, I can still sometimes learn about a subject or topic I am doing at school				
40	I think that people who do not have access to the Internet at home are at a disadvantage with their school work				

Section E Our future with the Internet

You are asked to imagine what might be happening at some time in the future, say in 5 to 10 years time.

Some statements are given in the table below. For each one put a tick in the box that you feel most comfortable with.

	Statement	Definitely	I mainly think so	No, not really	Certainly not
41	Within the next 5 to 10 years, more school related work will be undertaken via the Internet				
42	Because of the Internet, teaching and lessons will change				
43	Because of the Internet, more learning will be able to take place at home				
44	Because of the Internet, people will be able to choose where and when they learn				
45	Because of the Internet, school pupils will be able to take on greater responsibility for their own learning				

46. Please write down how you think the Internet is going to change education.

If you **DO NOT use the NRICH website** there are no more questions for you to answer but **please go to the last (blue) page** to fill in a few details about yourself.

If you **DO use the NRICH website** then it would be helpful if you continued with the next section.

Section F Using the NRICH Website

This final section is concerned with how and why you use the NRICH website. It also asks you a little about what you think about the problems which you can find on the NRICH site.

47. Do you see the website:

(please tick) only at school ☐ only at home ☐ both at home and at school ☐

48. If you use the website at home do you think that your maths teacher knows that you do?

(please tick) yes ☐ no ☐ not sure ☐

49. If the answer is yes, is it because:

(please tick) you have told your teacher ☐ your teacher suggested that you use the website ☐

50. Please write in this box some reasons why you choose to use the NRICH website (either at school, or home, or both).

51. If you are doing maths from the NRICH website at home, do you try the problems:

(you may tick more than one)

alone ☐ with a group of friends ☐ with a friend ☐

with brothers/sisters ☐ with adults in your family ☐

52. When you choose a problem to do, do you generally find it:

(please tick) fairly easy ☐ just right ☐ slightly difficult ☐ very difficult ☐

For the statement below put a tick in the box that you feel most comfortable with.

	Statement	Definitely	I mainly think so	No, not really	Certainly not
53	I like to spend some of my time out of maths lessons doing more maths for myself				

54. In this box, please give some reasons for your choice in question 53 above

55. Do you think you could decide to learn some of your maths from the NRICH website by yourself and then talk about it with your teacher? When would you be able to do this?

There are now no more questions left for you to answer, **but please go to the last (blue) page** to fill in a few details about yourself.

Section G Individual Details

In this section you are asked to give a small amount of some general information about yourself.

(I). How old are you? __________ years

(II). What year at school are you in? Year __________

(III) Are you Male ☐ Female ☐

It would be very helpful if you would give your name. *Remember that all the information you give is confidential will only be read by the researcher and you will not be referred to by name*.

If you feel happy to give your name,
please write it here: ____

It would also be helpful to interview just a very few of you in a group of four so that you can talk more about the information you have given on the questionnaire. You need to know that the interview would be tape recorded so the researcher can listen to what you say more than once.

We can only interview if you have written your name so we know who you are!

Would you be happy to be interviewed? Yes ☐ No ☐
(please tick)

THANK YOU VERY MUCH for filling in this questionnaire.

Section B Doing Puzzles and Games in Maths

This means work you do in maths lessons that asks you, for example, to solve a puzzle, or to do a logic problem, or to play a game that needs some maths or logic to find out how to win.

	Statement	Definitely	I mainly think so	No, not really	Certainly not
5	Doing puzzles and games in maths makes me feel I am doing maths in a different way to my other maths lessons				
6	Doing puzzles and games in maths is more interesting than my other maths work				
7	Doing puzzles and games in maths is less fun than other maths				
8	Doing puzzles and games in maths makes me think for myself more than when I am doing other maths work				
9	It is harder to learn maths by doing puzzles and games				
10	When doing puzzles and games, more of us work together than with other maths work				
11	When doing puzzles and games in maths, I feel I am working in a more problem solving way than at other times				
12	Puzzles and games are generally more challenging than other maths questions				
13	When doing puzzles and games in maths, I try harder to solve the problems than with other maths questions				

14. *Use the box below if you have anything more you would like to say about doing maths puzzles and games.*

Section C Maths Feelings

Not everyone in this world likes maths or finds it easy or interesting. Here you are giving your own thoughts and feelings so it is impossible to give a wrong answer.

How you understand Maths

This part is about maths rules and methods e.g. adding a zero when multiplying by 10 or being told that two minuses make a plus

Some statements are given in the table below. For each one put a tick in the box that you feel most comfortable with.

	Statement	Definitely	I mainly think so	No, not really	Certainly not
18	If I am given a rule or method I can generally remember it forever				
19	I like to be told the rule and method to be used, without any explanation, so I can simply use it				
20	I am interested in knowing why the rule or method works				
21	If the rule or method is explained I generally understand how the rule and method works				
22	Making up my own rules and methods helps me to understand the work				

23. Put a circle round the letter A or B below, for the statement that you think is the more important of the two:

A. Understanding the mathematics so you have a way for doing it for yourself

B. Remembering the rule or method

How you help yourself to learn your maths

This part is asking you to describe how you feel you help yourself to learn your maths. Below there are several statements that different people have given before.

For each of the statements please tick one of the boxes which best matches how much you use each way to help you learn your maths.

		Often	Quite often	A bit	Never
24	I ask my teacher to explain it again to me				
25	I ask a friend to show me what they are doing				
26	I talk about it with other people in my class				
27	I ask someone at home to help me				
28	I try and quietly work it out for myself				
29	I explain the work to others in my class to help them				

30. If there is some other way you use, please write it down here:

Research into Mathematics and the Internet: School and/or Home

Background Information

My name is Libby Jared and I am a lecturer in Mathematics Education at the Faculty of Education, University of Cambridge.

In 1996 - around the time I think that you were moving from KS1 to KS2 - I was one of a ‘gang of four’ who helped to set up the NRICH website with the primary aim to have a monthly set of challenging mathematics problems accessible on-line for anyone to try (if they wanted to). At that time, the ‘new’ Internet seemed a good way for making mathematics become more accessible to lots of people – the technology had some natural advantages for what it could offer in the way of open problems and other resources. The problems could be taken off the Internet and used by schools in lessons and mathematics clubs and/or by people at home.

Now in 2007, some (not all) students (from under 11s to sixth formers) appear keen (though some prefer to keep quiet about it) to spend time at home doing some of the problems. Others would prefer to do other things and never visit the website out of school. Either of these two scenarios are equally interesting to me – NRICH is not there to force anyone to do anything!

However I would like to find out a little more about the reasons for choosing between ‘to do or not to do’ and in investigating the role that Internet resources might or might not play in establishing ‘learning places’ in the future. Looking at the NRICH site your school gets a proud mention several times because some members are doing the problems somewhere (school? home?). Your teacher has kindly allowed me to visit and have a chat with you - if you are willing - by way of a questionnaire and through some group interviews.

Your participation in both the questionnaire and interview is purely voluntary - even though I am hoping that you will help me out! I need to give you the assurance that anything you say or write will be kept confidential and used only for the purposes of my research. In any publication relating to this research all names will be referred to by a pseudonym. If you are willing to be interviewed it is best for you to sign what is known as a consent form. (You show consent for the questionnaire by filling it in).

The questionnaire appears overleaf. The interview consent form and a draft* of the interview questions (you can think of replies in advance, if you wish to) are also included. *During the interview, depending upon your replies I might ask some supplementary, follow-up questions.

Libby Jared. 08.10.07.

Code				

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Mathematics and the Internet: School and/or Home

You are free to choose to leave some replies blank, if you so wish. Everything you write on this questionnaire (including your name) will be kept confidential and read only by me. You will remain anonymous in any reporting of the results of the research.

Thank you for your time and effort in helping me with my research.

Your name (optional but helpful): ____________________________________

1. Please write yes or no as appropriate in each of the two columns below for whether you did / you do use any mathematics based websites pre 6th form and currently now you are in the 6th form:

	Before 6th form	Now in 6th form
at home		
at school during ‘normal’ lessons		
at school as extra curricular e.g. maths club, competitions		

2. If you have in the past, or currently use, mathematics website, please list, with reasons for doing so, the main ones in the box below:

3. For roughly how many hours per week (including zero) do you use the Internet at home:

	Hours
for homework	
for work which is in some way connected to school subjects but which you would like to pursue further just for yourself	
for leisure and pleasure	

I have previously given out the following statements in a web questionnaire – which formed part of longer one than this one! The statements and responses are 'rough and ready' as you can only give a spontaneous tick – but you might be asked more about your thoughts in any interview.
For now, please place a tick in the box you feel most comfortable with – note there isn't a not-sure column.

The Internet Effect (5 to 10 years into the future) on Learning

	Because of the Internet …	Strongly agree	Mainly agree	Mainly disagree	Strongly disagree
4	… within the next few years, more school related work will be undertaken using it				
5	… teaching and lessons will change				
6	… more learning will be able to take place at home (than at present)				
7	… people will be able to choose where and when they learn				
8	… students will be able to take on greater responsibility for their own learning				

Problem Solving in Mathematics

For this section I am trying to focus on more open-ended mathematics problems (puzzles, games) that would not be considered as 'routine textbook or examination work' – more the sort of problem you would find on the NRICH website or competitions.

The statements here are simplistic and open to many different interpretations – more to be discussed in interview!

	Undertaking this type of problem-solving work …	Strongly agree	Mainly agree	Mainly disagree	Strongly disagree
9	… feels different for doing maths than in other maths lessons				
10	… is generally more interesting				
11	… is less fun				
12	… makes me think for myself more				
13	… makes learning maths harder				
14	… encourages group work				
15	… makes me feel I am working more mathematically				
16	… is generally more challenging than other maths questions				
17	… makes me try harder				

Mathematics and You!

This is the last part to the questionnaire – but please note it is spread over two sides.

It is not meant to be an essay – though please write as much as possible, answering as many of the questions below as you feel you can plus any others that you might like to add!

What are your feelings about mathematics – maybe comparing with other subjects? What motivates you?

How important is it to have a good memory in mathematics? How useful is it to remember rules and methods as opposed to understanding?

more on the next page!

How do you feel you learn your mathematics? e.g. what role do you see your teacher having in this? what role do you have? how independent do you feel you are in organising your own learning? who do you turn to for help?

How important is it to you to work with others? What are the advantages/disadvantages of doing this?

Anything else?

THANK YOU

FreeOnlineSurveys.com

Mathematics and the Internet at school and at home

Welcome and thank you for thinking about filling in your replies to my questions.

Who am I?
I am Libby Jared, a lecturer in mathematics education at the Faculty of Education, University of Cambridge.
I was one of the four people who helped to set up NRICH because I had always liked solving mathematics problems and the Internet seemed a good way for making problems accessible to lots of people.
I have two major research interests - seeing what other people think about solving mathematical problems and finding out whether or not work is different now that the Internet is very widely used in some parts of the world.

What is this questionnaire about?
The purpose of this questionnaire is to ask you about the way you do your mathematics related work - with or without the Internet - at home and at school.

The questionnaire has six sections, each one on a different page - the first two ask you about the Internet, the next about using NRICH, the fourth is asking a little about yourself, and the last two focus on learning mathematics in school.
It would be helpful if you could answer as many questions as possible.
When a question asks you to put an answer in a box please try to write down as much as you can.

Please note:
Responses to this questionnaire will be used in my research
Your responses will only be seen by me and kept confidential
Data published in reports which relates to any responses will be anonymous

THANK YOU

and thanks also to NRICH for making the link from their site to my questionnaire

Section One
Using the Internet at Home

If you do not use the Internet at home, you will not be able to answer the questions in this section. But do not worry, later on in the questionnaire there are other sections which have lots more questions for you to do.

1) The table below lists different activities for which some people use the Internet at home. Choose the box to show how many hours each week you usually spend on it.

	Over 7 hours each week	Between 3 to 7 hours each week	From 1 to 3 hours each week	Less than one hour each week	Never
Sending and receiving messages from friends and family					
Finding out about things which have been set for homework					
Using websites which have school type work but are not directly related to any set homework					
Searching for general information, for example train times, shopping, booking tickets ...					
Downloading music or graphics					

2) If you use the Internet at home for other things not given above, please write down the activities in the box below and give a time in hours that you usually spend each week† on it.

3) Please choose the option below to say how many HOURS ON AVERAGE EACH WEEK you spend using the Internet at home.	
Less than 1 hour	
From 1 to 3 hours	
Between 3 to 7 hours	
From 7 to 14 hours	
Over 14 hours	

4) For each of the statements in the table below, choose the option that you feel you are most comfortable with.

	Definitely	I mainly think so	No, not really	Certainly not
I think that the Internet can help me with some of my homework				
Even when I am using the Internet for some other reason than homework, I can still sometimes learn about a subject or topic I am doing at school				
I think that people who do not have access to the Internet at home are at a disadvantage with their school work				

Section Two
Our future with the Internet

You are asked to imagine what might be happening at some time in the future - say in 5 to 10 years time.

5) For each of the statements in the table below, choose the option that you feel you are most comfortable with.

	Definitely	I mainly think so	No, not really	Certainly not
Within the next 5 to 10 years, more school related work will be done using the Internet				
Because of the Internet, teaching and lessons will change				
Because of the Internet, more learning will take place at home (than at present)				
Because of the Internet, people will be able to choose where and when they learn				
Because of the Internet, school pupils will be able to take on greater responsibility for their own learning				

6) In the box below, please write down how you think the Internet is going to change education.

Section Three
Using the NRICH Website

This section is to do with how and why you use the NRICH website. It also asks you a little about how you feel about the problems you find on the NRICH website.

7) Please select the option for where you use the website	
only at school	
only at home	
both at home and at school	

8) If you use the website at home,† do you think that your mathematics teacher knows that you do?	
yes	
no	
not sure	

9) If the answer to the last question was yes, you think that your mathematics teacher does know, is it because:	
you have told your teacher	
your teacher suggested that you use the website	

10) Please write in the box below some reasons why you choose to use the NRICH website (either at school, or home, or both).

11) For this question you may choose more than one option. If you are doing mathematics from the NRICH website at home, do you try the problems:	
alone	
with a group of friends	
with a friend	
with brothers/sisters	
with adults in your family	

12) When you choose a problem to do, which one of the following best describes how you find the problems overall:	
fairly easy	
just right	
slightly difficult	
very difficult	

13) For the statement below, choose the option that you feel you are most comfortable with.

	Definitely	I mainly think so	No, not really	Certainly not
I like to spend some of my spare time out of school doing mathematics type problems				

14) In the box below, please write down some reasons for your choice above.

15) Do you ever decide for yourself to learn some of your mathematics using† NRICH or any other mathematics website without your teacher knowing?

yes	
no	

16) If yes, please write in the box below why you decide to do this.

17) If yes, you do learn some mathematics for yourself, do you then talk about it with your teacher?

always	
sometimes	
never	

Section Four

General Information

This section asks a few general questions about you and your school.

Please answer the following questions by writing in the boxes below.

18) How old are you?	
Less than 11 years old	
Aged 11 to 13	
Aged 14 to 16	
Aged 17 to 18	
Aged 19 to 22	
Over 22 years old	

19) Are you	
Male	
Female	

20) Which country do you live in?

21) Which school year or school grade are you in?

22) Please select the option which indicates the mathematics group or set you are in at school:	
Higher/Upper set/group	
Middle set/group	
Lower set/group	
There are no sets/groups at school	

23) For each of the statements in the table below, choose the option that you feel you are most comfortable with.

	Definitely	I mainly think so	No, not really	Certainly not
I feel I am quite good at all subjects				
I feel I am better at mathematics than I am at most other subjects				
I feel I am worse at mathematics than I am at most other subjects				

24) In mathematics lessons, do you sometimes work on questions taken from the Internet?	
yes	
no	
not sure	

25) Do you belong to a school mathematics club?	
there is no mathematics club	
yes	
no	

26) Does your school use e-mail or the Internet to send information home?	
yes	
no	
not sure	

Section Five

Doing puzzles and games in mathematics lessons

In mathematics lessons, work can be from a text book which is often a set of practice questions from an exercise. The following statements are not about this type of work, but instead are describing the sort of mathematics questions you find on the NRICH site - work which you can be given in lessons but involves you trying to solve a puzzle or a logic problem or playing a game which needs some maths or logic to find out how to win.

27) For each of the statements in the table below, choose the option that you feel you are most comfortable with.

	Definitely	I mainly think so	No, not really	Certainly not
Doing puzzles and games in mathematics makes me feel I am doing mathematics in a different way to my other mathematics lessons				
Doing puzzles and games in mathematics is more interesting than my other mathematics work				
Doing puzzles and games in mathematics is less fun than other mathematics				
When doing puzzles and games in mathematics I think for myself more than when I am doing other mathematics work				
It is harder to learn mathematics by doing puzzles and games				
When doing puzzles and games more of us work together than with other mathematics work				
When doing puzzles and games in mathematics, I feel I am working in a more problem solving way than at other times				
Puzzles and games are generally more challenging than other mathematics questions				
When doing puzzles and games in mathematics, I try harder to solve the problems that with other mathematics questions				

28) Use the box below if you have anything more you would like to say about doing mathematics puzzles and games.

Section Six
Understanding and Learning Mathematics

29) For each statement in the table below, choose the option that you feel you are most comfortable with.
The statements often include the word rule or method. Two examples of a rule are being told that to add a zero on the end to multiply by 10 (not always true!) and that two negative signs together make a positive. A method would involve you being told to do this, followed by this and this... - with little explanation as to why the method works.

	Definitely	I mainly think so	No, not really	Certainly not
If I am given a rule or method I can generally remember it forever				
I like to be told the rule or method to be used, without any explanation, so I can simply use it				
I am interested in knowing why the rule or method works				
If the rule or method is explained, I generally understand how the rule or method works				
Making up my own rules and methods helps me to understand the work				

30) Decision Time. From the two statements below, select THE ONE that you think is the more important of the two.

Understanding the mathematics so you have a way of doing it for yourself	
Remembering the rule or method	

31) You will be pleased to know that this is the final question! It is asking you to describe how you feel you help yourself to learn your mathematics. The statements given below are ways that different people have suggested in the past.
For each of the statements in the table below, choose the option that you feel you are most comfortable with.

	Often	Quite Often	A Bit	Never
I ask my teacher to explain it again to me				
I ask a friend to show me what they are doing				
I talk about it with other people in my class				
I ask someone at home to help me				
I try and quietly work it out for myself				
I explain the work to others in my class to help them				

32) If there is some other way you use, please write it in the box below.

THANK YOU VERY MUCH FOR TAKING THE
TIME TO FILL IN THE QUESTIONNAIRE

Section	Rationale	Question	Proposed Analysis
One Using the Internet at Home To consider young people's home Internet use, both school subject based and more general [Not reported in Thesis]	Use as an 'ice-breaker' and gentle start, with the assumption that, as the survey had by necessity been accessed by Internet users, they would find it easy to answer the questions. Questions set to elicit general information to add to published studies about the use and time such people give to Internet related activities and to gain thoughts from a group of people, living in a post-internet-invention era, on issues surrounding the Internet in terms of access, equality and the role it played in supporting homework.	1. Five activities (combination of school and leisure) listed with five time options per week - from 'never' to 'over 7 hours' (top specification implying on average more than one hour per day). 2. Opportunity to list (with time per week) other activities not included above. 3. Specify from five time options per week listed, the average number of hours per week (from 'less than one hour' to 'over 14 hours' (highest time span implying on average more than two hours per day). 4. Four levels of agreement to three statements - the first two seeking views as to whether the Internet is 'helpful' to 'school work' and the third whether non access is a disadvantage.	1 and 3. Frequency tables for each time span to calculate: mean, range, inter-quartile range, max and min. 2. Additional activities noted. 4. Frequency tables for each option (percent) and collapsed to two categories of agree/disagree. Age and gender groups for comparison tests.

Section	Rationale	Question	Proposed Analysis
Two Our Future with the Internet To imagine what might be happening at some time in the future (5 – 10 years) [Not reported in Thesis]	This section continues to probe young peoples' perceptions but moves the context to be both within home and school and the focus on teachers' practices and self-learning. Would respondents feel that the Internet could make them more independent in their choice in how and where they study? The future limited to the next 5 to 10 years provides a realistic visionary framework.	5. Four levels of agreement to five statements: (a) more school related work undertaken using the Internet (b) teaching and lessons would change as a consequence (c) more learning would be undertaken at home (d) the opportunity to choose where and when they learn (e) pupils able to take on more responsibility for own learning 6. Open response for how Internet might change education.	5. Frequency tables for each option (percent) and collapsed to two categories of agree/disagree Age and gender groups for comparison tests. 6. Responses entered into Nvivo for coding.

Section	Rationale	Question	Proposed Analysis
Three Using the NRICH website How and why the website is used and thoughts about the type of problems available on the site [Reported in Chapter 4]	As the survey could only be accessed through NRICH website then some questions explicitly concerning NRICH would serve to determine (a) current practices of such a website and (b) crucially whether the findings a decade earlier, that there were young people deciding to do mathematics at home (without telling their teacher), still held. If they were then further research into the home workers could be undertaken. The questions were devised through the experience of two evaluations in the early days of NRICH [Jared 1997,1998].	7. Three discrete options as to where the website was used. 8. Three discrete options as to whether the respondent's mathematics teacher would know they were using it. 9. Two discrete options as to how the teacher knew (if they did). 10. Open response as to why they choose to use the website. 11. Selecting as many relevant options (of five) with whom, at home, the problems are tried (including the option of by self). 12. Four discrete options as to the general degree of difficulty that the problems present. 13 & 14. Four levels of agreement in liking to spend time out of school doing mathematics problems and open response for selecting degree of liking 15, 16 & 17. Dichotomous question on whether choose to self learn mathematics from the Internet without teacher knowing and if yes, then open response for reasons in doing so and three discrete options for doing so.	7, 8, 9, 11, 12, 13, 15, 17 Frequency tables for each option. 10, 14 & 16 Responses entered into Nvivo for coding. Using groups [defined below] to emerge from q7, further analysis on frequency of responses to questions in this section: Home Only (oh) Home and School (hs) School Only (os) Home People (hp) formed by joining oh and hs Throughout: also by age and gender groups for further comparison.

Section	Rationale	Question	Proposed Analysis
Four General Information To collect background data on individuals including self-perceived ability and perceived technology uses within a school setting [Questions 18 to 23 reported in Chapter 4, Questions 24 to 26 not reported in Thesis]	Acts as a link between the key respondents having their own choice for doing NRICH and the mathematics found in school. Data gathered within this section enables (a) school-aged individuals to be isolated and (b) use of subsets (see final column in row above) for analysis. Comparison between schools would be impossible and any request for specific set levels invalid. As survey is freely and openly available, judgments of attainment levels are without teacher reference hence need for respondents to self-report providing an alternative perspective to the more usual form of teachers' judgments. The final questions seek to see if systems are in place to accommodate the widening of learning locations.	18, 19, 20 & 21: age, gender, country, school year/grade. 22 & 23. Four discrete options as to general position of set including no setting. Four levels of agreement to three statements on self-perceived attainment: overall and mathematically relative to other subjects. 24. Three discrete options (including 'not sure') for whether school mathematics work was sometimes taken from the Internet. 25. Three discrete options (including 'no club') for there being a mathematics club. 26. Three discrete options (including 'not sure') for whether school used electronic communications with parents.	18, 19 & 21: Frequency tables for each option Results used in analysis elsewhere 20. Different countries noted 22, 23, 24, 25, & 26: Frequency tables for each option Also by groups to emerge from q7.

Section	Rationale	Question	Proposed Analysis
Five Doing puzzles and games in mathematics lessons To investigate pupils' perceptions of doing this type of mathematics [Reported in Chapter 4]	Evidence from earlier stages showed that many pupils enjoyed doing mathematics when there was a puzzling, game or involved problem solving element to the work. Whilst pupils may see a stated reason (practice, needed for tests!) for doing 'boring' routine exercises, the 'fun' and challenging element often prevails [see LRI]. Nine statements (not necessarily discrete) relating to ideas that respondents had suggested in previous questionnaires and interviews. Hypothesized positive statements were given in seven (not (c) or (e)), with a deliberate negative opposite (c) provided as a system check. No anticipated outcome was made for statement e - a negative response could be persuasive to convince greater adoption.	27. Four levels of agreement as to whether doing puzzles and games in mathematics: (a) felt different (b) was more interesting (c) less fun (d) made one think more (e) made mathematics harder to learn (f) allowed more people to work together (g) for working in a more problem solving way (h) was more challenging (i) made one work harder 28. Open response to add any other comments.	27. Frequency tables for each option (percent) and collapsed to two categories of agree/disagree Age and gender groups and by groups to emerge from q7for comparison tests. Using responses to q27, count agreements to hypothesised result (positive agreement to all except (c) where negative agreements expected) for eight of nine statements (omitting statement e, harder to learn) to assign a problem-solving characteristic [See Table 3.5 Section 3.4.4.1]. 28. Responses entered into Nvivo for coding. Throughout: Age and gender groups and by location groups [findings from q7] for comparison tests.

Section	Rationale	Question	Proposed Analysis
Six Understanding and Learning Mathematics To investigate preferences for: (a) instrumental or relational understanding, and (b) the ways used to learn mathematics [Reported in Chapter 4]	The first part tests the hypothesis that 'home' workers would (a) be more interested in working in a relational understanding way (b) be able to (relationally) understand and (c) like to be independent - though possessing these attitudes would not preclude others from having the same feelings. There is also the hypothesis that the way that mathematics is learned, be it through some degree of (a) rote or memory and (b) asking for help and discussing work with others, will also have relevance in distinguishing characteristics for the 'home' worker. The terms 'relational' or 'instrumental' are not explicitly used as could be uncommon. Statements included for those who would like to (relationally) understand but may not have the ability to do so.	29. Four levels of agreement to five statements about being taught rules and methods in mathematics: (a) remember it forever (b) want to just use without explanation (c) interested in knowing why works (d) if explained can generally understand (e) making up own helps understanding 30. Select preference from two statements: better to understand/remember. 31. Four levels of frequency to six statements about how help self to learn mathematics: (a) ask teacher to re-explain (b) ask friend to show (c) talk about it to others in class (d) ask for help at home (e) try quietly for self (f) help others by explaining 32. Open response to add any other comments (both for this section and generally).	29. Frequency tables for each option (percent) and collapsed to two categories of agree/disagree. 30. Frequency table for each choice. Use selection of understanding or remembering to be of more importance (q30), and responses to statement 2 of q29 (likes to be told rule without explanation) to assign characteristic code. [See Table 3.5 Section 3.4.4.1]. 31. Frequency tables for each option (percent) and collapsed to two categories of more/less often. Use selection of responses to specific statements to assign characteristic code [See Table 3.5 Section 3.4.4.1]. 32. Responses entered into Nvivo for coding Throughout: Age and gender groups and by location groups [findings from q7] for comparison tests.

Pilot Group Interviews with Pupils – July 2005

Thank you for filling in my questionnaire and for coming along to this interview. I will find it very useful if I could get you to talk about some of the parts of the questionnaire which need more discussion than just ticking a box. As your are altogether you can a group you can talk about your answers between yourselves as a discussion. It would though be helpful that before you speak each time, you say your name, so I will know who has said what when I listen to the tape afterwards.

One area of school maths that I am interested in finding out more about to do with what you think about doing maths Games and Puzzles in maths lessons.

Questions Focus 1: Favourite Maths Topic.

On the questionnaire you were asked to write down a n all time favourite maths topic.

Can you say what you wrote down for your favourite maths topic?

On the questionnaire I was wondering what the reasons might be for your choice. Can you say what your reasons were?

When during your school life, did you do the topic you chose?

Question Focus 2: Maths Puzzles and Games

On the questionnaire, I asked you to explain why you thought that doing maths games and puzzles was the same or different from what you usually did maths lessons.

Did anyone think it was different? Could you say why you thought this?

Did anyone think it was much the same as usual? Could you say why you thought this*?*

Question Focus 3: Perceived attainment when doing Maths Puzzle and Games

My next question is to ask you whether you think you are better worse or just the same at doing this type of work (maths games and puzzles) than other parts of maths

Do any of you feel you do better with this type of maths? If so what do you feel you gain (if anything) when your teacher gives you such questions to do?

If worse, why?

And why would you choose the option that it is just the same?

Question Focus 4: Feel that there is sufficient time given in lessons for this sort of work

I was also wondering whether you feel you do enough of this type of work (maths games and Puzzles) in your maths lessons

The quick question is to say do you?

The longer question is to ask you to say why you feel you do do enough, or you do not do enough.

Another area that I am interested in is to find out how you like to learn your maths.

Question Focus 5: Rules, Methods and Understanding

On the questionnaire I asked several questions about using rules and methods in maths and whether you liked to understand what you were actually doing in maths. So I have three of questions to ask. The first is:

Why is it (or isn't it) good to understand the rules and methods in maths?

The second is:

Do you think that understanding the maths, is more or less important than remembering rules and can you explain why you think this?

And the third:

If you think back to what we have just been talking about I was wondering whether you could describe how, in school, you feel you learn the maths you do.

The last four questions are hopefully quite short and a little more general.

Question 6: *Do you ever think of trying a maths problem (not set by your teacher) for yourself at home? If yes, why? If not, why not?*

Question 7: *Would you ever think of finding any maths problems to do if you could access them via the Internet at home?*

Question 8: *Do you think that school is the place where you do your learning or do you think that there are other places where you spend your time learning things?*

Question 9: *Do you think that sometimes you can decide what you want to learn, rather than relying on an adult to tell you?*

Pilot Interviews with Teachers – July 2005

Preamble

The first area that I am interested in seeking your view on, is to do with what I called maths games and puzzles on the questionnaire or in fact the type of problems found on the NRICH site.

Question Focus 1: Using problem-solving activities in the classroom

Do you use NRICH-type problems within in your maths lessons. Why?

or if the response is no

Would you like to use NRICH-type problems within your maths lessons.? Why?

If not covered during the response

> Do you feel that (some) pupils' achievements are different when they are working on problem solving questions rather than when they are working in a more traditional lesson?
>
> What do you think your pupils gain by working with these problem solving questions?

Question Focus 2: Time availability on the timetable for such activities

Do you feel that the school curriculum allows time for problem solving work?

Subsidiary questions:

Is it just sufficient?

Is the curriculum sufficiently flexible to allow you the freedom to do problem solving questions as and when?

Is there a greater or lesser freedom now that you are using the Numeracy Framework?

Do you feel that your pupils would like to do more problem-solving question?

Question Focus 3: Departmental philosophy for such activities

What do you think that other members of your department feel about mathematics problem solving?

Is there a departmental policy on this?

Are you encouraged to pursue this pedagogical (teaching) approach?

A second area of interest is the way that maths might be taught (and thus how pupils might learn).

Question Focus 4: Rules, Methods and Understanding

One way of teaching maths is telling pupils what and how to do – rules and methods - but either not including the why, sometimes including the why or always attempting to include the why.

Can you talk a little about your views on this, what you do and why you do things the way you do?

A third and final area that I am keen to investigate, is the nature of ICT within school and the potential of the new technologies to bring about change – both for the teacher and for the pupils. So I have a few questions on this.

Question Focus 5: ICT facilities and use of such in school

Can you say a little bit about the ICT facilities within your school?

Dependent upon the responses, there will be subsidiary questions to gain further insight:

How much is ICT used in maths lessons? What and how is it used?

What do you feel your pupils think about the amount of ICT they do overall in school, particularly in maths?

How much does the maths dept make of the Internet for using in the classroom. Are pupils expected to use it for (maths) homework?

Question Focus 6: Internet access and electronic links between home and school

Does the school communicate with parents electronically?

Could you give a rough (or accurate) estimate of how many homes you think have a computer?

Can you give an estimate for what you think might be the number of homes that have Internet access? Can you do this because you have asked or is it just a guess?

Question Focus 7: Does the teacher know what the pupil is doing?

I was very interested in an early finding of NRICH when, in its second year, it appeared that there were several pupils finding and doing the problems at homes. However, I do not know if they told their teacher if they were doing these problems or not.

So I was wondering whether you could imagine any of your pupils (past or present) who might decide to choose to do some NRICH type maths problems at home? If they did, would they be likely to keep it a secret and if so what reasons do you think they would have for doing this?

E-mail Interview: male NRICH participant for five years aged 17 from EU country

[May 2007]

Firstly your e-mail address does not tell me your country but I believe you live in […], so I must say a big ‘Welcome to the EU.’

Secondly I would like to say congratulations for all the solutions you have submitted. I am very impressed by your work.

I have thought up five questions that I would like to ask you. I am sorry that I have to write them in English and not […], so please ask me if you would like me to be explain more about any question. You do not need to answer any question which you do not want to.

Question One
I hear that you started using NRICH questions when you were around 11 years old. Can you tell me some reasons why you started doing Maths questions from the Internet – and maybe why you have continued to do them for many years now.

Question Two
What type of mathematics problems do you do at school? How do they compare with those you do from the NRICH site? Is there a different way of working at your school maths?

Question Three
How do you know or learn any new mathematics that is needed to answer the problems?

Question Four
Without the Internet, do you think that you would have had the same chances to develop your maths ability?

Question Five
Do other your classmates do as much maths as you?

I think this is plenty of questions for now. I chose Group Photo for my favourite NRICH puzzle and in the notes I say why I chose it. If you look you may find out a little more about me.
My teacher training students call me by my first name, Libby, and if you would feel comfortable with calling me Libby you may.

Thank you again for letting me ask you these questions. Is there anything you want to ask me?

Interview with Scott on 07.09.07

I will briefly explain what my research project was about and why I would like to conduct the interview. Before I do I would like to tell you that

- During the interview you need not answer any question you would prefer not to.
- You are free to ask me any questions before, during and at the end of the interview.
- Anything you say will be kept confidential and used only for the purposes of my study. When I write about this work in the future, your name will always either be referred to by a pseudonym or omitted altogether, i.e. you will remain anonymous.

So now a little bit about my project.

I am very interested in talking to some people like yourself who do some maths types of puzzles or number games outside of your normal maths lessons. I enjoy doing lots of maths and playing around with numbers that was not always like the maths questions I was given in school. Nowadays the Internet can have these types of problems so I am wondering whether maths lessons might one day change and have more puzzles but I will have to wait and see.

For now I am looking forward to talking to you as Mrs [teacher] told me that you were enthusiastic about doing lots of number things.

So if you are ready, I would like to begin.

Q1. I hear that you enjoy doing number puzzles at home:
Can you tell me why you like doing them?
What sort of number puzzles do you do?
Do you have a favourite? Will you tell me a little bit about it.

Q2. Did your teachers know that you were doing these number puzzles at home?
If yes, how, did you talk about them much at school?
If no, why did you decide not to tell them?

Q3. Do you do any similar number puzzles at school?
If yes, do others enjoy doing them as much as you?
If no, why do you think you don't have them in your lessons?

Q4. What sort of maths did you like doing at your primary school?

Q5. Do you sometimes find out how to answer questions by yourself?
If you do, how do you go about this?

Q6. I do not know if you find any number puzzles on the Internet. Do you?
If yes, what are these like?
If no, do you think that you might try and find some when you are older?

Q7. Do you think that teaching and school will be different because of the Internet in say ten years time?

Q8. Is there anything which you would like to ask me?

I am very grateful to you and your Mum for allowing me to come and talk to you. You have both kindly given me a lot of your precious time.

Interview Questions with Sixth Form Boys, October 2007

Interview Question 1

What is a favourite area of mathematics for you? What in particular made you opt for it?

Interview Question 2 [will depend upon whether you (a) use NRICH at home (b) use other maths websites (cipher maths has been mentioned!) or (c) do not use any maths websites at home]

For 2 (a) - those who do NRICH problems at home (and maybe use other websites too):
I am wondering whether the way you do the mathematics in class from a textbook is different from the problems you see on the NRICH site. Could you say a few words about the two sources. For example:

How do you feel you work with each?
What are the advantages/disadvantages of each source?
What do you feel you 'get out' of doing NRICH type problems?
Is it important to you that your teacher, or your friends know that you do these problems at home?

For 2 (b) – users of other website(s) but not NRICH
What is the challenge for you that makes you use the website(s)? Why this (these) and not a more general problem-solving site such as NRICH?

For 2 (c) – not using websites at home
Why don't you use any maths websites at home?

Interview Question 3

Do you feel that using the Internet encourages you (or could encourage you) to have a greater independence in organising your learning (in general, and in mathematics)?

Interview Question 4

The questionnaire had some tick boxes for agreement (or not) to what might be happening with the Internet and schooling in the (near-ish) future. Do you think that teaching and school will be different because of the Internet in say ten years time? How? If not, why not?

Interview Question 5

In the questionnaire I was trying to get your thoughts on problem-solving work – work which might be a little out of the ordinary from a normal mathematics lesson. Could you share your thoughts about some of the aspects I was trying to probe: e.g. is it more 'fun'? more challenge? greater stickability? opportunities for group work? is it any harder? is it detrimental to exam work?

Interview Question 6

This will be based around parts of the **'Mathematics and You'** *section – discussing some of the points you made.*

This Appendix contains samples of pages each extracted from one of the three workbooks created in order to manage the data automatically produced by the web-survey software. As explained in the chapter, the three separate workbooks were created to collate the data according to the location, gender and age of the respondents.

The full text of the web-survey questions is contained in Appendix 3.5.

Table A shows an extract from the data collated by location presenting the responses from those who do NRICH problems only at home (oh). Findings from the analysis of the data for Questions 8, 9, 15 and 17 (teacher knowing) are reported in Section 4.3.1 and those for Question 11 (working practices) in Section 4.4.2 and Appendix 4.2.

Table B is an extract collated by location and gender presenting the responses from females who do NRICH problems both at home and at school (hs). Findings from the analysis of the data for Question 23 (self-assessment) are reported in Section 4.2 as part of the dataset overview. Findings for Question 27 (Puzzles and Games) are reported in Section 4.4.1 and Appendix 4.1. Findings for Question 29 and 30 (Understanding) are reported in Section 4.4.3 and Appendix 4.3.

Table C is an extract collated by age and location presenting the responses from those aged 17 to 18 who do NRICH problems only at home (oh – indicated by ‘h’ in response to Question 7). The extract also contains responses to Questions 27 and 29.

In both Tables B and C the responses to Questions 27 and 29 are encoded: 1 meaning ‘definitely’, 2 ‘I mainly think so’, 3 ‘no, not really’ and 4 ‘certainly not’. Responses to other questions presented in this Appendix are generally encoded (where necessary) by an abbreviation of the text of the option box in the web-survey.

Work on NRICH only at home Extract from Sheet 2																					
	Place	Gender		Age				t know		m club	int wk	sp're time	self learn	tell t	working pattern					NRICH	sch com
Reply N0	Q20			<11	11to13	14to16	17to18	Q8	Q9	Q25	Q24	Q 13	Q15	Q17	Q11a	Q11b	Q11c	Q11d	Q11e	Q12	Q26
163	Eng	M				x		no		yes	ns	2	yes	s	alone					vd	ns
169	Eng	M					x	ns		no	no	1	yes	nev	alone					sd	no
175	Eng	M			x			no		nc	ns	3	yes	nev	alone					jr	no
186	India	M				x		no	~~u~~	nc	yes	3	yes	nev	alone		wf			jr	no
192	UK		F				x	no		no	no	4	yes	nev	alone					sd	no
210	Eng		F			x		no	~~t~~	nc	ns	4	yes	s	alone					jr	yes
215	Eng		F	x				no		nc	no	1	yes	nev	alone				afam	fe	ns
217	US		F		x			no		no	yes	3	yes	s	alone		wf		afam	sd	yes
222	Eng		F		x			no		nc	ns	2	yes	s	alone					fe	no
242	Eng		F		x			no		nc	yes	4	no	~~nev~~	alone					sd	no
243	UK		F			x		ns		nc	yes	2	no	~~nev~~	alone					vd	no
259	Eng		F	x				yes	t	nc	yes	2	no		alone					jr	ns
272	India		F			x		no		yes	ns	2	yes	s			wf			jr	no
277	NZ	M				x		no		yes	no	2	yes	s	alone					fe	no
278	Swe	M					x	no		nc	yes	1	yes	s	alone					sd	yes
309	F/Wal		F			x		yes	u	nc	no	2	yes	s	alone					jr	no
332	Eng		F				x	no		nc	yes	1	no		alone					jr	no

Table A Extracted from sheet showing oh web survey respondents with coded answers to questions including those relating to working alone on NRICH problems.

Work on NRICH at both Home and School Extract from Sheet 3 (Gender files)

	Place	Gender		Sch yr	Age				Set	m club	Self ability			Puzzles & Games									Rule & Methods					
Rep no	Q20	Q19		Q21	<11	11 to 13	14 to 16	17 to 18	Q2 2	Q25	23 a	23 b	23 c	27 a	27 b	27 c	27 d	27 e	27 f	27 g	27 h	27 i	29 a	29 b	29 c	29 d	29 e	Q30
7	S'pore		F	Sec 1		x			m-a	nc	3	1	3	1	1	3	1	3	1	2	1	1	2	2	1	2	1	Un
19	UK		F	13				x	m-a	nc	1	1	4	2	3	3	3	3	3	2	3	3	1	4	1	2	2	Un
56	Eng		F	11			x		H	nc	1	3	4	2	2	3	1	1	2	1	1	1	2	3	1	1	2	Un
66	Eng		F	7		x			H	nc	2	3	2	2	1	4	2	3	2	2	3	2	3	3	2	2	2	Rem
68	Eng		F	8		x			Mid	no	2	3	4	1	1	3	1	4	1	3	2	1	2	3	1	1	3	Un
70	Eng		F	7		x			H	no	2	3	3															
81	Eng		F	7		x			H	no	2	3	3	2	2	3	3	3	2	3	3	3	2	3	2	2	2	Un
91	Eng		F	13				x	H	nc	1	1	4	2	3	4	2	3	3	2	2	3	2	4	1	1	2	Un
139	uk		F	13				x	H	yes	2	1	4	1	2	3	3	2		3	4	2	2	4	1	1	1	Un
162	UK		F	13				x	m-a	nc	3	2	4															
191	Eng		F	6	x				Mid	no	2	3	1	3	2	3	3	3	2	3	3	3	3	3	3	2	1	Un
196	Eng		F	6		x			H	yes	1	2	4	1	3	3	3	3	2	2	4							
209	Eng		F	7		x			H	no	1	3	3	1	1	3	1	4	1	1	3	3	3	4	1	1	2	Un
263	Wales		F	8		x			H	yes	2	3	2	2	1	3	3	3	3	3	3	3	3	4	3	1	4	Rem
303	Eng		F	7		x			H	nc	2	2	3	2	1	3	3	3	2	3	2	3	2	3	1	1	2	Un
304	Eng		F	7		x			H	nc	3	3	2	1	1	4	2	4	1	2	3	1	2	3	4	2	3	Rem

Table B Extracted from sheet of respondents by gender, including responses to questions relating to self-assessment, puzzles and games, rules and methods and understanding

Extract of Aged 17 to 18 from all ages workbook																			
	n	Q7	Q19	Q20	Q27a	Q27b	Q27c	Q27d	Q27e	Q27f	Q27g	Q27h	Q27i		Q29a	Q29b	Q29c	Q29d	Q29e
1	18	h	M	Eng	1	2	3	2	2	1	1	2	2		2	4	1	1	1
2	32	h	M	Eng	1	1	3	4	2	2	2	3	2		2	4	1	1	1
3	35	h	M	Eng	3	3	3	3	3	3	2	4	3		1	3	1	1	1
4	37	h	M	Eng	1	2	3	3	3	2	2	3	3		2	3	1	2	2
5	38	h	F	UK	2	2	3	2	3	2	2	3	3		2	4	1	2	3
6	40	h	F	India	2	2	3	2	2	3	2	2	2		2	3	2	2	3
7	87	h	M	UK	3	3	4	2	2	3	2	4	4		1	4	1	1	1
8	99	h	F	Eng	1	2	3	2	3	1	1	3	3		1	4	1	1	1
9	119	h	F	Eng	2	3	2	4	2	1	2	4	3		2	4	1	1	3
10	123	h	F	NI	1	1	3	1	3	3	1	2			2	3	2	1	2
11	124	h	F	Eng	1	1	3	1	3	3	2	2	3		2	4	1	1	2
12	128	h	M	Eng	3	3	3	2	2	2	3	3	3		3	4	1	1	3
13	134	h	F	Eng	2	4	1	3	1	4	3	3	4		2	4	1	2	2
14	138	h	M	UK	3	1	3	2	3	2	2	2	2		2	4	1	1	1
15	141	h	F		2	2	3	2	2	3	2	3	3		3	3	2	2	2
16	169	h	M	Eng	2	2	3	3	3	3	2	3	2		2	4	1	1	2
17	192	h	F	UK	4	4	4	4	4	4	4	4	4		1	1	2	2	2
18	278	h	M	Swe	3	3	3	3	4	2	3	2	3		1	4	1	2	2
19	332	h	F	Eng	1	1	4	1	2	2	2	1	2		2	3	2	1	3

Table C Extracted from sheet of respondents by age, including responses to questions relating to puzzles and games, rules and methods

Appendix 3.9 Example of open response imported from Nvivo with categories of location, gender, UK related school key stage, and non UK country noted

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Question 16 Web Questionnaire

Q16 invites an open response from the previous question which had asked do you decide for yourself to learn some of your mathematics using NRICH or any other mathematics website without your teacher knowing - if yes, please write in the box below why you decide to do this.

Key: Wq-Rx Respondent number from Web Questionnaire M Male F Female
HO Home only NRICH user HS Home and School NRICH user SO School only NRICH user
KSx Age of respondent according to relevant Key Stage year in English school system
[KS2 Years 3 to 6, ages 7 to 11; KS3 Years 7 to 9, ages 11 to 13; KS4 Years 10 &11, ages 14 to 16;
KS5 Years 12 & 13, ages 17 to 18(+)]

Wq_R7 (HS Singapore F KS3) I think I can study on my own and not to do only whatever teacher tells us to do.	Independent Desire
Wq_R12 (HO M KS4) I love doing maths so want to learn more and become better	Pursue Interest Improve
Wq_R18 (HO M KS5) Because more often then not my teachers take too long going over the same topics for weeks on end	Needs not being met (pace & unchallenging)
Wq_R19 (HS F KS5) In order to help solve a problem, or just to find more about something	Improve Pursue Interest
Wq_R22 (HS M KS5) A teacher can never teach you something from all angles. It's good to have different viewpoints, and different methods of approaching problems. By learning some maths before lessons, it makes it easier to understand it when it's finally taught in school. Also, I don't get the opportunity to do Further Maths at my school, so I enjoy learning new things on my own. It gives me a feeling of satisfaction when I can grasp something, and I know I've done it on my own	Expand Experience Get ahead Independent Desire
Wq_R26 (HO KS4 no gender given) Just to look at stuff like infinity, 0/0 etc.	[No code]
Wq_R29 (HO M KS4) I do not wish to be forced to jump through hoops. I would rather set my own pace, especially where I see questions that remain unanswered at the current level of thinking I am supposed to conform to	Independent Desire Needs not being met (pace & poor teaching)
Wq_R30 (HS M KS5) It is often true that one person's point of view is different than another, therefore some people may find it easier to read around a subject from another, more clearer, aspect to themselves	Expand Experience
Wq_R32 (HO M KS5) If it interests me.	Pursue Interest

Appendix 3.9

Example of open response imported from Nvivo with categories of location, gender, UK related school key stage, and non UK country noted

Wq_R33 (HS M KS3) You can do it in your own time with no one hassling you.	Independent Desire
Wq_R37 (HO M KS5) It's fun…	[No code]
Wq_R40 (HO India F KS5) School teaching is inadequate.	Needs not being met (poor teaching)
Wq_R50 (HO F KS4) I think that we should also make an effort to expand our knowledge / understanding of various subjects if we have got facilities such as the internet	Independent Desire
Wq_R56 (HS F KS4) Less waste of class time for others.	Needs not being met (pace)
Wq_R70 (HS F KS3) *[Note: Similar comments to R70]* Because then I will learn more and get a good education meaning that I get a good job.	Improve
Wq_R75 (HO F KS4) I like trying to find different ways to learn and by doing this i can find things which i may come up to later on at school but would like to know some knowledge of them before i do them.	Expand Experience Get Ahead
Wq_R81 (HS F KS3) *[Note: Similar comments to R70]* Because then I can learn more about maths and improve and get a good education which means then when I grow up I will get a good job.	Improve
Wq_R87 (HO M KS5) You can learn things teachers don't know, and in a more relaxed 'oh that's interesting' way.	Independent Desire Expand Experience
Wq_R95 (HS M KS3) To help me with my homework.	Improve
Wq_R97 (HO F KS4) I do it so that I can learn maths outside of the classroom and away from the school environment as well as in lessons.	Independent Desire
Wq_R98 (HO F KS3) Yes if I'm unsure about something I use a website to look it up on and give me a mini test and I do come across other stuff on the website that I have not done before and if it looks interesting I look into it.	Improve Pursue Interest
Wq_R99 (HO F KS5) I'm just often interested in it and doing 5 AS levels. I don't get much spare in school time.	Pursue Interest
Wq_R103 (HO M KS3) It CAN BE quite fun - but only sometimes.	[No code]

Appendix 3.9

Example of open response imported from Nvivo with categories of location, gender, UK related school key stage, and non UK country noted

Wq_R119 (HO F KS3) I like to know all of the maths behind a problem, not just what the exam specification wants you to know.	Independent Desire Expand Experience
Wq_R121 (HO F KS4) So I can understand the next topic we cover more easily.	Get Ahead
Wq_R122 (HS M KS5) The maths teachers at my school don't teach me for the most part - I'm teaching myself for 14 of the 18 maths modules I'm studying, and so it makes sense to consult resources online to broaden my knowledge. Sometimes the textbooks are not hugely detailed or in depth, and it can certainly be nice to achieve a deeper level of understanding in a topic that interests me. Also I have an interest in olympiad mathematics, and have studied from various sources outside of school to help me in this respect too	Needs not being met (working beyond forced Independent Desire) Expand Experience Expand Experience Pursue interest
Wq_R123 (HO F KS5) If I find something that interests me in a question that someone has asked I'll check up a bit more about it. I usually don't feel the need to tell my teacher...we're too busy getting through M3 at the minute! But if I really don't understand something, I do ask him because I prefer direct contact with somebody rather than through the internet. I find it a lot easier to understand something when it's explained in person	Pursue Interest/Expand Experience No need to tell [Comment here shows Semi-Independence as will fall back on teacher if necessary]
Wq_R124 (HO F KS5) Mathematics is taught very slowly at my school and in a way that generally confuses me (ie 'learn this formula' when you have no idea where it comes from). it is much easier to learn it myself - and to justify everything and probably is also better preparation for university mathematics.	Needs not being met (poor teaching, pace, forced Independent desire Expand experience
Wq_R128 (HO M KS5) It isn't stuff covered on syllabus, so they do not need to know.	Expand experience No need to tell
Wq_R134 (HO F KS5) I read about a topic on somewhere like NRich and want to find out more, I'm self teaching certain modules and the textbook explanations don't quite cut it!	Pursue Interest/Expand Experience Forced Independent Desire
Wq_R135 (HS M KS5) My teachers know that, while it is obviously nice to broaden the syllabus, it is foolish to do so at the expense of the basic understanding of key concepts by other members of the class. I like to broaden my horizons in a way that does not rely on input from my teacher.	Needs not being met (working beyond/pace) Independent Desire
Wq_R138 (HO M KS5) I am an independent learner, sometimes school mathematics moves too slowly.	Independent Desire Needs not being met (pace)

Appendix 3.9 Example of open response imported from Nvivo with categories of location, gender, UK related school key stage, and non UK country noted

Wq_R139 (HS F KS5) Not usually nrich though, usually wikepaedia, cut-the-knot, etc. because I'm curious! and my teachers don't need to know, they know I love mathematics but are not so interested in discussing anything not on the Alevel syllabus	Pursue Interest Independent Desire No need to know Expand experience
Wq_R143 (HO F KS5) I want to try to do well in the BMO, for which I need to know some things which I'm not taught at school.	Expand experience
Wq_R147 (HO F KS4) School work is sometimes... too easy? XD Got bored	Needs not being met
Wq_R148 (HS M KS4) Easier, quicker.	[No code]
Wq_R149 (HS M KS2) Because I don’t really learn allot at school.	Needs not being met
Wq_R162 (HS F KS5) Sometimes I need maths not covered in the lessons to complete question e.g. STEP questions	Expand Experience
Wq_R163 (HO M KS4) It explains high level maths very well	Expand Experience
Wq_R175 (HO M KS3) To help my understanding and improve my marks	Improve
Wq_R180 (HS Netherlands M KS5) Mathematics is mostly to do with practice. Teacher is for guidances, hints, and instant inspirations.	[No code]
Wq_R181 (HS M KS4?) To broaden my mathematical knowledge.	Expand Experience
Wq_R186 (HO India M KS4) It makes no difference, whether my teacher knows or not.	No need to know
Wq_R191 (HS F KS2) Well most of the time I tell her what I do in my spare time which is learning on the computer.	[No code]
Wq_R192 (HO F KS5) It interested me and it wasn't on the syllabus.	Expand Experience
Wq_R196 (HS F KS2) I need to learn but my teacher does not need to know.	No need to know Independent desire
Wq_R198 (HS Italy M KS3) To widen my knowledge in mathematics.	Expand Experience
Wq_R215 (HO F KS2) Well I don't see why my teachers should know about all the maths I do at home and I'm to shy.	No need to know +

Wq_R222 (HO F KS3) `I go to a state school which isn't exactly very good (although our current teacher is brilliant, next year we are getting a teacher who basically teaches me nothing) at teaching g+t pupils, so we aren't stretched. By doing work at home I know I am covering all the syllabus.`	Needs not being met (going beyond, pace)
Wq_R263 (HS F KS3) `I don't really know. Sometimes just because I am bored and have nothing to do.`	[No code]
Wq_R272 (HO F India KS4) `It is me who is leading my life, the teacher is, for sure, a guideline but she need not say everything that is perfectly right as she is also a human. I know to choose between good and bad.`	No need to know
Wq_R277 (HO New Zealand M KS4) `So I can learn faster of the subjects in Maths that my teacher is going to teach in a few years later.`	Get ahead
Wq_R278 (HO Sweden M KS5) `Out of interest, and to prepare for new things that eventually will come up.`	Pursue interest Get Ahead
Wq_R297 (HS M KS3) `Because I feel that I am being taught 1-on-1 instead of being in a class.`	[Expand Experience]
Wq_R301 (SO M KS3) [As school only location, assumed using websites and learning some mathematics at home but not NRICH] `I think I learn more of different work websites.`	Expand Experience
Wq_R304 (HS F KS3) `To revise and get better.`	Improve

Memo for Web-Survey Question 16

Introduction

Q16 invites an open response from the previous question which had asked do you decide for yourself to learn some of your mathematics using NRICH or any other mathematics website without your teacher knowing - if yes, please write in the box below why you decide to do this.

56 respondents choose to make an open comment including one [Respondent 301] from the ‘school only’ group. This comment has been considered to be valid as the location groups were devised on using NRICH problems whereas this questions explicitly states ‘other websites’. Thus a ‘school only’ response is possible if the respondent uses websites, other than NRICH [without the teacher knowing].

Two comments [Respondents 70 and 81] are very similar – possibility of answering the survey together.

The [poor] quality of comments from seven respondents meant that no code was allocated to the response made.

Devising the codes

In the first phase of open coding, codes of ‘motivation’, ‘interest’ and ‘independence’ were, with hindsight, quickly considered to be superfluous, since no study would be undertaken unless there was some motivation and interest to do so. Similarly, choosing to do ‘extra work’ must bring with it a degree of independence. All three codes were therefore inherently present.

However some comments conveyed that the respondent choose to be independent for a particular reason [*‘I enjoy learning new things on my own’* Respondent 22] and thus were allocated the code **Independent Desire**. Nonetheless it was (as with all coding) difficult at times to always categorise this. *‘I’m self teaching certain modules’* [Respondent 134] is an example where Independent Desire could be prefaced with the word **Forced** i.e. there was some necessity for the respondent to take on independent learning. Whether they wanted to or not is indeterminable. A different decision was made with comments such as *‘My teachers don’t need to know’* [Respondent 139]. Although this *infers* an independent desire, it was eventually allocated a separate code **No Need to Know** to highlight the reference to the teacher specifically.

Although it is explained above that ‘interest’ would be one code that would be inherently present, just as with ‘independence’ there seemed to be some comments where ‘interest’ was made more explicit than the general pervasion considered above. Hence comments that included the word ‘interest’ explicitly [‘*if it interests me*’ Respondent 32] were allocated the code **Pursue Interest**. However the code was also assigned to other comments that emphasised the pursuit such as ‘*because I’m curious*’ [Respondent 139]. A further code of **Expand Experience,** reasoning that some ‘interest’ comments were suggestive of something more than mere pursuit. An example of this the comment *‘consult resources online to broaden my knowledge’* [Respondent 122]. Whether this is intrinsically different to simply pursuing an interest is open to debate, but the comment is more specific in the reason to pursue the interest making is possible to find a distinction. At one stage, the code ‘Added value’ was used for comments such as *‘it’s good to have different viewpoints and different methods of approaching problems’* [Respondent 22] to imply that more was on offer in a wider context, but when returning to reconsider the codes the description of ‘Added Value’ was considered synonymous with ‘Expand Experience’.

Other codes seem more entrenched within the mathematics lessons. Some comments indicated that they choose to do additional work at home to *'become better'* [Respondent 12] and *'help my understanding and improve my marks'* [Respondent 175]. Such examples were coded Improve. Some respondents reported on a desire to learn topics before they were introduced in class: *'So I can understand the next topic we cover more easily'* [Respondent 121]. To these were allocated the code **Get Ahead**.

The final 'parent' code was named **Needs Not Being Met** allocated to comments that clearly indicated a necessity for the individual to be proactive. This could be due to **Poor Teaching**: *'Mathematics is taught …in a way that confuses me'* [Respondent 124]. Lessons can often not be at the correct **Pace:** *'sometimes school mathematics moves too slowly'* [Respondent 138] Linked with this is a clear indication that some respondents are working at a level **Beyond** the majority of the class *'basic understanding of key concepts by other members of the class'* [Respondent 135].

Independent Desire
    [Forced] Independent Desire
No Need to Tell

Pursue Interest
Expand Experience

Improve
Get ahead

Needs Not Being Met
    Poor Teaching
    [Incorrect] **Pace**
    [Working at a level] **Beyond** [the majority of the class]

Additional Observation
It was interesting to see how considerate (or tolerant) some respondents were of school situations that do not meet their needs – either because they were the 'odd' ones ahead of the majority of the class, or that the work that there were interested in was not on the examination syllabus.

Examples

The full comment of Respondent 135 [used in part to exemplify the code labelled Beyond] is '*My teachers know that, while it is obviously nice to broaden the syllabus, it is foolish to do so at the expense of the basic understanding of key concepts by other members of the class. I like to broaden my horizons in a way that does not rely on input from my teacher'.* [Please note that the full coding for this comment was Needs not being met (Pace/Beyond) and Independent Desire].

Respondent 143: *I want to do well in the BMO, for which I need to know some things which I'm not taught at school'* [Coded Expand Experience, even though it could be argued that Needs are not being met in school, but 'off the curriculum' mathematics].

Problematical to Code
As indicated in the section explaining the derivation of codes and indeed exemplified by the last comment above [Respondent 143], there is likely to be fine distinctions to be drawn in deciding which code should be allocated in open responses. The comment below is indicative of this.

Respondent 297: *'Because I feel I am being taught 1 to 1 instead of being in a class'* [Coded Expand Experience, rather than Needs not being met as there is no indication whether the desire to be taught 1 to 1 emerges from requiring attention or the work being undertaken in class is not at the correct level].

A final comment on Independence …
Respondent 123: ‘*If I find something that interests me in a question that someone has asked I’ll check up a bit more about it. I usually don't feel the need to tell my teacher...we're too busy getting through M3 at the minute! But if I really don't understand something, I do ask him because I prefer direct contact with somebody rather than through the internet. I find it a lot easier to understand something when it's explained in person’*. [Semi Independent Desire – as will ‘fall back’ on teacher if necessary]

Appendix 3.11 Example of individual file made from respondents' data generated from the web-survey software originated as a spreadsheet plus vignette

Recording Respondent Data

55 respondents selected for inclusion in web-survey dataset had responses recorded as in the first two tables below [37, 14, 4 from 'home only', 'home and school' and 'school only' groups respectively]. Four respondents then had their responses turned into a prose account as a vignette [including the illustrative respondent chosen included here].

Record Number 16R 54 H [Web Respondent Number 54. Sixteenth Member in Home Only Records]		
General Data		
Questions: 19, 20, 21, 22	Female. UK. Aged 14 to 16. School Year 10. Higher Maths Set(?)	
Questions: 7, 11, 8, 25, 24, 26	Home Only and on her own or with adult members in her family. Not sure if teacher knows. No maths club in school, work is not taken from the Internet in maths lessons, no e-communications between home and school	
Questions: 15, 17, 13	Decide to learn some maths from websites without teacher knowing, never talks to teacher about the work, definitely likes spending some of her spare time doing maths type problems	
Self-Perceived Ability		
Question 23	Good at all: mainly Better at mathematics than most other subjects: not really Worse at mathematics than most other subjects: certainly not	
Puzzles & Games		
Question 27	Definitely	different work, more interesting
	Mainly	thinking more, try harder to solve problem
	Not really	harder to learn, more challenging, working in a more problem solving way, working together more
	Certainly not	less fun
	Hypothesised Agreement Score: 5/3	Characteristic: Positive
Rules and Methods		
Question 30	More important Understanding	
Question 29	Definitely	interested in knowing why it works, remembers forever, generally understands if explained, making up own rules helps understanding
	Mainly	[none selected for this category]
	Not really	[none selected for this category]
	Certainly not	likes to be told
		Characteristic: Relational
Working Patterns (in classroom)		
Question 31	Often	work quietly alone, explains to others in class
	Quite Often	[none selected for this category]
	A bit	asks a friend to explain, ask for help at home, talks to others in class
	Never	asks teacher to explain
		Characteristic(s): Insular, Explainer

Appendix 3.11 Example of individual file made from respondents' data generated from the web-survey software originated as a spreadsheet plus vignette

Record Number 16R 54 H

[Web Respondent Number 54. Sixteenth Member in Home Only Records]

The Internet

Questions: 1, 3	Messages: 1 to 3 hours General info: Never Music & Graphics: Never Homework: <1 Other schoolwork: <1 Average time per week: 7 to 14	
Questions: 4, 5	Definitely	Internet can help with homework, people without access are at a disadvantage more school related work will be undertaken using the Internet, teaching and lessons will change
	Mainly	Internet can help learn about other school type things when just looking, people will be able to choose where and when to learn more learning will take place at home, students can take on more responsibility for own learning
	Not really	[none selected for this category]
	Certainly not	[none selected for this category]

Record Number 16R 54 H

[Web Respondent Number 54. Sixteenth Member in Home Only Records]

Written responses

Q6: How the Internet is going to change education	A lot of school work might be posted by the teacher, completed and sent to the teacher via the internet
Q10: reasons why you choose to use the NRICH site (school, home or both)	Some interesting questions and articles, and AskNRICH is a good place to ask and answer questions, as well as discuss maths
Q14: Reasons for agreement level on liking to spend some of spare time out of school doing maths problems	UKMT mentoring scheme has good questions, and doing Nrich questions help keep my brain working for the next maths challenge/Olympiad
Q16: Reasons for deciding to learn some maths for self from websites without telling teacher	To help with questions outside of school work
Q28: Additional comments about doing puzzles and games	Sometimes a question can be more challenging that a puzzle, but not most of the time
Q32: Any other comments	[No reply]

Vignette – [made for only four respondents in total]

Interpretative Commentary from Responses: Distinction between home and school life

This respondent is a 14 to 16 year old female, currently in Year 10 in a school in England, who feels (modestly given comments below) that she is quite good at all subjects and not just mathematics. She does NRICH problems only at home either by herself or with a family member: "[It has] *some interesting questions and articles".* She is unsure as to whether her teacher knows this. She sometimes takes the initiative to learn things for herself. She definitely enjoys spending some time out of school doing mathematics problems *"to help with questions outside of school work"* but never talks to her teacher about this. It would appear that a distinction has been made here between what is needed to be done for school-work and that beyond: "*UKMT mentoring scheme has good questions, and doing NRICH questions helps keep my brain working for the next maths challenge/olympiad*". She definitely believes in understanding rules and methods for herself and therefore does not countenance less. She enjoys doing puzzles and problems in mathematics, finding them more interesting and working in a way different to other work: "*sometimes a question can be more challenging than a puzzle, but not most of the time".* She does not really agree that is harder to learn mathematics through doing this work and whilst she sees herself working harder at this type of work, she does not see herself either working in a more problem solving way or collaborating more with friends.

There is no maths club at school. There is no use made of the Internet in mathematics classes.

Having a UKMT mentor marks a highly attaining capability. Independence and ability are further illustrated by her choice of 'never' asking her teacher to explain and rarely seeking help from anyone else. There is a preference 'often' for working quietly on her own. She will also often explain the work to other class members.

She appears (as the numbers given total less) to use the Internet on average for between 7 to 14 hours per week, though never for every day information or downloading music. Her greatest use is for contacting friends and family. She does make use of the NRICH conference board: "*AskNRICH is a good place to ask and answer questions, as well as discuss maths".* Although less than one hour per week was spent on using the Internet for homework, she is definite that the Internet is an aid for helping with homework and those without access will be at a disadvantage. She thought that 'definitely' more school related work would be Internet based and that teaching and lessons would change. A little less strength was given to the other three statements though she is the first of the three so far described who felt that people might have greater independence in undertaking more work and having a choice of learning place. This reflects her experience of having a 'second home' for some of her mathematical studies.

Findings: doing puzzles and games type of work in mathematics lessons

1. Introductory presentation and discussion of responses

As highlighted in LRI, it is widely acknowledged that there is no precise and universal definition of problem-solving. However, the web-survey respondents had been given a clearer concept of the type of problem and manner of problem-solving that was to be under scrutiny by the context provided by NRICH. The initial School-based Investigations, including taped interviews [see 1-5 of Table 3.2 p60], revealed that many pupils enjoyed doing mathematics when there was a puzzling, game, or problem solving, element to the work. As also shown in the LRI [Sections 2.2 & 2.3 pp39-44], whilst pupils may understand reasons stated (e.g. practice, needed for tests/examinations) for doing 'boring', routine exercises, the 'fun' and challenging element has a strong appeal for many.

Question 27 of the web-survey [see Appendix 3.5] directly addressed sub-question (i) of RQ2 by eliciting respondents' perceptions of doing problem-solving in mathematics lessons with a focus on undertaking puzzles and games in class. Nine statements were given, each seeking a response to one of four categories of (dis)agreement. Table A presents the percentage in agreement arrived at by amalgamating the two categories of agreement.

Statement meaning that doing puzzles and games in mathematics	% for agreement
27c. is more fun*	90%
27a. is doing mathematics in a different way	86%
27g. is working in a more problem solving way	80%
27b. makes for more interesting work	77%
27e. is not a harder way to learn mathematics*	77%
27f. allows for working together more	68%
27d. makes you think more	60%
27i. means trying harder to solve problems	52%
27h. more challenging than other mathematics questions	45%

* to present these as positive statements required respondents to select the two disagreement categories

Table A: Agreement responses to statement

Six of the nine statements had more than two-thirds of the respondents in agreement in line with the research reported in the LRI. Thus the respondents viewed by undertaking Puzzles and Games in mathematics, they felt that were doing mathematics in a different way (86%), the work was more interesting (77%) and more fun (90%). In addition, the work is undertaken more collaboratively (68%), involves working in a more problem solving way (80%) than in other mathematics lessons and does not make it harder to learn mathematics (77%). This last figure means that respondents would not agree that the examination system requires 'normal' work, in contrast to the often expressed opinion of teachers [see LRI p41]. However, of course, this finding cannot be taken as evidence to suggest that such 'normal' work can be replaced by this more 'interesting' work.

The general view expressed in the literature is that problem-solving offers greater challenges than other mathematics, requires deeper thinking skills and greater enthusiasm for working on these types of problems brings with it greater perseverance [Fennema & Romberg 1999, Schoenfeld 1994]. Thus it might be expected that these views would be reflected in the results given in Table A for statements 27h, d, and i respectively. Whilst two of these remain over the 50% agreement mark, it would be reasonable to expect a higher level of agreement. It might be suggested that these findings call into question some of the reasons given in the literature for including such work in the curriculum, but this would be an unwarranted inference. The data in Table A is derived from a question requiring one box to be ticked without explicitly probing the reasons behind the choice and, furthermore, three-quarters of respondents belong to the two home categories. In these categories, three-quarters stated[1] that they liked to spend time out of school doing mathematics problems and also perceive themselves to perform well in Mathematics [see Table 4.3 in chapter text]. For such people, this work might not necessarily be 'more challenging', nor might they 'have a need to think more' or 'work any harder' to find the solution.

The results for the 'fun' statement in Table A are consistent with those from classroom-based studies [e.g. Brown, Brown & Bibby 2007; Nardi & Steward 2003; Miller, Parkhouse, Eagle & Evans 1999; Boaler 1997, Schoenfeld 1989, Hodgen, Küchemann, Brown & Coe 2009] all of which report pupils' desire for 'fun' in lessons. Some of the web-survey open responses[2] directly echo expressions reported in the published studies: *"I have to say it [NRICH] is much more fun than doing a normal maths lesson!"* [R103-M-oh-KS3], and *"I personally find puzzles and games more fun and people say that if children have fun in there [sic] lessons then they learn more"* [R98-F-oh-KS3]. In direct agreement with findings from

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[1] Q13 Appendix 3.5

[2] Q28 Appendix 3.5

Nardi & Steward [2003], there are further, discerning[3], responses that portray a considered perception of what might constitute fun related to an individual’s view, liking and attainment in mathematics: *“I think that puzzles may seem the fun option but written questions take just as much thought and can be as fun”* [R105-F-oh-KS3] and *“I'm afraid whenever we've done puzzles and games in maths it’s always been a poor attempt to make maths ‘fun’. Those who dislike maths see through this and hate logic puzzles anyway, those who do like maths hate the lack of mathematical content, and resent having to work with unco-operative people. But I'm sure some certain puzzles would be beneficial”* [R134-F-oh-KS5].

2. Comparative Analysis of Responses

A detailed breakdown by location group of the responses to each of the statements in WQ27 was made [see Table B] and subjected to further analysis whose results are presented in Table C. This Table indicates the percentages of each group that ticked one of the two agreement options for each statement and shows the results of chi-squared tests, where subsets were sufficiently large, when there were significant differences between groups. In four of the five cases where the differences were significant, it was due to the selection of the more emphatic agreement of the two choices by the **os** group. Further investigation revealed only one significant gender difference (5.56 $p<0.025$) in the responses to WQ27 statements, for 27i Males (65% in agreement that they try harder to solve the problems) and Females (42% agreement).

Since this thesis is on mathematics studied beyond the classroom door, a detailed examination of the perceptions of **os** people is outside its remit. Nonetheless, the comparisons presented in Table C relating to the **os** group permits the inference, in keeping with the OfSTED [2006, 2008] reports, that they *perceive* that they would find this type of work more conducive to their mathematical study than what is presently provided.

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[3] Furthermore, as early as the first pilot questionnaire PQ1 [July 2004] whilst one year nine pupil commented *“we are learning AND having fun”* [respondent‘s own capitals, Upper Set] a second categorised the games and puzzles undertaken in mathematics lessons as *“they are just fun games not learning games”* [Upper Set].

WQ 27 Doing puzzles and games in mathematics lessons ….

	No. of respondents			Definitely			I mainly think so			No, not really			Certainly not		
	oh	hs	os	oh	hs	os	oh	hs	os	oh	hs	os	oh	hs	os
27a Diff Way	52	32	28	42.3	34.4	25.0	36.5	59.4	64.3	21.2	6.3	10.7	0.0	0.0	0.0
27b More Int	52	32	28	30.8	31.3	53.6	42.3	37.5	39.3	25.0	28.1	7.1	1.9	3.1	0.0
27c Less Fun	52	32	28	5.8	3.1	0.0	9.6	6.3	0.0	61.5	68.8	46.4	23.1	21.9	53.6
27d Think More	52	32	28	17.3	28.1	10.7	46.2	31.3	42.9	30.8	37.5	39.3	5.8	3.1	7.1
27e Harder to learn	51	32	28	3.9	12.5	3.6	23.5	15.6	7.1	54.9	50.0	57.1	17.6	21.9	32.1
27f More collab	52	31	28	15.4	29.0	42.9	40.4	41.9	42.9	42.3	29.0	14.3	1.9	0.0	0.0
27g Work more PS way	52	32	28	28.8	28.1	17.9	57.7	46.9	57.1	13.5	18.8	21.4	0.0	6.3	3.6
27h More challenging	52	32	27	13.5	15.6	0.0	32.7	31.3	40.7	46.2	40.6	51.9	7.7	12.5	7.4
27i Try harder	51	30	28	11.8	23.3	28.6	37.3	33.3	25.0	41.2	30.0	42.9	9.8	13.3	3.6

Doing puzzles and games in mathematics …	% agreement to statement			
	All	oh	hs	os
27a is doing mathematics in a different way	86	79	94	89
27b makes for more interesting work	77	73	69	93
27cs more fun	10	15	9	0
27d makes you think more	60	63	59	54
27e s not a harder way to learn mathematics	23	27	28	11
27f allows for working together more	68	56	71	86
27g is working in a more problem solving way	80	87	75	75
27h more challenging than other mathematics questions	45	46	47	41
27i means trying harder to solve problems	52	49	57	54

Table B Web survey results to WQ 27 (percentage) by location: home only (oh), home and school (hs), school only (os) and percentage to positive agreement

	Percentage in agreement ‘definitely’ and ‘I mainly think so’			Comparisons	χ^2 value and p score
	oh	hs	os		
27a. It feels I am doing mathematics in a different way	79	94	89	**oh** more emphatic that it was a different way of working by opting for ‘definitely’ rather than ‘mainly’ than **os**	4.15 $p<0.05$
27b. It is more interesting than other mathematics work	73	69	93	**os** opted for ‘definitely’ more interesting over all the three other options than **hp**	4.63 $p<0.05$
27c. It is less fun than other mathematics	15	9	0	**os** more adamant not less fun than **hp** by opting for ‘certainly not’ less fun rather than ‘mainly think so’.	9.5 $p<0.005$
27e. I feel it is harder to learn mathematics doing this type of work	27	28	11	**os** disagreed that it was harder to learn more than **hp** ** data small in one category*	3.34* $p<0.1$
27f. More of us work together when we are doing this work	56	71	86	**os** agreed that they worked together more than **hp**	2.94 $p<0.15$
27d. I think for myself more when I am doing this type of work	63	59	54	No significant differences	
27g. I feel I am working in a more problem solving way	87	75	75		
27h. These problems are more challenging than other mathematics questions	46	47	41		
27i. I try harder to solve the problems**	49	57	54		

** This was the only statement that showed a significant gender difference

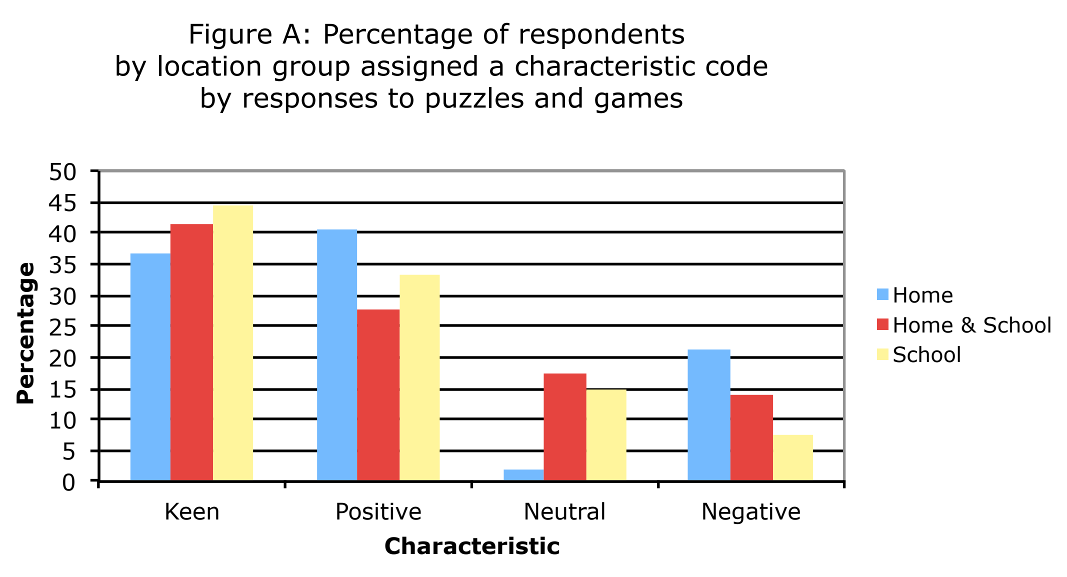
Table C Data Comparison between the three location groups: home only (oh), home and school (hs),combined to home people (hp) and school only (os)

3. Rudimentary Characterisation of Respondents

This section presents an analysis of the same dataset as the previous one, but instead of comparing the groups’ responses to *each* of the nine statements, the individuals in the groups are examined in terms of the profile of their *summarised* responses to the statements[1]. Chapter Three [Table 3.5 p70] explained how respondents were characterised according to their selection of level of agreement to statements in WQ27 into four categories which were discrete subsets within the groups. Table D and Figure A display the number of respondents in each of the three different location groups (**oh**, **hs** and **os**) assigned to the four characteristic categories.

	Keen		Positive		Neutral		Negative	
Only Home	19	37%	21	40%	1	2%	11	21%
Home & School	12	41%	8	28%	5	17%	4	14%
Only School	19	44%	9	33%	4	15%	2	7%

Table D: Number and Percentage of Respondents allocated to each characteristic category



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[1] As reported in Chapter Three, statement e, harder to learn, was excluded as an expected choice was more difficult to determine leaving eight statements for this characterisation.

The *descriptive* statistics in Table D firstly, show up the ‘polarisation’ in the **oh** group’s characterisation with 21% ‘Negative’, i.e least in agreement with the expected choices, and only one individual ‘Neutral’. This is an expression of the same phenomenon that led to the ‘discerning’ comment implying a more serious consideration as to what constitutes fun quoted in Section 1 above.

Secondly, since the **os** group are inherently those showing less apparent interest in the subject by not doing problems at home, they might not be expected to be predominantly present in the two favourable categories, Keen and Positive. Furthermore, the **os** group ‘wins’ in the Keen category and outdoes **hs** in the Positive one. This again can be seen as an expression of the ‘disaffection’ referred to at the end of Section 1.

Findings: Ways of Working in Mathematics Lessons

1. Introductory presentation and discussion of responses

Web-survey WQ31 [see Appendix 3.5] directly addressed sub-question (ii) of RQ2: *How do these students seek help with school mathematics?* by probing their working practices in the classroom situation, a setting in which no one is ostensibly alone. Table A summarises the responses to WQ31 by providing a breakdown of the percentage scores for each of the four frequencies of working practices. It also contains the combined percentage scores of selecting *'often'* or *'quite often'* frequencies, listed by location group. Table B indicates where chi-squared tests produced significant differences between the various groupings based on location and gender.

2. Comparative Analysis of Responses

WQ31 was clearly intended to refer to a classroom situation, where it might be assumed that the teacher would be the main / frequent source of help. However, Table A shows teachers to be one of the less popular sources of help with only around one quarter of respondents in each of the location or gender groupings stated that they '*often / quite often'* asked their teacher for re-explanations. This may be due to a general dislike of asking teachers to explain again or, given the self-perception of high performance of the respondents [see Table 4.3 in chapter text], it might be inferred that their teachers' help was frequently not required. A further possibility is that the teachers concerned were so effective that one explanation from them was usually sufficient.

The percentages of respondents choosing 'help from home' were also relatively small, perhaps again due to self-sufficiency, although there was a significant difference ($p < 0.05$) between the gender groups with females (35%) more likely to ask than males (16%). Similar significant gender differences (females > males) occurred for the options 'asking a friend' and 'talking to others'.

WQ 31 How do you 'best' learn your mathematics? I …

	No. of respondents			Often			Quite Often			A Bit			Never		
	oh	hs	os	oh	hs	os	oh	hs	os	oh	hs	os	oh	hs	os
31a ask a teacher to explain	52	31	28	7.7	12.9	3.6	15.4	12.9	25.0	53.8	58.1	64.3	23.1	16.1	7.1
31b ask a friend to show me	52	31	28	9.6	3.2	35.7	17.3	22.6	25.0	50.0	48.4	32.1	23.1	25.8	7.1
31c talk to others in class	52	31	28	9.6	12.9	25.0	32.7	41.9	46.4	40.4	22.6	28.6	17.3	22.6	0.0
31d ask someone at home	53	31	28	9.4	16.1	10.7	11.3	16.1	21.4	30.2	16.1	50.0	49.1	51.6	17.9
31e try to work it out myself	52	31	28	61.5	51.6	14.3	25.0	22.6	39.3	13.5	22.6	32.1	0.0	3.2	14.3
31f explain to others in class	52	31	28	48.1	32.3	25.0	40.4	38.7	32.1	9.6	25.8	32.1	1.9	3.2	10.7

Doing puzzles and games in mathematics …	% for Often/Quite Often			
	All	oh	hs	os
31a ask a teacher to explain it to me again	25	23	26	29
31b ask a friend to show me what they are doing	35	27	26	61
31c talk about it to other people in my class	53	43	55	71
31d ask someone to help me at home	27	20	32	32
31e try to work it out for myself	75	87	75	53
31f explain the work to others in the class to help them	76	88	71	57

Table A: Web survey results to WQ 31 (%) by location: home only (oh), home and school (hs), school only (os) and % for Often/Quite Often combined

WQ31: This question is asking you to describe how you feel you help yourself to learn mathematics…

Statement	Percentage opting for Often and Quite Often					Comparison made between groups option for more frequent (i.e. often and quite often) and less frequent (i.e. a bit and never)	χ^2 value and p score
	Location			Gender			
	oh %	hs %	os %	M %	F %	Groups of significant difference	
31a. Ask a teacher to explain it to me again	23	26	29	29	23	no significance	
31b. Ask a friend to show me what they are doing	27	26	61	22	42	**os** more likely to ask a friend than **oh** **os** more likely to ask a friend than **hs** **female** more likely to ask than **male**	8.76 $p<0.005$ 7.34 $p<0.01$ 4.67 $P<0.05$
31c. Talk about it to other people in my class	42	55	71	35	55	**os** more likely to talk to others in class than **oh** **os** more likely to talk to others in class than or **hp** **female** more likely to talk to others than **male**	6.2 $p<0.025$ 5.02 $P<0.05$ 4.47 $p<0.05$
31d. Ask someone to help me at home	21	32	32	16	35	**female** more likely to ask for help at home than **male**	4.86 $p<0.05$
31e. Try to work it out for myself	87	74	54	76	73	**oh** more likely to try to work it out than **os** **hs** more likely to try to work it out than **os**	10.55 $p<0.005$ 2.73 $p<0.01$
31f. Explain the work to others in the class to help them	88	71	57	76	76	**oh** more likely to explain than **os** **oh** more likely to explain than **hs** [but no significance between hs and os]	10.24 $p<0.005$ 4.01 $p<0.05$

Table B: Ways of Working Web-survey Results to WQ31 by location: home only (oh), home at school (hs), school only (os) and by gender

Table A also shows that, with the exception of the asking teacher and asking at home choices, there are significant differences between combinations of location groups in the respondents' choices. In simple terms **os** group respondents are more likely to seek help through interaction with a classmate or friend. The 'home' respondents are more likely to work things out for themselves and more likely to provide explanations for others. These latter two findings support the inferences already made from Table 4.3 [p75] that 'home' respondents have an inherent motivation and interest in the subject leading to greater knowledge and experience in relation to their classroom peers.

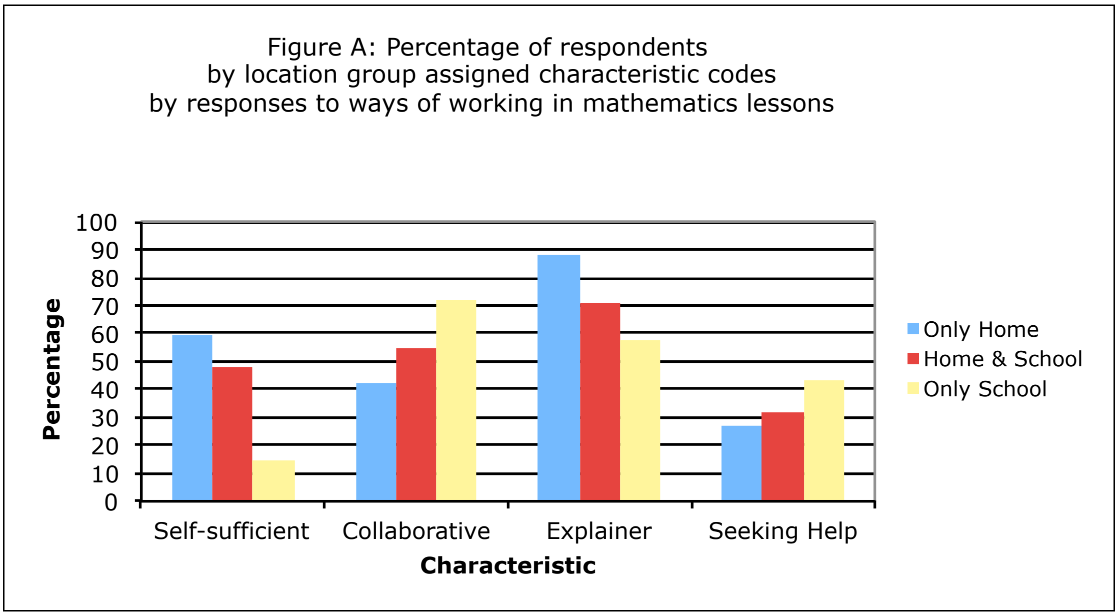
3. Rudimentary Characterisation of Respondents

The dataset from WQ31 is analysed a second time by examining the individuals in the location groups in terms of their frequency choices for each of the five working practices probed. Table 3.5 [p70] set out the criteria used for determining the characteristic codes: **Self-sufficient**, **Collaborative**, **Explainer** and **Seeking Help** (abbreviated to SS, C, E and SH respectively in Figure B below). Table C and Figure A display the number of respondents in each of the three different location groups assigned to each of the four characteristic codes. Given that the allocation was made on distinct statements, there is the potential for each respondent to be assigned none, some or all of the codes.

Thus, although the results presented in Table C and Figure A come from the same dataset as Tables A and B, the criteria for assignment of codes means that Collaborative and Explainer correspond exactly to single statements 31c & f with their *'often/quite often'* data as given in Table A. However, Self-sufficient corresponds to only the *'often'* responses to statement 31e and Seeking Help is an amalgam of different frequencies from the three remaining statements [see Table 3.5 p70]. This presentation by codes provides a different viewpoint that serves to emphasise the properties of, and relationships between, location groups already present in the data and thus provides further evidence to support inferences already made earlier from Table 4.3 [p75] and Table A, viz. the **oh** group are self-sufficient and have knowledge and experience that enables them to explain and the **os** group are collaborative and seek help through interaction with their peers. As far as the **hs** location group is concerned, Figure A highlights the incidental observation, from which no inferences are drawn, that the group is sandwiched between the **oh** and **os** for all four characteristics.

	Self-sufficient		Collaborator		Explainer		Seeking Help	
Home Only [52]	31	60%	22	42%	46	88%	14	27%
Home & School [31]	15	48%	17	55%	22	71%	10	32%
School Only [28]	4	14%	20	71%	16	57%	12	46%

Table C: Number and Percentage of Respondents allocated to each characteristic code



The association of ‘collaborative’ with the statement ‘talk about it with others in class’ is possibly the most complex or contentious as it is not possible to be certain what ‘talk’ is going on. Although being ‘collaborative’ is clearly connected to ‘talking about the work’, the talking may be in either an ‘Explaining’ or ‘Seeking Help’ way. Nevertheless, difference in totals for the three location groups for the three related characteristics leads to the inference that the respondents could distinguish between the roles.

Combinations of Characteristics

As already explained, the sub-groups associated with the characteristics are not discrete and thus respondents could be given more than one characterisation. For the 52 **oh** respondents there were 113 assignments, for the 31 **hs** 64 and for the 28 **os** 52. There is little difference between the location groups in the mean number of characteristics assigned per respondent: 2.17, 2.06 and 1.86 respectively.

Table D shows the numbers of characteristic codes assigned to respondents and the resulting percentage of respondents with that number in each location group. All three groups had the same modal score of two assignments; out of the total of 111 respondents just one (an **os** respondent), had no assignment, and two others (both **oh**) had all four.

Number of codes	Four		Three		Two		One		Zero	
Only Home [52]	2	4%	17	33%	20	38%	13	25%	0	0%
Home & School [31]	0	0%	10	32%	14	45%	7	23%	0	0%
Only School [28]	0	0%	7	25%	11	39%	9	32%	1	4%
Total (all groups) [111]	2	2%	34	31%	45	41%	29	26%	1	1%

Table D: Number and percentage of characteristic codes assigned to each location group

Around three-quarters of respondents are assigned more than one characteristic code. Table E shows the numbers of respondents in each location group assigned two characteristic codes, also expressed as a percentage of the group, for all six combinations of two codes. Figure B presents the percentage scores as a bar chart. The total number of respondents with any combination of two characteristics cannot exceed the smaller of the totals for each of the two. Thus the maximum possible totals for combinations can only equal the second largest number in each row of Table C which are: 60%, 55% and 57% respectively. Table E shows that in fact the combination of any two characteristics is associated with less than half of the specific location group.

	oh [52]		hs [31]		os [28]	
Self-sufficient & Collaborative	15	29%	4	13%	3	11%
Self-sufficient & Explainer	24	46%	13	42%	2	7%
Self-sufficient & Seeking Help	5	10%	1	3%	0	0%
Collaborative & Explainer	19	37%	13	42%	8	29%
Collaborative & Seeking Help	4	8%	5	16%	9	32%
Explainer & Seeking Help	12	23%	4	13%	8	29%

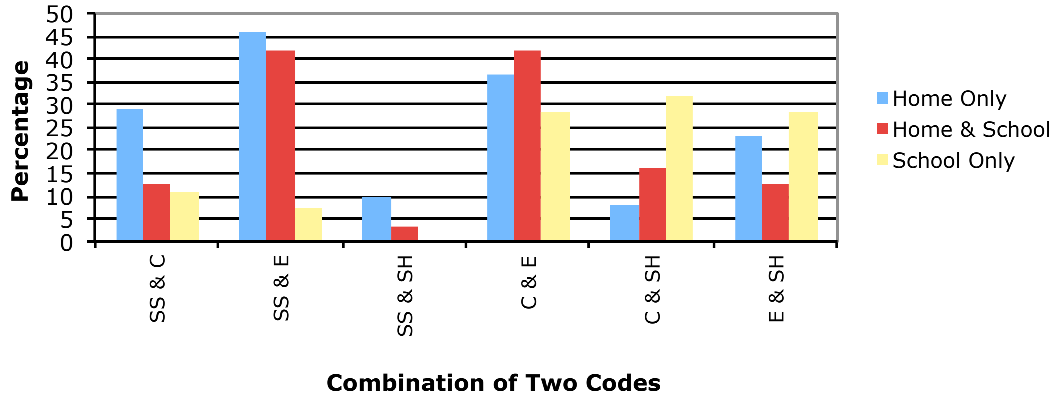
Table E: Number and percentage of respondents assigned a combination of two characteristic codes

However, Table F shows the combinations as percentages of the maximum possible total rather than of the whole location group size and illustrates the distortions resulting from the very small sub-sets, especially in the shaded area, combined with the relative frequencies of the Explainer (high) and Seeking Help (low) codes.

	oh [52]	Max. poss.		hs [31]	Max. poss.		os [28]	Max. poss.	
Self-sufficient & Collaborative (SS & C)	15	22	68%	4	15	27%	3	4	75%
Self-sufficient & Explainer (SS & E)	24	31	77%	13	15	87%	2	4	50%
Self-sufficient & Seeking Help (SS & SH)	5	14	36%	1	10	10%	0	4	0%
Collaborative & Explainer (C & E)	19	22	86%	13	17	76%	8	16	50%
Collaborative & Seeking Help (C & SH)	4	14	29%	5	10	50%	9	12	75%
Explainer & Seeking Help (E & SH)	12	14	86%	4	10	40%	8	12	67%

Table F: Number and percentage of respondents assigned a combination of two characteristic codes

Figure B: Percentage of respondents
by location group assigned a two code combination



Although it is not justifiable to base any further specific claims on such small group sizes [Gorard 2001], around three-quarters of respondents are assigned more than one characteristic code. Thus it is reasonable to infer that respondents can work in different ways at different times according to the classroom activities. So, for example, although it has been shown that the home groups work substantially at home on their own and score highly on the Self-sufficient code, this does not preclude them from collaborating at other times. Furthermore, the strength of the results in this section might seem to lie with the ‘ordinariness’ of the outcomes, the unsurprising nature of what might be declared as intuitively obvious. The results show how respondents from the three different location groups view the ways they work when doing mathematics in the classroom. The results can also be viewed as exemplifying scenarios of the individual’s mindset discussed in LRI [p49]. The analyses for the home groups is of particular importance in positioning, from how they work in other contexts, the type of people who use the AskNRICH web-board intensively that is investigated in the Main Study. Indeed, characteristics that might be described as ‘self-sufficiency’, ‘explaining’, ‘seeking help’ and ‘collaborating’, are all essential to full participation in AskNRICH.

Findings: Perceptions of Understanding Mathematical Content

1. Introductory presentation and discussion of responses

Web-survey WQ29&30 [see Appendix 3.5] directly addressed sub-question (iii) of RQ2: *What are students' perceptions about the relative merits of rules, methods and understanding?* WQ29 gave five statements, with four degrees of agreement, relating to aspects of how 'best' to learn mathematics, selectively probing in a relatively simplistic way the respondent's desire to veer towards relational or instrumental understanding [Skemp 1976]. WQ30 asked respondents to choose between two statements (understanding the mathematics or remembering the rule) by stating which one they considered to be the more important.

2. Comparative Analysis of Responses

Table A provides a full breakdown by location group of responses to WQ29, indicating the percentages choosing to agree with each statement. Table B provides these overall agreement percentages by location group along with the choice of which is the more important statement for WQ30. The only significant difference found using chi-squared comparison tests between the three location groups' responses to WQ29 [is the more adamant choice of 'definitely' for 29c, *'interest in knowing why'*, between both home groups **oh** and **hs** in comparison with the **os** group. However, the very small number of responses in one option of the **oh** group raises questions about the validity of even this result. No gender differences were found on any the statements. Both the **oh** and **hs** groups are significantly different from the **os** group in selecting understanding over remembering in WQ30 [see Table B]. This was more marked for the **oh** with $p<0.005$ as against $p<0.1$ for **hs**. Moreover, even within the **os** group there was a majority of 2:1 in favour of understanding!

Thus results are again consistent with studies reported in LRI and open responses quoted at the end of Section 4.3.3 [p81]. Respondents want to know why a rule or method works, they believe that they understand the explanation and they have a dislike for just being told what the rule is.

WQ29 How do you ‘best’ learn your mathematics ….

	Number of replies			Definitely			I mainly think so			No, not really			Certainly not			% agreement statement			
	oh	hs	os	oh	hs	os	oh	hs	os	oh	hs	os	oh	hs	os	All	oh	hs	os
29a Can remember Rule	53	31	28	25	19	14	57	68	61	19	13	21	0	0	4	81	81	87	75
29b Like to be told	53	31	28	6	6	7	11	10	25	45	45	64	38	39	4	21	17	16	32
29c Like knowing why	53	31	28	66	61	25	26	16	54	6	19	21	2	3	0	85	92	77	79
29d Understand rule/methods	52	31	28	56	61	36	42	35	57	2	3	4	0	0	4	96	98	97	93
29e Make up own rules/methods	52	31	28	38	29	18	35	45	39	23	19	36	4	6	7	69	73	74	57

Table A: Web survey results to WQ29 (percentage) by location: home only (oh), home and school (hs), school only (os) and percentage to positive agreement

WQ29 How do you 'best' learn your mathematics	Percentage in agreement [‘definitely’ and ‘I mainly think so’]			Comparisons	χ^2 value and p score
	oh	hs	os		
29c. I am interested in knowing why the rule or method works	92	77	79	**Significant difference: os less adamant in being interested:** percentage of **os** people selecting the ‘definitely’ (25%) rather than ‘I mainly think so’ (54%) compared with 66% and 26% respectively for **oh** & 61% and 16% for **hs** groups’.	17.15 $p<0.005$ though oh total frequency [of 4] for negative, strictly speaking too small
29a. If I am given a rule I can generally remember it forever	81	87	75	No significant differences	
29b. I like to be told the rule or method to be used so that I can remember it forever	17	16	32		
29d. If the rule or method is explained, I generally understand how the rule or method works	98	97	93		
29e. Making up my own rules and methods helps me to understand the work	73	74	57		
WQ30 **Select the one [understanding or remembering] that you think is more important of the two**					
Understanding	89	84	64	Significant differences comparing **oh** and **os** groups χ^2 6.98 $p<0.005$ and **hs** and **os** groups χ^2 3.02 $p<0.1$	
Remembering	11	16	36		

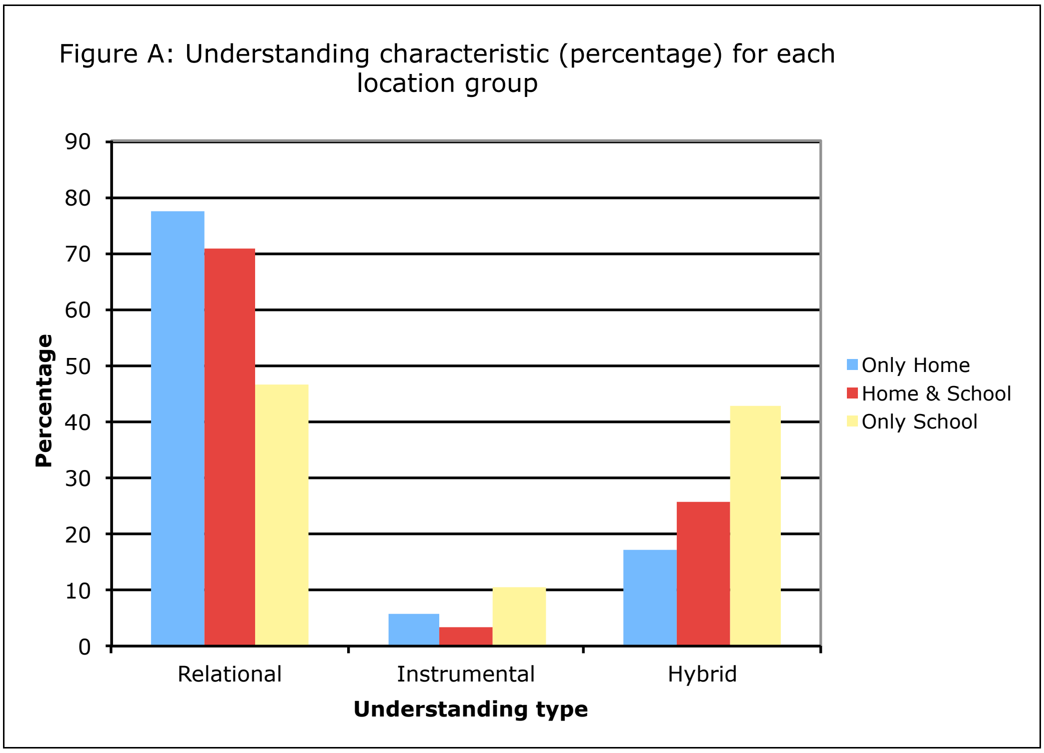
Table B: WQ29 & 30: Data Comparison between the three location groups: home only (oh), home and school (hs), and school only (os)

3. Rudimentary Characterisation of Respondents

Respondents were assigned characteristic codes based on their selection of statement 29b *'likes to be told a rule without explanation'* **and** their choice of understanding/remembering in WQ30. Table 3.5 [Section 3.4.4.1] set out the criteria used for determining the characteristic codes: **Relational, Instrumental** and **Hybrid**. Table C and Figure A display the percentages of respondents in each of the three different location groups assigned to each of the three characteristic codes for each discrete subset of the group.

	Home Only	Home & School	School Only
Relational	77	71	46
Instrumental	6	3	11
Hybrid	17	26	42

Table C: Percentage distribution by location group of 'understanding' characteristics



Comparison of Tables B and C shows that introducing the Hybrid characteristic in effect merely removes a few respondents from the 'understanding' and 'remembering' groups.

This is reflected in significant differences remaining between **oh** and **hs** compared with **os** location groups' characteristic codes that have the same confidence levels ($p<0.005$, $p<0.1$) and increased χ^2 values (7.89, 3.67) respectively. However, the most noticeable feature of the analysis remains the extremely small percentages assigned the Instrumental code in all location groups. Even in the highest location group, **os**, this code is still only assigned to 11% of its members. The **os** group is also more evenly divided between Relational (46%) and Hybrid (42%), whereas both other groups have percentage scores for Relational in the seventies.

Two further pieces of qualitative data connect with the quantitative analysis above. Firstly, the open response: *"Mathematics is taught… in a way that generally confuses me (i.e. 'learn this formula' when you have no idea where it comes from)"* [R124-F-oh-KS5], first given when discussing reasons for working at home [Section 4.3.3 p81], highlights the desire and indeed the need for relational understanding. The second is the comments taken from a paper-based questionnaire and face-to-face group interviews [see PQ4 and Tasks 9&10, Table 3.2 p60] following up on aspects of the web-survey with fifteen high-performing A-level students. The comments [see Appendix 4.5] are in response to a question modelled on WQ29&30 on the importance of memory in mathematics. Despite the examination focus of the students, the responses are divided 12:3 in favour of relational understanding: whilst memory is more prominent, responses tended to suggest that whilst a good memory would suffice [with the formula book] to pass the examination, understanding would better place someone to study further.

The pervasive effect of examination-dominated curricula is illustrated by similar sentiments expressed in an interview with Scott aged 11, just after leaving primary school. For this young, enthusiastic puzzle solver, sharing secret pleasures with his teacher appears to bring unwelcome consequences as it might precipitate such work becoming one of his academic targets. Furthermore, Scott questions whether the teacher’s perception of a fun puzzle would coincide with his own. Scott’s concerns are also indicative of a common belief [see LRI] that, even at such an early stage of school study, education has become so prescriptive and target-oriented that everything is viewed and measured only in terms of targets.

Int: Did you used to talk to your primary school teachers about doing puzzles at home?

Scott: *No, not really. Just like to keep it to myself.*

Int: You like to keep it to yourself. Do you know why you like to keep it to yourself?

Scott*: Just ‘cos I don’t like get [pause] get given any targets or anything like that.*

Int: Oh right.

Scott: *So I can do it out of my free will, not like say, like it’s not like homework say like you have to do it for the next day so you have do then in four hours or be too late to do it.*

Int: I’ve found other people who don’t want to tell their teacher and I’m interested in thinking about why this is. Even if your teacher knew you might be able to do more fun problems in school, you think that might not be what you would want.

Scott: *Yes. Cos to find puzzles for [unclear] they might think it would be fun for me but it might not be fun for me actually.*

Int: So you’ve got to choose which ones you find are fun.

Scott: *Yes.*

Figure A: Extract from Interview with Scott – September 2007

Responses from questionnaire completed by a sixth form A level mathematics class attending an All Boys School.

Question: How important is it to have a good memory in mathematics? How useful is it to remember rules and methods as opposed to understanding.

(The comments have been part-ordered, according to the responses, so that the focus shifts from a real need to understand at the beginning to memory being (almost) sufficient at the end).

Alistair

The formula book should mean that you don't need to remember complicated formulae. I think that understanding is far better than remembering especially in the long run. If you understand you are able to build on what you know far easier than remembering a formula.

Ben

Good memory would be a useful tool in mathematics and is very useful for mental maths. It is useful to remember rules and methods but not entirely necessary. It is a lot better to learn and understand the rules and methods.

Gareth

In lower years, I think that it was possible to answer mathematics questions purely based upon factual recall. For A-level mathematics, a good understanding is required, as each question involves the use of more than one mathematical concept in logical succession.

Harry

1. Not important as long as 2. You remember rules and methods. If you are able to work through similar questions and understand how to do those then you can do all maths questions.

Ian

I feel that, when initially taught, it helps to simply learn rules. However once many problems have been solved using a given technique, a deeper understanding of the problem develops which is very important in maths.

Michael

Memory is important but understanding will help more.

Oliver

Having a good memory can be successful in terms of scoring well in exams. But to enjoy the subject and to have a true aptitude for it one must understand it.

Callum

It is certainly useful to have a good memory – remember rules/equations/ special properties of numbers. However it is better to be able to derive and understand formulae for example.

Nathan

It is very useful, in order to progress. Understanding is required but a bit of both really.

David

Mathematics is about understanding. But it does require some degrees of good memory. It is easier to memorise some of the difficult formulas rather than trying to understand their formation. Well, at least I find it slightly easier to memorise some formulas so that when dealing with problems I would find them more straight-forward.

Edward

In mathematics, formulas are usually given when required. I do however believe that at a simple level (which sometimes includes much of maths A level) maths can be done using memorised methods. However, at a more advanced level a little more intuition is usually required.

Findlay

Important – without remembering rules and formulae then more questions would be impossible. But an understanding is also important to make the questions easier to complete.

Lennie

Very important. More time in exams instead of looking at formulae booklet. Ultimately exam is based on remembering and not particularly on understanding.

Jack

It is very important because in the exam quick and easy methods are preferred and these can be learnt. I would prefer to remember the rules and methods as opposed to understanding the subject so that I can pass the exam.

Keir

Extremely important to remember rules, otherwise very hard.

<table>
<tr><td>Paper author(s)</td><td>Date of publication</td><td>Theoretical Background / Collaborative Learning Model [stated in Table 1 p12]</td><td>Précis of description of instrument / analysis as reported by De Wever et al. [pp13- 23]</td></tr>
<tr><td>Henri</td><td>1992</td><td>Cognitive and metacognitive knowledge</td><td>Unit of analysis: messages divided into statements corresponding to ‘units of meaning’.
Uses definition of interaction if three stages are present (communication, first response, response back to first response)
Five dimensions of categorization enumerated:
participative divided into (i) overall participation: total number of messages; (ii) active participation: number of statements relating directly to learning.
social: statements not related to formal subject matter content
interactive divided into two interactive types: direct explicit and indirect implicit with responses and commentaries distinguished for each type; and non-interactive: independent statements
cognitive: elementary clarification, in-depth clarification, inference, judgment, strategies [furthermore distinguishes between surface and in-depth processing in evaluating these cognitive skills]
metacognitive (declarative) knowledge and metacognitive (self-awareness) skills</td></tr>
<tr><td colspan="4">AskNRICH: Number of messages per thread is indicated on the ‘front page’ of each of three mathematical levelled discussion boards. All messages within the thread should be related to learning. Social statements should be made primarily in the MathsTalk forum established specifically for this purpose, though there is overlap e.g. when discussing performance pre and post competition papers. A thread starts with a poster posing a problem that they require help with. In the ensuing stages, interactions will take place between those offering help and the original poster (plus any others) interested in developing their personal knowledge and skills (metacognition). During this process the discussions are likely to be of a cognitive nature, between ‘teacher’ and ‘student’ learning processes. A further difference to Henri’s theoretical model (the instrument was never empirically tested) is that, given the mathematical nature of AskNRICH there is opportunity for group (social) co-construction of knowledge, that De Wever et al.’s paper reports that others have suggested is lacking in Henri’s analysis [p13], although ultimately the thread should end when an individual is satisfied that they have received the means to complete the solution and there will be an AskNRICHer who is able to provide sufficient help to an individual and who will not need to co-construct knowledge.</td></tr>
<tr><td>Newman, Webb & Cochrane</td><td>1995</td><td>Critical thinking</td><td>Content analysis based on Garrison’s [1991] five stages of critical thinking and Henri’s [1992] cognitive skills.
Unit of analysis can be phrases, sentences, paragraphs or whole messages to illustrate indicator.
Ten categories:
relevance, importance, novelty, outside knowledge, ambiguities, linking ideas, justification, critical assessment, practical utility, width of understanding</td></tr>
<tr><td colspan="4">AskNRICH: As categorized here there are difficulties in identifying these categories per se though the theoretical concepts that support this instrument – group learning, deep learning and critical thinking - are present (or have the opportunity to be present) amongst the participants of AskNRICH. The group is dynamic and exists in variable numbers determined by the original poster’s question. There can be threads where more than one person offers an alternative method or strategy to solve the mathematical problem which culminates in an ‘expert’ also learning something new. Messages that convey problem resolution by the original poser (with the help of others) can illustrate deep thinking. Components of critical thinking are also present in the mathematical problem resolution sense, though the definitions of the ten categories given above fit ‘hot-topic’ type of discussions more easily.
Importantly Newman et al. claim that noting the indicators relies on subject knowledge and thus necessitates identification by an expert in the domain.</td></tr>
</table>

Zhu	1996	Theories of cognitive and constructive learning – knowledge construction	Unit of analysis: message Eight categories: social interaction (vertical & horizontal) questions (Type 1 information seeking, Type 2 provide information, seek opinions start dialogue) answers (in response to Type 1 questions) information sharing discussion (elaborating and sharing ideas) comments (non-interrogative statements concerning readings) reflection (evaluation, self-appraisal, relating or linking messages, adjusting learning goals and objectives) scaffolding (guidance or suggestions)
AskNRICH: The theoretical framework here is certainly relevant to the aims of AskNRICH, though as with Newman et al. there is some difficulty with the definition of group. Given the starting point of the thread by a poster who requires help to answer a mathematical question and the ethos of AskNRICH not to simply give the solution but to ‘scaffold’ the poser’s learning, then questions (both types), answers, information sharing, discussion, and scaffolding can be present. Vertical social interaction as proposed by Zhu [p14] ‘looking for more capable peers’ is a situation that predominates but horizontal social interaction ‘all contributing equal knowledge’ inferring ‘no-one knows the answer’ is not the situation in the two sections of the web-board that is the focus of this research. Similarly, comments and reflections do not easily fit, each having more relevance to a ‘hot-topic’ debate, although the last of these (reflections) may have relevance, though not always shown, by the original poser.			
Gunawardena, Lowe & Anderson	1997	Social Constructivism - knowledge construction	Unit of analysis: message Initially distinguished two types of learning for analysis: (i) basic type of learning, learning by accretion or pooling of knowledge, (ii) adjustment learning – arising out of cognitive conflict Went to five to include different stages of negotiation: sharing and comparing information discovery and exploration of dissonance/inconsistency negotiation and/or co-construction of knowledge testing and modification of previous proposals statements of agreement and application of newly-constructed knowledge
AskNRICH: The assigned theoretical framework (each time indicated in column 3 of this table) is similar to Zhu’s but AskNRICH essentially only has the first basic type of learning and that only in part and with reinterpretation: the pooling of knowledge is to provide knowledge construction for the individual, the originator of the thread, rather than any knowledge co-construction of the group that, if it occurs, is incidental. Furthermore, in relation to type (i) the individual may or may not already understand the concept and if type (ii) is invoked it is often because the individual has already experienced ‘cognitive conflict’, ‘dissonance’ in attempting to solve a problem and which they wish to resolve and thus is the very reason they are posting for help. Planning for cognitive conflict within mathematics lessons is now widely encouraged [Swan, 2001; Tanner & Jones 2000] and it can at times be a pedagogical strategy adopted by AskNRICH helpers and for example in considering the special case or counter example within a proof. The five stages of negotiation have little in common with the main activities within AskNRICH.			

<table>
<tr><td>Bullen</td><td>1997</td><td>Critical thinking</td><td>Unit of analysis: message
14 week Bachelor Degree course, 13 students, 1 instructor, 207 messages.
Four categories of critical thinking skills with a series of + and – indicators within each category
clarification
assessing evidence
making and judging inferences
using appropriate strategies and tactics</td></tr>
<tr><td colspan="4">AskNRICH: Similar comments as those made under Newman et al. above also apply here. Although the general/surface assumption of meaning to the category and indicator headings could be considered relevant to AskNRICH, the assigned definitions/explanations of instances do not fit the AskNRICH situation. Again, for AskNRICH the nature is not where one is looking at a group working together debating a hot-topic but rather aiding an individual to understand a mathematical problem to which at the time of asking they do not know the solution.</td></tr>
<tr><td>Fahy, Ally, Crawford, Cookson, Keller & Prosser</td><td>2000</td><td>Social network theory – interactional exchange patterns</td><td>Unit of analysis: sentence
Promote a holisitic approach to transcript analysis
Developed a Text Analysis tool (TAT)
Focus on two network concepts:
structural (size, density and intensity of the social network)
interactional exchange patterns (kinds of content exchanged in interaction and the exchange flow/directness of resulting interaction)</td></tr>
<tr><td colspan="4">AskNRICH: This part of the NRICH site was set up to be a social network and thus in the way described here there is again similar sounding terms for considering AskNRICH. A primary research focus on AskNRICH is in analysing the mathematical content within the threads and as such does not fit well with the network theory, although included within this mathematical analysis there will be consideration in part of the interactional exchange patterns. Nevertheless it is currently difficult to see how to relate this instrument to proposed research.</td></tr>
<tr><td>Veerman & Veldhuis-Diermanse</td><td>2001</td><td>Social Constructivism - knowledge construction</td><td>Instrument applied to four consecutive studies, three had unit of analysis as message whilst one used a thematic unit. Participants numbered 40, 30, 20 and 14 students during 6 to 12 weeks of an undergraduate course with 2040, 1287, 952 and 1088 messages analysed.
Two categories of message (with interest in first):
(i) task-related (new ideas, explanations, evaluation)
(ii) not task-related messages</td></tr>
<tr><td colspan="4">AskNRICH: De Wever et al. cite Veerman & Veldhuis-Diermanse beliefs that collaborative learning is one pedagogical method that can stimulate and motivate students to negotiate information and complex problems from varying viewpoints [p18] this again appears more relevant to ‘hot-topic’ debate than working through a mathematical problem that one person has asked to be solved. Although AskNRICH should only have task-related messages, it is really only explanations from the three suggested here (under (i)) that appears to have relevance. De Wever et al. also report that others have combined the three task-related components here with the five stages of Gunawardena et al. and comments made above under Gunawardena et al. are likewise relevant here.
This instrument has been used by others e.g. Schellans & Valke 2005 with 230 students during 12 week, first year university course involving 230 students and 1428 messages; by De laat & Lally (2004) analyzing workshop in virtual e-learning masters’ program, transcripts of discussions of 7 professionals during 10 day period of 160 messages.</td></tr>
</table>

<table>
<tr><td>Rourke, Anderson, Garrison & Archer</td><td>1999</td><td>Community of inquiry – social presence</td><td>Unit of analysis: Thematic
Two studies both graduate level courses: 11-14 students, 2 moderator students and 1 instructor; 90 and 44 messages coded.
Three main categories of responses:
affective, interactive, cohesive</td></tr>
<tr><td colspan="4">AskNRICH: Detailing three separate instruments (this and the two others immediately below) isolates the three types of presence that Garrison et al. (2001) have argued constitutes together a Community of inquiry – though it is Garrison and his team who have undertaken the separation into three publications. Even with strict posting guidelines and a separate area for social ‘chit-chat’ AskNRICH’s objective in helping someone to come to understand can at times show these aspects of social presence listed above – e.g. in encouraging the original poser in their quest</td></tr>
<tr><td>Garrison Anderson & Archer</td><td>2001</td><td>Community of inquiry – cognitive presence</td><td>Unit of analysis: message
Two studies; 13 and 2 weeks in duration; 11 students, 2 moderator students and 1 instructor; 51 messages analysed.
Four phases:
initiation (triggering event)
exploration (brainstorming, questioning and information exchange) integration (constructing meaning)
resolution (of problem created by triggering event)</td></tr>
<tr><td colspan="4">AskNRICH: The nature of the group makes it again necessary to make some adaptation/reinterpretation of these phases in relation to AskNRICH. There is always a triggering event in the form of the first message posted to the thread – a question is asked. Exploration is limited (adapted) to the exchange taking place on the web-board between helper and poser so questioning is generally more on ‘do you understand’ or ‘is that correct’ rather than people working through a ‘hot-topic’. Likewise the original poster, posing the question, at least is likely to construct meaning and will in ascertaining the solution have resolution – but both are likely to be in a different way than suggested. The dictionary definition of cognition ‘the faculty of knowing or perceiving things’ is certainly present in some form.</td></tr>
<tr><td>Anderson, Rourke, Garrison & Archer</td><td>2001</td><td>Community of inquiry – teaching presence</td><td>Unit of analysis: message
Study Graduate level, one instructor, 13 week duration; 139 student and 32 instructor messages analysed.
Function of teacher has three major roles:
designer of educational experience
facilitator & co-creator of social environment conducive to learning
subject matter expert knows more than most learners and can therefore ‘scaffold’ by direct instruction</td></tr>
<tr><td colspan="4">AskNRICH: For the mathematical exchanges taking place on AskNRICH it is difficult to delineate the cognitive with the teaching presence. In AskNRICH there is no course or syllabus therefore there is no designer, all participants have the opportunity to be a facilitator and all ‘sign up’ to a learning atmosphere. There is no one subject matter expert but by volunteering to scaffold learning in some way (but generally not be unambiguous direct instruction) there are some within the group who are more expert than others.</td></tr>
</table>

<table>
<tr><td>Järvelä & Häkkinen</td><td>2002</td><td>Social Constructivism – perspective taking</td><td>Unit of analysis: message
Three aspects:
postings (theory, new point or question, experience, suggestion, comments)
levels of discussions (higher-level, progressive, lower-level)
perspective taking discussion stages (ego-centric, subjective role-taking, self - reflective, third person and mutual, in-depth and societal-symbolic)</td></tr>
<tr><td colspan="4">AskNRICH: De Wever et al. states that the foundation of Järvelä & Häkkinen's theoretical framework specifically has 'the idea of apprenticeship in thinking' [p 20]. Postings of AskNRICH can be analysed by following regular posters over a period of time to investigate this aspect. However aspects described here are less transparent than others proposed so far. For AskNRICH e.g. not including some form of knowledge construction would fail to give the whole picture.</td></tr>
<tr><td>Veldhuis-Diermanse</td><td>2002</td><td>Social Constructivism - knowledge construction</td><td>Used both written and read notes for first step; meaningful units for step two and first coding system, whole messages for step three and second coding system.
Three step method:
Analysis of participation and interaction
Different learning activities (cognitive, affective, metacognitive)
Quality of constructed knowledge based on observed learning outcome [as also used by Schrire, 2005, whose interactive representation is reviewed in Chapter Five of this thesis].</td></tr>
<tr><td colspan="4">AskNRICH: Although the words knowledge construction are explicit here, the instrument described here would be difficult to implement with the type of group using AskNRICH and the reasons why an individual is seeking and/or giving help. 'Observed learning outcomes' is somewhat problematical even if later analysis of AskNRICH consider that learning is taking place. See Pena-Shaff & Nicholls below for further elaboration.</td></tr>
<tr><td>Lockhorst, Admiraal, Pilot & Veen</td><td>2003</td><td>Social Constructivism – learning strategies</td><td>Unit of analysis equates to unit of meaning (as with Henri above)
Five different instruments based on five perspectives:
Participation (number of statements, centrality of poster to network)
Nature of content (content related, procedural, social and nil)
Interaction (threads or chains of semantically or conceptually connected messages)
Information processing from surface to deep across a ten point scale (repeating, interpreting, argumentative, adding new elements, explaining, judgmental, asking questions, offering solutions, offering strategies questioning)
Procedural statements analysed against six categories (evaluative, planning, communication, technical, description and rest)</td></tr>
<tr><td colspan="4">AskNRICH: Pragmatically, the list here seems easier to implement if appropriate than the one suggested by, for example, Veldhuis-Diermanse; although not a check-list, it is broken down into smaller descriptions which 'sound' as if they have a greater potential to link to AskNRICH. In this respect it is reminiscent of Henri's descriptive foci which De Wever et al. report that it was based on [p 21]. However the work was based on the learning strategies exchanged amongst the group rather than the individual but for AskNRICH it is primarily the individual who is personalising a learning strategy as a result of a series of teaching strategies offered by others.</td></tr>
</table>

Pena-Shaff & Nicholls	2004	Social constructivism – knowledge construction	Basic unit of analysis is sentences within a message, but also paragraphs to maintain sentence meaning Study involved undergraduates, graduates and university employers working together over three weeks, analyzing 152 messages from 35 students. Distinguish eleven categories: question, reply, *clarification, interpretation, conflict, assertion,* consensus building, *judgment, reflection,* support and other - with those italicized considered most directly related to the process of knowledge construction
AskNRICH: By this stage there is (probably inevitably) a growing similarity with, and repetition of, the terminology appearing in each instrument e.g Henri, Zhu, Guanwardena et al., Lockhorst et al.. Likewise often in relating any instrument to AskNRICH the comment that it is the nature of the (AskNRICH) group that differs to those described in all the studies reported here. AskNRICH has no 'hot-topic' in the sense of joint debate. As with e.g. Guanwardena et al. and Veldhuis-Diermanse the knowledge constructed is generally only with the individual, not as a group. Co-construction of knowledge may well exist but is essentially secondary.			
Weinberger & Fisher	2005	Social constructivism – argumentative knowledge construction	Four different process dimensions: (i) Participation ((a) quality and (b) heterogeneity) (ii) Epistemic (off- and on-task discourses with latter subdivided into three: (a) the construction of problem spaces (b) the construction of conceptual spaces and (c) the relations between the two (iii) Argument (single or qualifier) (iv) Social modes of co-construction (externalization, elicitation, quick consensus building, integration-oriented building and conflict-oriented consensus building)
AskNRICH: This has little fit with AskNRICH. Weinberger & Fisher's theoretical framework has argumentation as its basis and thus diverges in seeking behaviours and practices associated with such – another 'hot-topic'. This is completely different from the environment of AskNRICH which centres around someone asking a question who requires help in seeking the solution.			

Knowledge Construction

Reviewing the commentaries above about the relevance of each instrument with AskNRICH, there appears to be one major difference between AskNRICH and the CMCs instruments devised, be they theoretical or empirical. Although construction of knowledge is the key component to the AskNRICHer who asks for help, and such knowledge may subsequently be constructed with several helpers within a social (virtual) setting, the resulting social co-construction of knowledge is quintessentially different from any described in the papers reviewed. Thus although much of the terminology used throughout the studies sounds plausible to translate to AskNRICH the fundamental differences in purpose makes any matching problematical.

Context and Size of Empirical Studies

Where reported all studies referred to students of post-school age. All but Veerman & Veldhuis-Diermanse's 2001 study appear to be small in size. Given that the latter study analysed on-and off-task related messages, the higher numbers here could result from off-task (perhaps similar to two threads in AskNRICH which have a large number of messages that are a playful game and 'off-task').

> As ethnography has had such a part to play in my research it is worth outlining what I believe ethnography to be -… for a study to be called ethnography, it needs, at the very least, each of the following seven elements Walford [2001: 7].

The table lists the seven elements each accompanied by a summary of Walford's description [pp7-11] along with a discussion of their relation to this study of AskNRICH.

Element	Walford Description	Relationship to my study of AskNRICH
Culture	to understand the behaviour, values and meanings of any given individual (or group) necessitates taking into account cultural context. Need to make sense of what people are doing by asking 'What is going on here? How does this work? How do people do this? and hoping that these people will tell the researcher about the way things are being done.	It is precisely these [types of] questions that [in part] underpin my research and as someone who has been keen to engage in solving mathematical problems all my life I could *almost* be one of those people and thus in some ways *almost* part of the culture. But obviously I am not an AskNRICHer, participating in an activity that encroaches/complements 'school studies' and thus do not live in an 'average' AskNRICHer's world/culture, even if I have 'empathy' with them.
Multiple methods, diverse forms of data	The complexity and multifaceted nature of cultures, requires an openness to looking at many different ways even to get a rudimentary understanding. Different situations require frequent (different times) sampling including people, place and time to establish what and who counts as being part of a culture. Varied data sources [fieldwork] are only exhausted when no-one can identify other kinds of informants and questions.	Researching AskNRICH has relied heavily on written documents (one of the forms listed as an ethnographer's data source) with a limited number of interviews (face-to-face or by email) due to the 'remoteness' and anonymity of AskNRICHers. Thus data collection methods available for my study do not encompass as many as those suggested are needed for an ethnographic study, even though I have tried to view things from, and in, as many ways as possible.
Engagement	Observation in-situ, first hand; 'being there' as things actually happen; 'hanging around' and 'picking things up'; requirement of human connection and investment of time for as trust builds, more details of lives are revealed.	Logging-on and 'lurking' daily (or indeed several times a day) could almost be said to be 'hanging around' and indeed I gained greater familiarity with the AskNRICHers and felt I got to know them better (whatever that can mean for a virtual world), both of which could *almost* be considered observation in-situ, but that is rather extending the idea of being in-situ in a physical world/culture.

Element	Walford Description	Relationship to my study of AskNRICH
Researcher as instrument	Detailed and useful background information on a setting can often be subjectively informed. Whether this is a strength or a weakness is not an issue simply an inevitably of the act of conducting research in this way.	For this element I have argued within the chapter the dual relationship I have made between subjectivity and objectively. I have endeavoured to follow the processes of being as open and honest as possible about, and reflecting on, possible assumptions and values, whilst at the same time arguing that my professional experience has brought an expertise to the research that adds value to it.
Participant accounts have high status	Everyone's account of the world is unique, but an ethnographer can at least offer an account which can be critically and systematically examined through detailing and clearly articulating how it was generated. The ethnographer can position the participants' accounts and actions in the foreground and broker the information, even though ultimately remaining in charge of that account.	Although I have again endeavoured to act as an honest broker in reporting the AskNRICHer's world, inevitably I am unable to only sketchily include participants' accounts of the world, in some ways through their own words, in as much as they need to write down what they wish to say to others. But these words are in many ways restricted by the medium and limited to the task in hand. Whether I have been able to lessen the 'researcher-centric' view of the culture is debatable, though it has never been through a lack of trying.
Cycle of hypothesis and theory building	Developing a theory is often not so much an event as a process; as new data emerge, the researcher's sense of what is then required can change. Indeed the process can go beyond the data collection stage and thus in this situation is consonant with emergent design	This seems to capture entirely the eventualities of my own research and although, as is evident from the preceding consideration of elements, there are many aspects of ethnography that cannot be present in my research, I certainly have total affinity with this element.
Intention and outcome	As the participants are acting within the boundaries and situations of the study, generalising findings beyond the study should be viewed as suspect as statistical random sampling is rarely a feature of the research. Rather, the intention is to 'storytell', to construct a coherent account to provide a deeper and richer appreciation of the people who have been studied.	As is made clear in the setting out of this element, such 'storytelling' is true of other kinds of qualitative work. My aim in setting out on this research was to tell the AskNRICHer story (as I have come to know it), a tale worth telling. As such I again have total affinity with this element (even if it is because I am doing qualitative research per se, rather than defining my study as true ethnography).

Email correspondence with female user of the NRICH site

To: <…>
Subject: AskNRICH research

Dear […],
Emma [AskNRICH Moderator] has passed on your e-mail message to her in response for me trying to find some people to 'talk' to as it were as part of my research.

From looking within AskNRICH, I've calculated that you are now in Year 12 and over 16. So I think that as long as you are willing to talk to me via e-mail, I can talk to you. However you say that your Mum also dips into AskNRICH sometimes to see what is going on, so please feel free to invite Mum to add any two pennyworth as well.

It doesn't matter a jot whether you post on nrich now, as I am looking more for people who like you obviously find maths interesting (I note your ambition, which is great and good luck) and become proactive in using the Internet in some way to, dare I say it, enrich the subject. I am thinking (or hypothesising as they say) that, now we have the Internet available, there are more opportunities for people like you to find extra people to communicate with than just teachers.

I should say at this point that if I write about you in any of my research you would have a new, anonymous identity - either no name or a completely different one!

So if you are happy with all the above, can I ask you a few questions to start with and then if I think of others from your replies, maybe I could maintain a conversation for a short while. (You are free to choose not to answer particular questions if you don't want to). If you cover a later question in an earlier response it doesn't matter and please don't waste time repeating it in a different place. I will get the overall picture regardless as to where you reply.

Here goes:
1. Could you give a brief description about yourself, your school, your maths lessons and what you are doing for maths away from the classroom. Here (or in the next question) maybe you can mention the other websites and resources you use.

2. If you haven't covered it in q1, can you say how you learn your maths for yourself? Do you teach yourself new material and if so how do you do this?

3. Do you feel that the Internet has given you more opportunities than perhaps in the past when it wasn't around? (I'm sure you can't remember a time when it wasn't, but I can!)

4. Do you (or did you) do any of the NRICH maths problems on the main site either at home or school (or both or somewhere else)? If yes are they different from what you get (or got earlier) in school? What type of maths do you like doing the most and why?

5. What made you become involved with AskNRICH. How do you see things like this forum contributing to maths teaching and learning?

Do only reply when you have the time to spare. I am very grateful that you said I could ask you some questions. I know that doing A levels makes for a busy life.
Last January to celebrate NRICH being 10 years old, I chose the problem Group Photo for my favourite NRICH puzzle. If you go to Jan 07 and look in the notes for the problem I say why I chose it. I only mention this in case you want to find out a little more about me.

All my teacher training students call me by my first name, Libby, so do please follow suit.

with best wishes, Libby

E-mail correspondence with keen male AskNRICHer
October 2007

I think I will call you [posting name] as it's a good informal name and I enjoy reading all [posting name]'s posts (well and those from [real name]. I liked your quickness of humour in your 'here all this week' comment in reply to my quick message last week.

For my research I am wanting to talk (well through e-mail) to people who obviously do lots of mathematics for themselves (as well as in school) as I thinking that now we have the Internet available there are more opportunities for people like you to find extra people to communicate with than just teachers. (I should say at this point that if I write about you in any of my research you would have a new anonymous identity and not be referred to as either [real name] or [posting name]).

I need to say thank you for already completing my web questionnaire and for giving me such interesting and useful answers on it. It led me to look in depth as AskNRICH and as a maths educator I am 'knocked out' by the quality of it all – especially the posts that come from school students such as yourself.

Can I ask you a few questions to start with and then if I think of others from your replies, maybe I could maintain a conversation for a short while. (You are free to choose not to answer particular questions if you don't want to). You may find that you've answered part of a later question in an earlier bit so unlike some maths questions if you already have you don't have to do it again!

Here goes:

1. I know you have an entry on Who's Who and I pick up bits and bobs about people from messages but maybe you would could you give a brief description about your school, your maths lessons and any other maths activities that take place in your school. Then perhaps what is happening with maths outside school – competitions? university? a career afterwards?

2. On your questionnaire you say that you teach yourself (all of?) your maths and you have a fantastic feeling of freedom. Good for you. You also say that it needs a lot of motivation and I am sure it does! How do feel you learn any new material? How do you come to understand the underlying principles?

3. I know that you don't use the main NRICH maths site very much, probably as you are very busy doing working on the Olympiad problems. However if you have done any NRICH problems in the past (or know of them) are they different from what you get (or got) in school? Are they different from the Olympiad problems where you comment that there is often a trick waiting to be spotted? Your enthusiasm for doing mathematics shines through your replies on the questionnaire, but can I ask here what type of maths do you like doing the most and why? (I am not going to delve back into the messages this minute to see if you love or hate geometry!).

You may want to read the next few questions about AskNRICH altogether before giving me your responses.

4. I know you have been quite a stalwart of AskNRICH. When and How did you use it when you first found it? How much time do you think you spend on it in a typical or non-typical week?

You mentioned in your survey response about the good atmosphere of AskNRICH and I've seen, for example, how 'brilliantly' encouraging you were to '[posting name]' around Christmas when he(?) asked about a question in the Number Theory book that he had just started. All your advice about sticking with it was really good (if not better) than a teacher could give. Similarly what is also impressive about the posts on AskNRICH is how when someone asks a question the (school) person who answers (following Emma's strict posting rules) never provides the answer but gives a push in the right direction.

5. How do you think that you have been able to develop these skills on AskNRICH? By giving a push or a helpful hint, does this also help you in developing a deeper understanding of the work? How do you cope with posting messages when you do not know people other than in this virtual environment? How do you see things like this forum contributing to maths teaching and learning?

6. I was interested in your own request for help on a transformation question last November. For someone who is so kind and helpful to others I wondered why you started off with “I'm very new to this, so go easy on me in the explanation please”. Is it almost harder to ask a question than answer one?

7. Everyone on AskNRICH is very respectful of the posting rules that have to be there as it is an educational resource open to the whole wide world? It is good to see the more social exchanges (and lots of smiley faces etc. etc.) but is it sometimes a little restricting? Do you use other forums or e-means (Facebook? MSN?) to communicate with other maths folk?

Oh dear I have gone on and on here. I think I had better call a halt to this for the moment. Please forgive any typos!

In January 2007 to celebrate NRICH being 10 years old, I chose the problem Group Photo for my favourite NRICH puzzle. If you go to Jan 07 and look in the notes for the problem I say why I chose it. I only mention this in case you want to find out a little more about me (and I think it is a problem where there is no trick!).

Please only reply when you have time to.

with best wishes,
Libby

Interview Questions for Deputy Moderator August 2007

I am most interested in the Please Explain section as it is more suited to the main age of my study, though Onwards and Upwards could get a look in too!

Interview Question 1 (and if you want to bring in part of the last question (7) here, feel free).

Perhaps you could begin by saying a little on how you became involved with AskNRICH.

Interview Question 2

Could you briefly describe the types of people who post questions to AskNRICH. And who replies?

I'm trying to probe the possibility of a Community of Practice connection in the next two questions

Interview Question 3

I have discovered that there are different labels attached to people depending upon the number of posts they make. Do you have a sense of some school aged pupils who become prolific and veteran and see how they develop in the way they respond to others' questions. An alternative way of approaching this would be to ask whether when people start out they are 'apprentices' and others, the 'masters' reply, those 'apprentices' later become 'masters' too.

Interview Question 4

Some of the discussions are very impressive – trying never to tell the answer but offering guidance towards it. How are people 'trained' to behave in that way? Is there a sense of community?

Interview Question 5

I am assuming that it is the Internet that has been the tool to enable a resource such as AskNRICH (without considering the necessity of personnel!). How do you view this 'innovation' with 'children's' learning – in the classroom and within a home setting.

Interview Question 6

As a second year PhD student at Cambridge, I was wondering if you can say a little about how you were taught maths in school and whether there were differences when you became an undergraduate. Were you, say, at aged 11 to 16, independent in pursuing your own maths learning? How would you like to see maths being taught in school?

The example below [taken from the Three Threads reported in Chapter Nine] is given to illustrate the problematical nature of any attempt to interpret. Figure A shows four messages from the first of the three threads. P who has asked for help has now found a number (48) that, because it proves that a statement was not true, is known as a counter example. P has also been willing to share the counter example with anyone else who might be looking at the board. These are facts and the interpretation is essentially simply a reporting of the situation. However it is the other three messages that are of interest (and concern).

> P: thanks i ve got it now …for anyone who's interested one counter example is 48
>
> (4 minutes later) HelpA: or 24 ☺
> …
> (6 hours 57 minutes) HelpD: Not to be a spoil sport, but I don't think 24 quite cuts it as a counterexample ☺
>
> (12 minutes) HelpA: Sorry haha, I was thinking that all numbers 0 (mod 6) worked. Good job i didn't make that mistake when I took the paper last year!

Figure A: Message extracts from Thread One of Three Threads on the same problem.

HelpA's inclusion of the emoticon ☺ suggests (in an interpretative way) that they 'smugly' consider that 24 being smaller than 48 is a 'better' or 'more clever' counter example. The fact that actually 24 is not a counter example comes later with HelpD's comment. Here one interpretation of the text is to say that HelpD is letting HelpA down gently, not ridiculing an error by using 'not to be spoil sport', 'I don't think that 24 quite cuts it' and ☺. However if HelpA was being smug then they have their 'comeuppance'. Additionally though HelpD might be feeling equally smug in finding the error and pointing it out sarcastically and in rather a superior manner. HelpA's response of 'sorry haha' *could* be showing HelpA laughing at themselves for the error and with the same sharing attitude as P earlier would like to say why they had may the mistake.

An alternative interpretation would be to decide that competitive natures are to the fore, each trying to outdo the other. This idea is fuelled by HelpA's final sentence that makes it clear that he is actually 'quite clever' as he had already done this question (correctly) in an exam paper the previous year. Is it therefore a possible battle of egos and one-upmanship or friendly banter? The text alone is insufficient to determine the answer.

This was HelpA's 754th post (the first posts began to arrive in late 2005) made on Saturday 20th January 2007. It is interesting to note that by some fortuitous chance HelpA made a contribution in the second exemplar thread [see Figure B below and Chapter Nine]. Even more of a coincidence the post was made on exactly the same date one year later, where as one would say colloquially, 'the tables were turned' though this time with the Deputy Moderator (DM). It is intriguing that the number under scrutiny should again be 24, though the context is different! It is also worthy to note that as this was HelpA's 1512th post they had contributed some 750 posts over the year averaging out as two for *every* day in the year.

DM: So for your example, we could write 24=2x12, and, as you say, 12 is even. But we insist that we have to take *all* of the 2s, so we have to write 24=2^2x3. And 3 is odd. Otherwise we could take out another 2, and so on...

(18 minutes) Help A: 24 = 2^3x3 ☺

(68 minutes) DM: Yes, all right, fair enough. Hopefully [name] will understand what I was trying to say despite that. (Curiously, I thought that something was a bit odd when I wrote it, but still didn't spot it!).

Figure B: Message extracts from Exemplar Thread Two.

The DM's response shows a similar ready acceptance of 'looking foolish' and the mathematician's bane of still doing these 'silly' things even when feeling that something is 'wrong' (cf. intuition). Both sentences are delivered with some degree of humour, maybe befitting of the role of DM, whilst the sentence in-between conveys an additional 'irritation' comment that even though there was a silly error the underlying structure was perfectly correct and the person reading it would have nevertheless understood.

This account has been given here to convey that, whilst the interpretations were made using only text, the assiduous exploration of AskNRICH gave me much more information that could influence and improve the interpretations than anyone just reading the text in passing could gain.

[N.B. of the 22 threads mentioned in Chapter Six only 20 are listed here. One of length 6 and one of length 9 missing – deleted accidentally by AskNRICH team during archiving!]

Post title (number of posts in thread)	Start day of week/time time lapse before first response duration description date of first and last posts	Comment [as at mid-December 2007]
Random question (2)	Sunday 6.40pm 3 hours 28 mins 21 10 2007	Posted by someone who had been posting since the previous year Asking a general question about BODMAS which moderator dealt with
Similar areas (2)	Wednesday 5.30 pm 1 hour 47 mins 26 09 2007	A second thread started by poster of random question Ask whether his thought is correct. Reply says not and corrects. No further posts as yet
Bounds (14)	Tuesday day12.05am 7 minutes 6 days for all information. Thank you message 7 days later 16 10 2007 to 29 10 2007	First question raised by poster (who had joined in Nth term). Mother home-educating child Two prolific posters reply as do the deputy moderator and moderator. Poster constantly replies to messages and keeps the conversation flowing. No further posts as yet
Box Plots (4) Sun 4.20pm	Saturday 5.07pm 19 minutes Carried over 1 day 27 10 2007 to 28 10 2007	Made by new poster (who added a little more information 30 minutes later in a second post) Asking for help in adapting stem and leaf data to become a box plot. Moderator replied I'm not very clear about what you're doing, but I'll do my best to say a bit about box plots. Following (Sunday afternoon) poster replies: "thanks i no what im doing and ive finally finished my coursework! thanks for the help anyway" No further posts as yet
Can anyone explain standard deviation please? (21)	Sunday 1.26pm 7 minutes 2 days with initial poster until new poster asking for help 18 days later, lasting a further 2 days 2 9 2007 to 24 09 2007	Has posted before (in Archive 2006-2007). Aged 11 and teaching himself GCSE by a correspondence course. The question gains a great deal of attention and the developing explanations enabling the poster to make good progress. Towards the end a new poster joins in: "Hi, I rarely did maths but am finding this interesting …" and then gets information on Chebyshev's inequality (web link given). Poster continues to be an active member of the forum

Post title (number of posts in thread)	Start day of week/time time lapse before first response duration description date of first and last posts	Comment [as at mid-December 2007]
Circle Theorems (9)	Wednesday 6.16pm 1 hour 8 minutes Carried over 1 day 10 10 2007 to 11 10 2007	Made by new poster “What is the reason behind students learing [sic] about circle theorems? I am unable to find any real life situations where these will be used.” Nothing known about the poster in terms of age – pupil? Deputy moderator and moderator start and end the reply in positive vein. In between one prolific poster offers their own thoughts that are quickly rebutted by another prolific poster and the pair begin a ‘heated’ but very polite exchange of views. No further posts as yet
Completing the square (5)	Wednesday 10.44am 10 mins Carried over 1 day 26 9 2007 to 28 9 2007	Made by new poster Conversation exchange with one helper (whose at that stage was a relatively new poster but has since been prolific in offering help to others) tending towards social comments at the end One further post made
Currency question (9)	Friday 2.33pm 1 hour 30 mins 3 days 5 10 2007 to 8 10 2007	Same poster as for grid question above. Two helpers respond. Original poster gets to a solution that they are happy with. Moderator thinks she has seen this question before and wonders if the poster here has asked it elsewhere. Poster continues to be a current member of the forum
Extrapolating data??? (2)	Wednesday 5.36pm only response 13 days later (moderator) 10 10 2007 to 23 10 2007	Made by new poster An unusually long and unclear question that received no immediate responses. The moderator replied to by: “I'm not sure that we know enough about the scenario”, but did offer pointers for further work No further posts as yet
Grid Question (2)	Sunday 4.59pm 2 hours 1 min 30 9 2007	Made by new poster Moderator replies similar to that for Extrapolating data??? Poster continues to be an active member of the forum

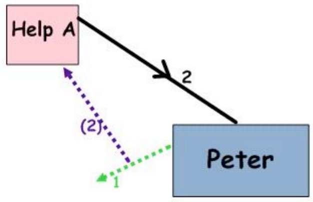
Post title (number of posts in thread)	Start day of week/time time lapse before first response duration description date of first and last posts	Comment [as at mid-December 2007]
Mathematics Investigations (6)	Thursday 9.26pm 10 hours 28 mins 6 days + one further message 13 days later (moderator) 4 10 2007 to 23 10 2007	Posted before (Undergraduate on non maths degree course posting on correct discussion board for content level) Asking for suggestions on what would make a good geometrical investigation. Conversation establishes at what level and purpose required. Finishes with moderator recommending search on NRICH question site Has posted since
Need to solve a sequence (4)	Thursday 10pm 1 minute 11 hours 14 minutes 27 10 2007 to 28 10 2007	Made by new poster (from US) Has 'Mystic Rose' investigation though maybe unaware of this. Numbers in the sequence seem to be following a pattern until 5th term. Some help offered with last posting suggesting it is a quartic (polynomial) No further posts as yet
Nth term (8)	Sunday 12.13pm 14 minutes 27 minutes with initial poster until new poster asking for help 15 days later, lasting a further 2 days 30 10 2007 to 17 10 2007	Made by new poster Six minutes later posts that he now has the answer but asks for help on another (harder) sequence. Two others reply (simultaneously) followed by a new poster who then asks whether her thoughts are correct (who subsequently posed the thread 'bounds' below. No reply in any part by the new poster No further posts as yet
Nth term formula help (2)	Sunday 5.28pm 50 minutes 4 11 2007	Made by new poster Advice from peer as to what to do next No further posts as yet
Numbers powered by zero (7)	Thurs 4.24pm 18 mins Next reply 10 days later for one further day 25 10 2007 to 06 11 2007	A returning 'new' poster (name appears in 2003-4 archive). Year 8 living in Iran. Sets the problem of (-25) and -25 raised to power zero and whether as teacher says result is always +1. Several different views and arguments to support choice but no real conclusion. Has posted again in February 08.

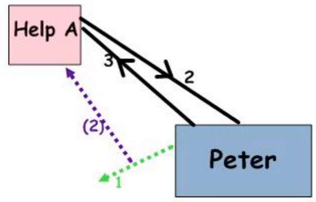
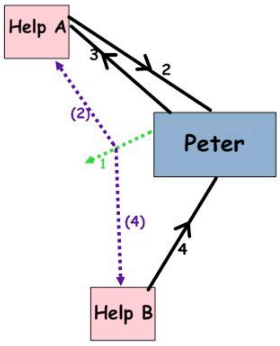
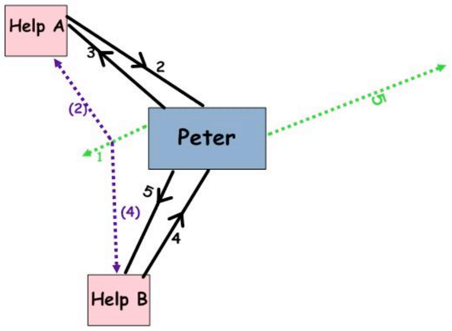
Post title (number of posts in thread)	Start day of week/time time lapse before first response duration description date of first and last posts	Comment [as at mid-December 2007]
Putting Fractions in Order (6) 1 day +)	Wednesday 6.20pm 5 minutes 6 hours 40 minutes One further message 13 days later (moderator) 10 10 2007 to 23 10 2007	Made by new poster (misposting by adding onto another problem and moved to become a new thread by moderator). Five different people responded the final remark from the moderator: “Does that help sort most of them out?” but no reply from the new poster No further posts as yet
Puzzle! (3)	Thursday 1.32am 7 hours 50 mins Closure 9 minutes later 25 10 2007	A social though mathematical question offered by a team member who had been ‘foxed’ by an unknown (to him) but common number problem/puzzle and wondered if others would be able to do it more quickly! Moderator replies with a suggestion. Although a social response follows, no-one else has joined in this conversation
Question – Need answer quick (8)	Thursday 8.50am 4 minutes 3 hours 28 minutes 23 08 2007	Posted two previous threads before this (both during August). Exchange with deputy moderator who tries to explain but poster is unable to work it out. No further posts as yet
Standard Deviation (3)	Wednesday 11.37pm 9 hours 52 mins Closure 2 hours 14 minutes later 17 10 2007	Made by new poster Checking that her work in calculating standard deviation was correct. When receives reply yes, sends a thank you message. No further posts as yet
Why is no one helping me on my sequences? (2)	Thursday 7.21pm 23 minutes 06 12 2007	Fourth thread started by new poster who had asked different three different sequence questions over three days. The second of these had been left unanswered for six days. The response here was to inform that the original question had now been responded to and this remains on the live discussion board with six further messages but none from the original poster. No further posts as yet

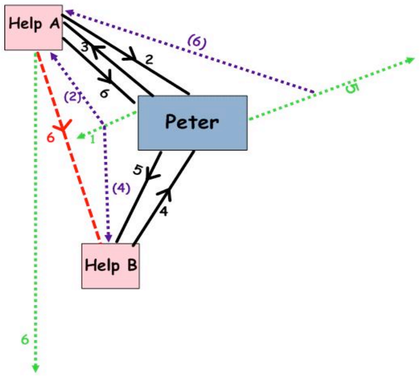
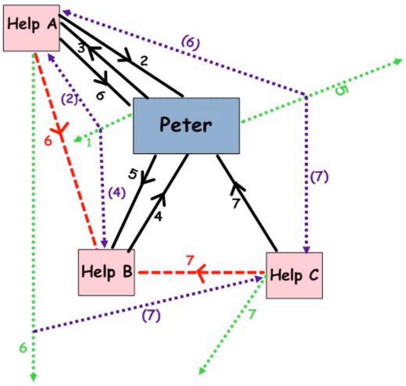
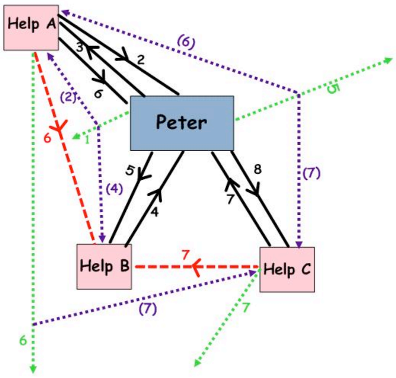
In this Appendix a sequence of Connection Diagrams, together with the associated post text, synopsis and response types, as each post of Thread One of the Three Threads arrives on the web-board.

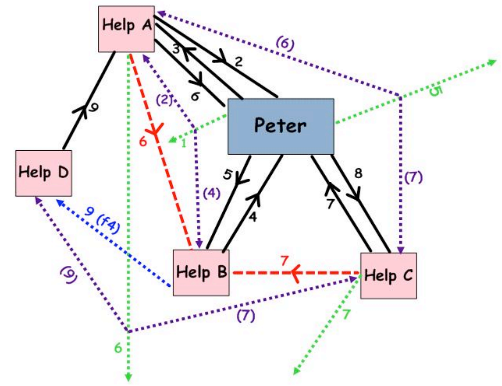
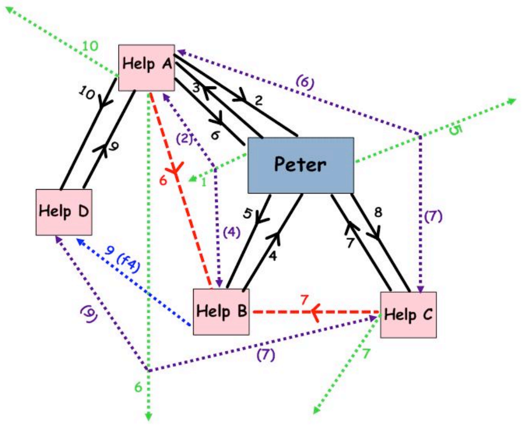
The entry for each post in this table follows the format below. The labelling of response type and the graphical conventions and symbols used in connection diagrams have been given in the main body of this chapter [p130].

Post No.	Participant and Post text		
	Response type	Connection Diagram	Synopsis of interaction / comments

1	Peter: Let n be an integer greater than 6. Prove that if n - 1 and n + 1 are both prime, then $n^2(n^2 +16)$ is divisible by 720. Is the converse true? i have managed to prove the first part of the question using the fact that all primes are of the form 6n-1 and 6n+1. when i tried to prove the converse i cant do it. i know that 2 and 3 divide n and n is of the form 2mod5 3mod5 or mod5. from here where do i go? thanks		
	OR [PUR by A & B]		Peter has completed the first part of question but cannot do second part in finding if converse is true
2	Help A: Do you think the converse is true?		
	PUR from OR Peter's Post 1 DR to Peter		Suggests starting with an intuitive approach – 'feeling' whether it is true or not true

<table>
<tr><td>3</td><td colspan="3">Peter: i presume that it isn't but im not very sure</td></tr>
<tr><td></td><td>DR to A</td><td>
</td><td>Responds by saying that he assumes that it not true, but is not sure</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>4</td><td colspan="3">Help B: If you look back over your proof, you used the fact that ALL primes are 6n-1 and 6n+1. However, is the converse of *this* true? Are all 6n-1 and 6n+1 prime? Using this, you can construct a counterexample.</td></tr>
<tr><td></td><td>PUR from OR Peter's Post 1
DR to Peter
[MR from A & C]
[FR from D]</td><td>
</td><td>Connects Peter's solution from the first part of the problem and suggests looking for a counterexample</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>5</td><td colspan="3">Peter: thanks i ve got it now, for anyone who's interested one counter example is 48.</td></tr>
<tr><td></td><td>DR to B
OR
[PUR by A & C]</td><td>
</td><td>Has found, and shares, 48 as a counterexample</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
</table>

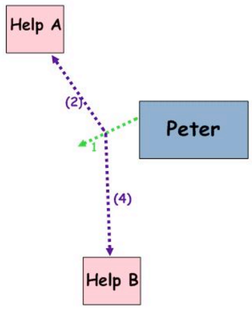
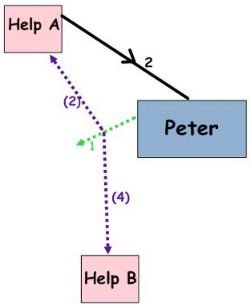
6	Help A: or 24 ☺		
	DR to Peter MR from B’s Post 4 OR [PUR by C & D]		‘Smugly’ (via emoticon) suggests 24 would also do (in fact it does not)
7	Help C: Or if you really want to do no work whatsoever when it comes to multiplication just use 720		
	PUR from Peter’s Post 5 DR to Peter MR from B’s Post 4 PUR from OR A’s Post 6 OR		Gives the ‘blindingly-obvious-once-someone-has-pointed-it-out’ solution of 720
8	Peter: lol i totally missed that		
	DR to C		Amused (lol - laughs out loud) at missing the obvious

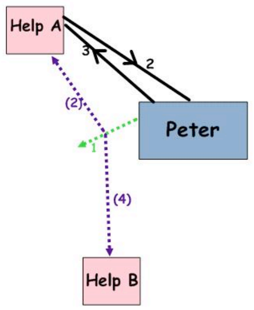
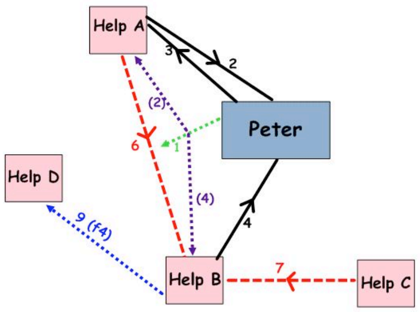
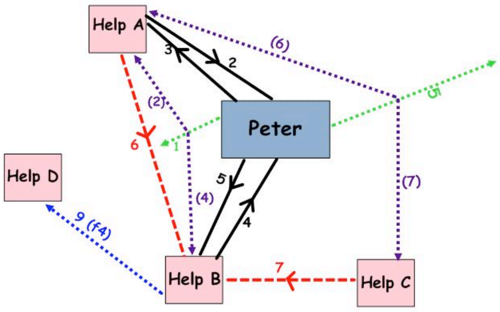
<table>
<tr><td>9</td><td colspan="3">Help D: Not to be a spoil sport, but I don't think 24 quite cuts it as a counterexample ☺</td></tr>
<tr><td></td><td>FR from B's Post 4
PUR from A's Post 6
DR to A</td><td>
</td><td>Politely suggests that 24 'does not quite cut' it as a counterexample</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>10</td><td colspan="3">Help A: Sorry haha, I was thinking that all numbers 0 (mod 6) worked. Good job i didn't make that mistake when I took the paper last year!</td></tr>
<tr><td></td><td>DR to D
OR</td><td>
</td><td>Laughs at own error and shares mistaken thoughts</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
</table>

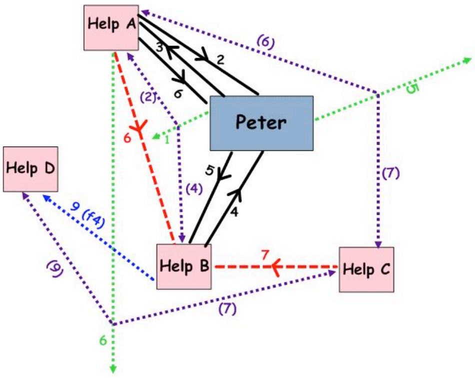
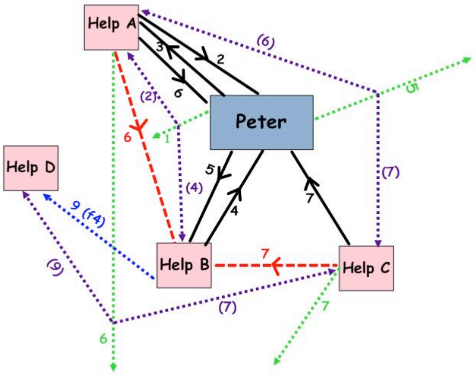
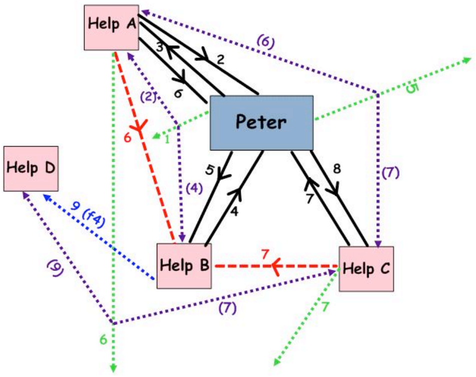
In this Appendix a sequence of Connection Diagrams, together with the associated post text, synopsis and response types, as each post of Thread One of the Three Threads arrives on the web-board. This alternative view differs from that of the previous appendix [Appendix 6.5] in that here it shows the situation when the post has arrived, including its later consequences within the thread.

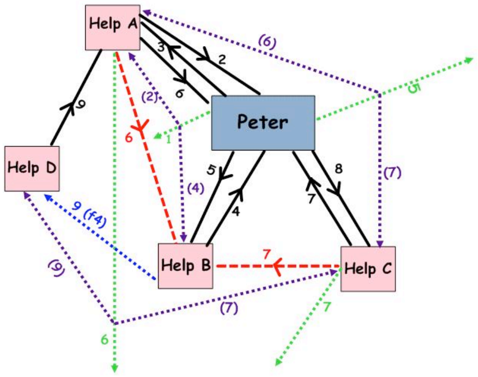
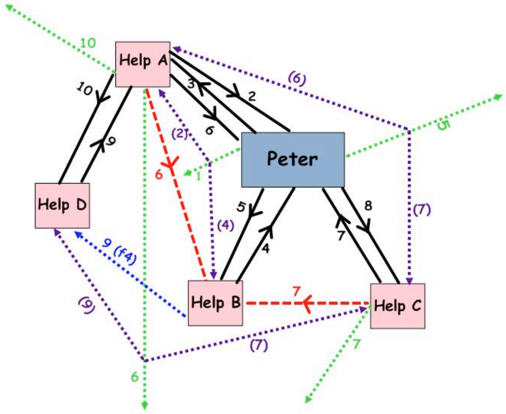
The entry for each post in this table follows the format below. The labelling of response type and the graphical conventions and symbols used in connection diagrams have been given in the main body of this chapter [p130].

<table>
<tr><td>Post No.</td><td colspan="3">Participant and Post text</td></tr>
<tr><td></td><td>Response type</td><td>Connection Diagram</td><td>Synopsis of interaction / comments</td></tr>
</table>

<table>
<tr><td>1</td><td colspan="3">Peter: Let n be an integer greater than 6. Prove that if n - 1 and n + 1 are both prime, then $n^2(n^2 +16)$ is divisible by 720. Is the converse true? i have managed to prove the first part of the question using the fact that all primes are of the form 6n-1 and 6n+1. when i tried to prove the converse i cant do it. i know that 2 and 3 divide n and n is of the form 2mod5 3mod5 or mod5. from here where do i go? thanks</td></tr>
<tr><td></td><td>OR
[PUR by A for Post 2 & by B for Post 4]</td><td>
</td><td>Peter has completed the first part of question but cannot do second part in finding if converse is true</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>2</td><td colspan="3">Help A: Do you think the converse is true?</td></tr>
<tr><td></td><td>PUR from OR Peter’s Post 1
DR to Peter</td><td>
</td><td>Suggests starting with an intuitive approach – ‘feeling’ whether it is true or not true</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
</table>

<table>
<tr><td>3</td><td colspan="3">Peter: i presume that it isn't but im not very sure</td></tr>
<tr><td></td><td>DR to A</td><td> </td><td>Responds by saying that he assumes that it not true, but is not sure</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>4</td><td colspan="3">Help B: If you look back over your proof, you used the fact that ALL primes are 6n-1 and 6n+1. However, is the converse of *this* true? Are all 6n-1 and 6n+1 prime? Using this, you can construct a counterexample.</td></tr>
<tr><td></td><td>PUR from OR Peter’s Post 1
DR to Peter
[A for Post 6 & C for post 7 provide MR]
[D for Post 9 provides FR]</td><td> </td><td>Connects Peter’s solution from the first part of the problem and suggests looking for a counterexample</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>5</td><td colspan="3">Peter: thanks i ve got it now, for anyone who's interested one counter example is 48.</td></tr>
<tr><td></td><td>DR to B
OR
[PUR by A for Post 6 & C for Post 7]</td><td> </td><td>Has found, and shares, 48 as a counterexample</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
</table>

<table>
<tr><td>6</td><td colspan="3">Help A: or 24 ☺</td></tr>
<tr><td></td><td>DR to Peter
B’s Post 4 allowed MR
OR
[PUR by C Post 7 & by D Post 9]</td><td>
</td><td>‘Smugly’ (via emoticon) suggests 24 would also do (in fact it does not)</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>7</td><td colspan="3">Help C: Or if you really want to do no work whatsoever when it comes to multiplication just use 720</td></tr>
<tr><td></td><td>PUR from Peter’s Post 5
DR to Peter
B’s Post 4 allowed MR
PUR from OR A’s Post 6
OR</td><td>
</td><td>Gives the ‘blindingly-obvious-once-someone-has-pointed-it-out’ solution of 720</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>8</td><td colspan="3">Peter: lol i totally missed that</td></tr>
<tr><td></td><td>DR to C</td><td>
</td><td>Amused (lol - laughs out loud) at missing the obvious</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
</table>

9	Help D: Not to be a spoil sport, but I don't think 24 quite cuts it as a counterexample ☺		
	B's Post 4 allowed FR PUR from OR A's Post 6 DR to A		Politely suggests that 24 'does not quite cut' it as a counterexample
10	Help A: Sorry haha, I was thinking that all numbers 0 (mod 6) worked. Good job i didn't make that mistake when I took the paper last year!		
	DR to D OR		Laughs at own error and shares mistaken thoughts

Asking questions

- Start a new thread for each question, unless they are closely related.
- Try to post in the right section. If you're not sure which is right for you, it doesn't matter too much, so just pick one; don't post it in every section as the team will see it whichever section it's in.
- Give your thread a title which indicates the maths involved. (Bad titles include *Help!!!*, *Another question*, etc.)
- Tell us a bit about where the question comes from; your textbook, a website, a competition, one you thought of yourself, etc.
- Don't expect us to do your homework for you - we'll give you a hand in the right direction, but we won't provide a list of answers.
- Try to avoid using attachments; not everyone can read them.
- It can help if you tell us what you are studying in maths. Sometimes the way we explain it will depend on what you know.
- Tell us what you've tried so far. If you just post a textbook question, we're likely to ask you what you've tried.
- Don't use text-speak or lots of abbreviations as these can make your post harder for other people to read. You are more likely to get a reply if people don't have to spend a long time decoding your question before they can answer!
- Be patient; you won't necessarily get an answer immediately, and posting again every hour won't help; the team will see your post when they next look.
- If you don't get a reply after a day or so, then it is fine to post again to the same thread to draw attention to it again.
- It's always nice when people tell us that what we've said has helped!

Answering other people's questions

- Don't just tell them the answer (tempting when you've just worked it out yourself).
- Give hints and explanations to help someone understand for themselves.
- If you're not sure whether what you are saying is correct, say so, so that others can check.
- Remember that the team will probably answer, so if you don't know, leave it to them!
- If someone has already started to help someone with their question, think carefully before joining in. It's often best to let the original poster respond before giving them more to think about.
- If several people are trying to solve the same problem, and you want to avoid giving things away to those still working on it, you can post your answer in white by typing "In white: \white{your answer}". Those who want to can select the text to be able to read it.
- Be tactful if someone is getting things wrong.
- Be careful about humour; a light-hearted comment about a silly mistake will not always come across how you meant it when it's in print.

Writing an entry for Who's Who

In this section, you can tell others a little about yourself. Anything posted to this section will be "queued", so that it is checked by a moderator before it appears. We will *not* allow you to post personal details like your address, school or e-mail.

The sorts of things you *can* include are:

- Country or region you live in
- The type of school you go to (for university students, you may say which university)
- The level of maths you are studying
- Your other interests

Number of Threads and Posts (retrievable on May 22nd 2008) for each section on web-board and mean length of thread for postings from 2005

Posts May 22nd	Please Explain		Onwards and Upwards		Higher Dimension	
	Threads	Posts	Threads	Posts	Threads	Posts
Active	43	370	141	1208	173	1085
Archive 07-08	69	570	468	4188	430	2385
Archive 06-07	236	1908	952	8546	667	4364
Archive 05-06	228	2518	1066	10658	781	6265
Archive 04-05	29	141	22	132	5	23
Archive 03-04	137	1083	207	1359	20	144
Archive 02-03	23	85	269	2253	0	0
	765	6675	3125	28344	2076	14266
Mean length of thread						
Active		8.6		8.6		6.3
Archive 07-08		8.3		8.9		5.5
Archive 06-07		8.1		9.0		6.5
Archive 05-06		11.0		10.0		8.0

Table 1 Postings to Onwards and Upwards: AUGUST 07 TO JULY 08

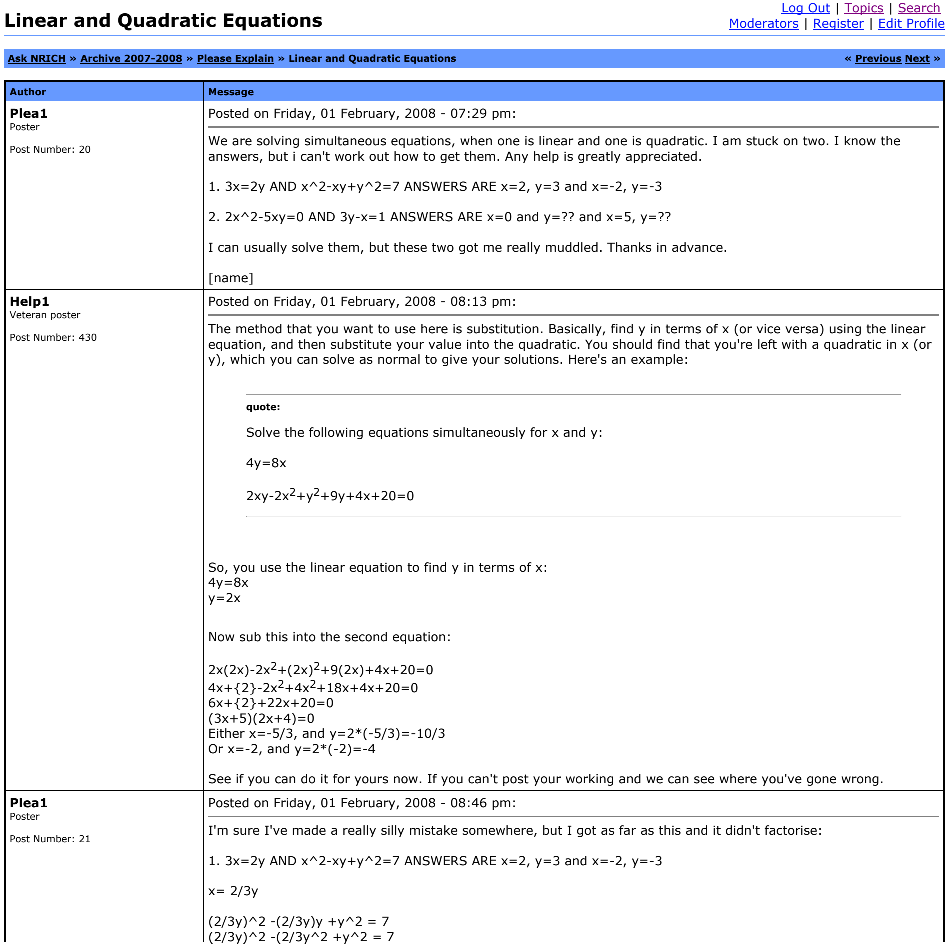
Month	Number of Posts	Number of Threads	Number of length 1-3	Number of length 4-8	Number of length 9-15	Number of length 15-25	Number over 25 with values
August	225	36	10	19	5	2	0
September	668	68	14	31	15	4	4 (35, 49, 51, 61)
October	592	78	20	35	15	7	1 (48)
November	930	109	34	41	24	9	1 (117)
December	*485	63	19	24	13	7	0
January	790	48	12	23	6	5	2 (55, 401)
February	482	58	12	32	8	5	1 (30)
March	**645	69	21	27	16	4	1 (201)
April	240	36	7	23	5	0	1 (35)
May	369	46	8	22	11	4	1 (27)
June	296	44	12	19	11	2	0
July	355	34	10	10	6	6	2 (34, 51)
Total	**6077**	**689**	**179**	**306**	**135**	**55**	**14**

* Excluding the one thread of 401 posts ** Excluding the one thread of 201 posts (but indicated in the extreme right hand column)

Table 2 Postings to Please Explain: AUGUST 07 TO JULY 08

Month	Number of Posts	Number of Threads	Number of length 1-3	Number of length 4-8	Number of length 9-15	Number of length 15-25	Number over 25 with values
August	88	9	0	4	4	1	0
September	34	5	2	2	0	1	0
October	80	13	4	5	4	0	0
November	100	12	2	7	1	2	0
December*	44	7	2	4	0	1	0
January	126	11	0	5	3	3	00
February	178	19	4	7	3	4	1 (26)
March**	188	17	6	4	5	0	2 (42, 54)
April	42	8	4	3	1	0	0
May	169	20	5	11	2	1	1 (63)
June	171	11	*Data not available				
July	59	10					
Total	**1279* (1049)**	**142* (121)**	**(29/121)**	**(52/121)**	**(23/121)**	**(13/121)**	**(4/121)**

In the two days between collecting the data for the total number of posts and threads per month and the decision to return and track the length of each thread, the board's moderator decided to move the data to the archive. In so doing, they inadvertently pressed the wrong key and the June and July threads were unfortunately and irretrievably deleted!



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Linear and Quadratic Equations

Ask NRICH » Archive 2007-2008 » Please Explain » Linear and Quadratic Equations « Previous Next »

Author | **Message**

Plea1
Poster

Post Number: 20

Posted on Friday, 01 February, 2008 - 07:29 pm:

We are solving simultaneous equations, when one is linear and one is quadratic. I am stuck on two. I know the answers, but i can't work out how to get them. Any help is greatly appreciated.

1. 3x=2y AND x^2-xy+y^2=7 ANSWERS ARE x=2, y=3 and x=-2, y=-3

2. 2x^2-5xy=0 AND 3y-x=1 ANSWERS ARE x=0 and y=?? and x=5, y=??

I can usually solve them, but these two got me really muddled. Thanks in advance.

[name]

Help1
Veteran poster

Post Number: 430

Posted on Friday, 01 February, 2008 - 08:13 pm:

The method that you want to use here is substitution. Basically, find y in terms of x (or vice versa) using the linear equation, and then substitute your value into the quadratic. You should find that you're left with a quadratic in x (or y), which you can solve as normal to give your solutions. Here's an example:

> **quote:**
>
> Solve the following equations simultaneously for x and y:
>
> 4y=8x
>
> $2xy-2x^2+y^2+9y+4x+20=0$

So, you use the linear equation to find y in terms of x:
4y=8x
y=2x

Now sub this into the second equation:

$2x(2x)-2x^2+(2x)^2+9(2x)+4x+20=0$
$4x+\{2\}-2x^2+4x^2+18x+4x+20=0$
6x+{2}+22x+20=0
(3x+5)(2x+4)=0
Either x=-5/3, and y=2*(-5/3)=-10/3
Or x=-2, and y=2*(-2)=-4

See if you can do it for yours now. If you can't post your working and we can see where you've gone wrong.

Plea1
Poster

Post Number: 21

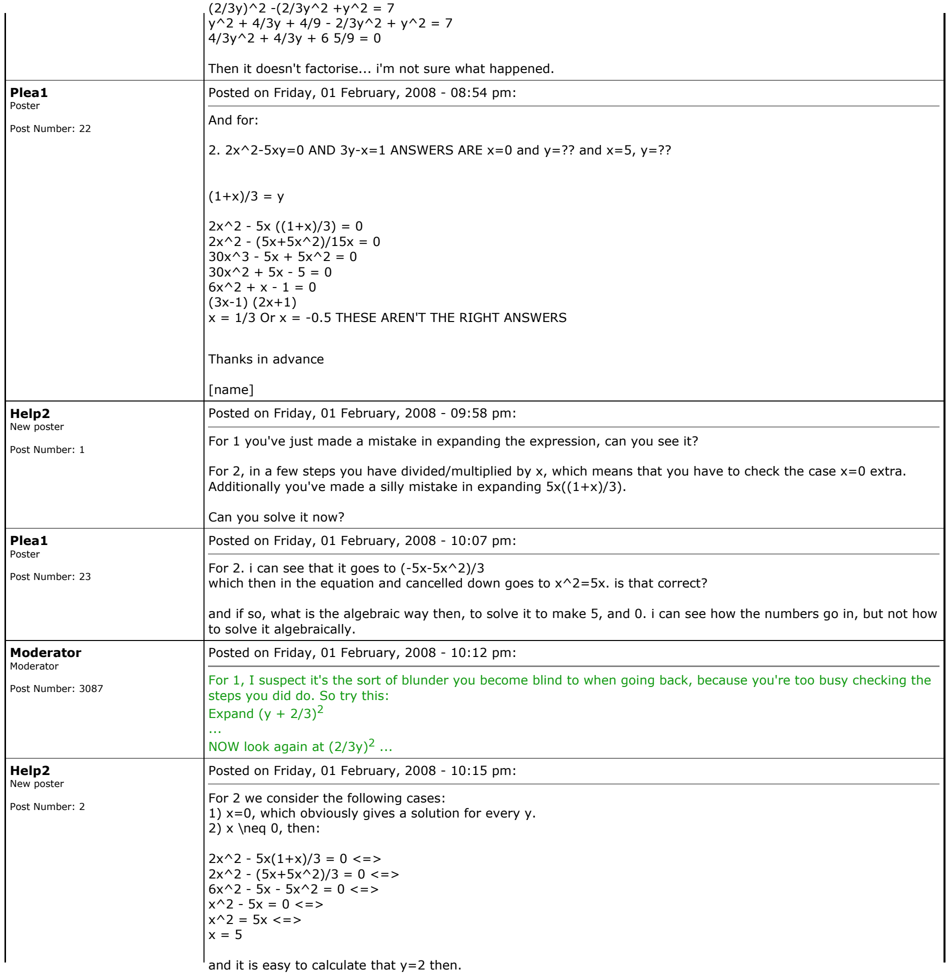
Posted on Friday, 01 February, 2008 - 08:46 pm:

I'm sure I've made a really silly mistake somewhere, but I got as far as this and it didn't factorise:

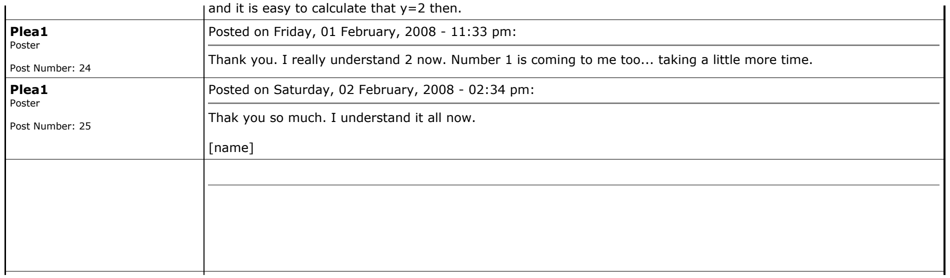
1. 3x=2y AND x^2-xy+y^2=7 ANSWERS ARE x=2, y=3 and x=-2, y=-3

x= 2/3y

(2/3y)^2 -(2/3y)y +y^2 = 7
(2/3y)^2 -(2/3y^2 +y^2 = 7



	(2/3y)^2 -(2/3y^2 +y^2 = 7 y^2 + 4/3y + 4/9 - 2/3y^2 + y^2 = 7 4/3y^2 + 4/3y + 6 5/9 = 0 Then it doesn't factorise... i'm not sure what happened.
Plea1 Poster Post Number: 22	Posted on Friday, 01 February, 2008 - 08:54 pm: And for: 2. 2x^2-5xy=0 AND 3y-x=1 ANSWERS ARE x=0 and y=?? and x=5, y=?? (1+x)/3 = y 2x^2 - 5x ((1+x)/3) = 0 2x^2 - (5x+5x^2)/15x = 0 30x^3 - 5x + 5x^2 = 0 30x^2 + 5x - 5 = 0 6x^2 + x - 1 = 0 (3x-1) (2x+1) x = 1/3 Or x = -0.5 THESE AREN'T THE RIGHT ANSWERS Thanks in advance [name]
Help2 New poster Post Number: 1	Posted on Friday, 01 February, 2008 - 09:58 pm: For 1 you've just made a mistake in expanding the expression, can you see it? For 2, in a few steps you have divided/multiplied by x, which means that you have to check the case x=0 extra. Additionally you've made a silly mistake in expanding 5x((1+x)/3). Can you solve it now?
Plea1 Poster Post Number: 23	Posted on Friday, 01 February, 2008 - 10:07 pm: For 2. i can see that it goes to (-5x-5x^2)/3 which then in the equation and cancelled down goes to x^2=5x. is that correct? and if so, what is the algebraic way then, to solve it to make 5, and 0. i can see how the numbers go in, but not how to solve it algebraically.
Moderator Moderator Post Number: 3087	Posted on Friday, 01 February, 2008 - 10:12 pm: For 1, I suspect it's the sort of blunder you become blind to when going back, because you're too busy checking the steps you did do. So try this: Expand $(y + 2/3)^2$ ... NOW look again at $(2/3y)^2$ ...
Help2 New poster Post Number: 2	Posted on Friday, 01 February, 2008 - 10:15 pm: For 2 we consider the following cases: 1) x=0, which obviously gives a solution for every y. 2) x \neq 0, then: 2x^2 - 5x(1+x)/3 = 0 <=> 2x^2 - (5x+5x^2)/3 = 0 <=> 6x^2 - 5x - 5x^2 = 0 <=> x^2 - 5x = 0 <=> x^2 = 5x <=> x = 5 and it is easy to calculate that y=2 then.



	and it is easy to calculate that y=2 then.
Plea1 Poster Post Number: 24	Posted on Friday, 01 February, 2008 - 11:33 pm: Thank you. I really understand 2 now. Number 1 is coming to me too... taking a little more time.
Plea1 Poster Post Number: 25	Posted on Saturday, 02 February, 2008 - 02:34 pm: Thak you so much. I understand it all now. [name]

UKMT Number Theory book

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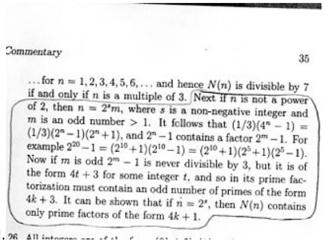
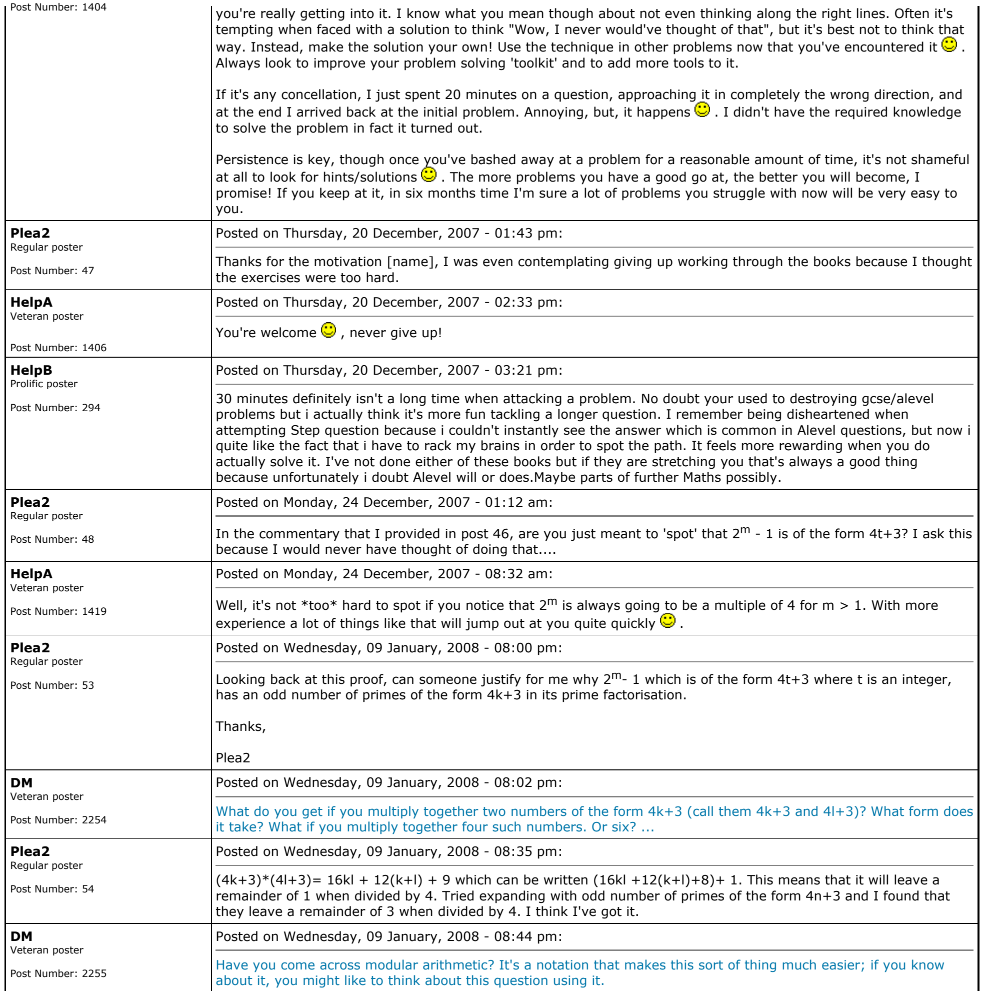
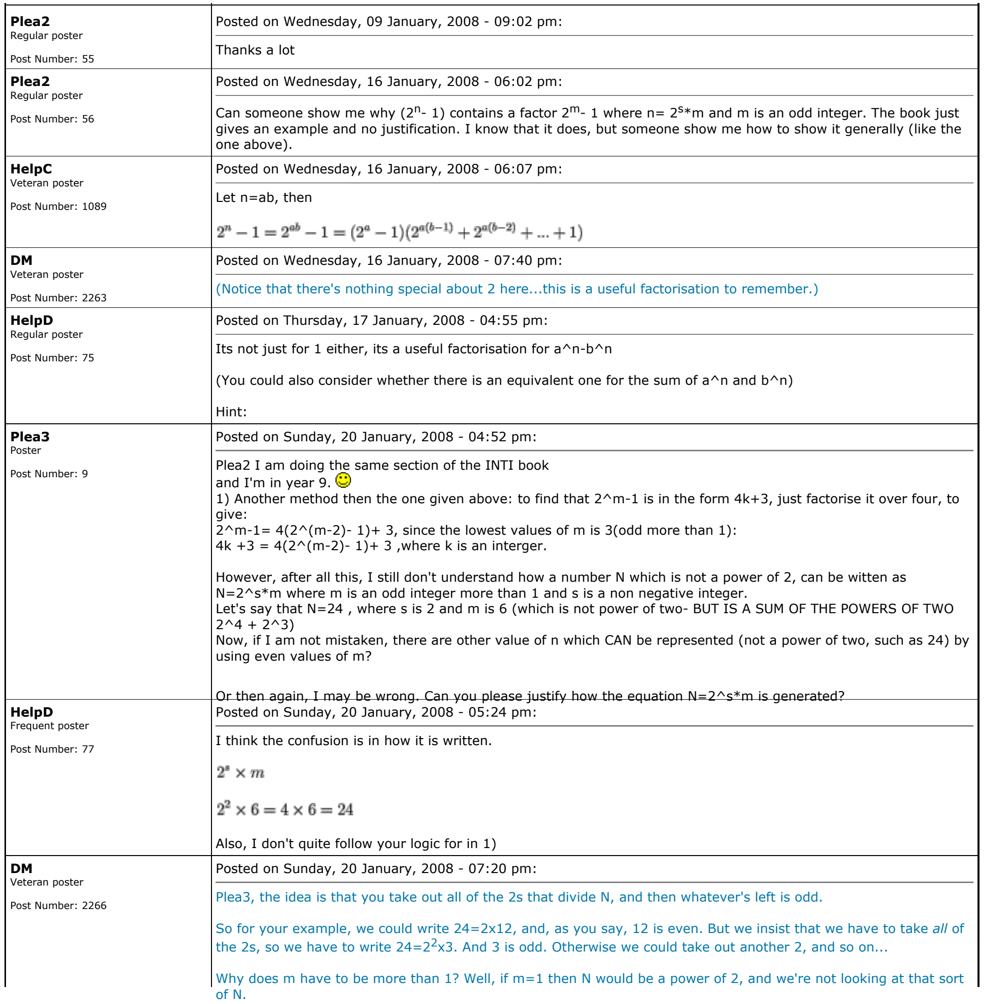
Ask NRICH » Archive 2007-2008 temporary extract » UKMT Number Theory book « Previous Next »

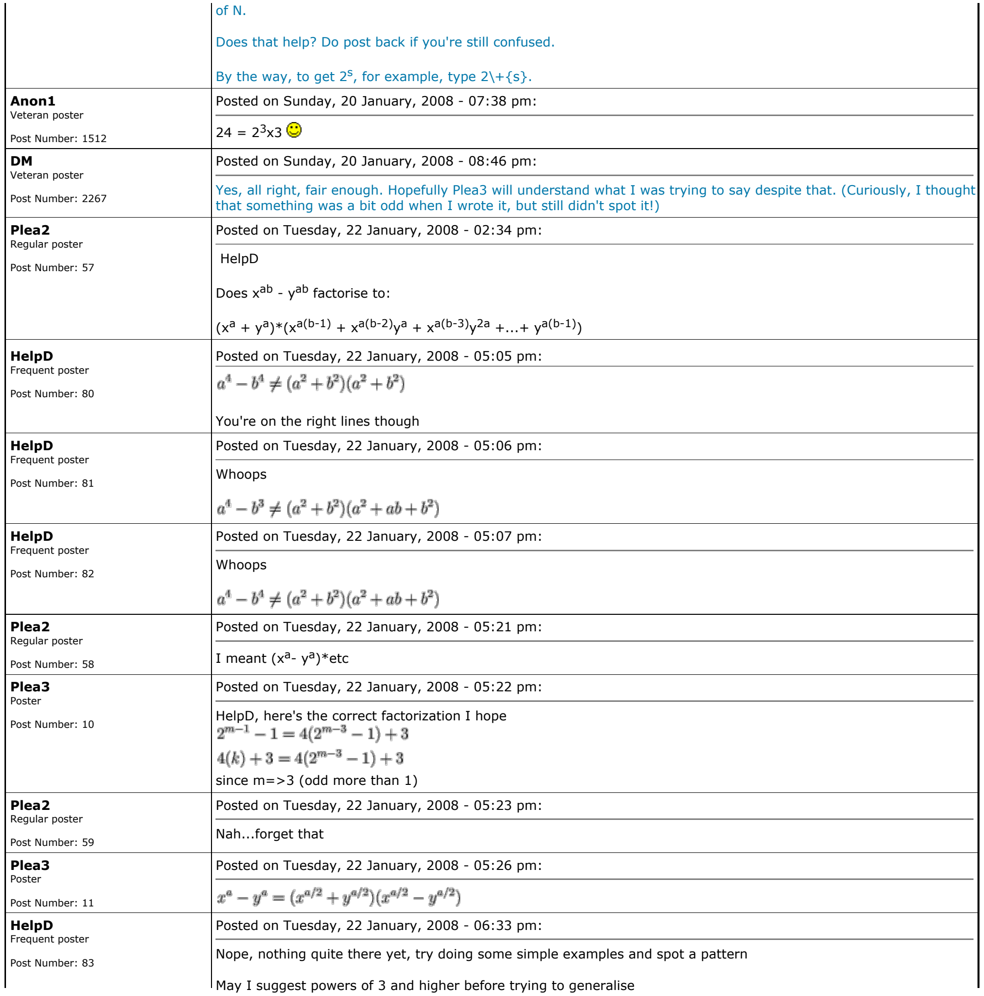
Author	Message
Plea2 Regular poster Post Number: 45	Posted on Thursday, 20 December, 2007 - 12:03 am: I just wanted to ask a few questions about the ukmt number theory book. How much prior knowledge does it assume? Are the exercises meant to be challenging? I put particular emphasis on the last question as I find the exercises quite tricky.
DM Veteran poster Post Number: 2244	Posted on Thursday, 20 December, 2007 - 08:11 am: Well, exercises aren't much fun if they're easy!
HelpA Veteran poster Post Number: 1398	Posted on Thursday, 20 December, 2007 - 08:22 am: I found the number theory problems to be fairly easy in comparison to the inequalities problems in the same book, and in comparison to the geometry problems in the other UKMT book. Just keep practising, and only look at the hints when you're really, really stuck - you'll gain more if you struggle with the question a bit before looking at the hint. They will become easier if you keep hammering away at problems 🙂 . I wouldn't have thought much prior knowledge is assumed - simply because a lot of readers won't have much (if any) experience with NT. Out of interest, how do you find the inequality problems, and if you have the geometry book, how do you find those problems?
Plea2 Regular poster Post Number: 46	Posted on Thursday, 20 December, 2007 - 01:18 pm: I haven't started the inequalities or geometry yet. I'm doing them one by one. I asked my original question about the difficulty of the exercises because I spent 30 minutes on one part of the exercises about primes and prime factors, (week 2). The task was to prove that when n is not a power of 2, the function (1/3)*(4^{n} - 1) would have a prime factor of the form 4k+3. After 30 minutes of struggling with the question, I looked at the commentary and was extremely put off to know that I hadn't even been thinking along the lines of the solution. (It involved writing n as 2^{s}*m where m is an odd integer and s is a non negative integer)  *Commentary* 35 . . . for $n = 1, 2, 3, 4, 5, 6, \ldots$ and hence $N(n)$ is divisible by 7 if and only if n is a multiple of 3. Next if n is not a power of 2, then $n = 2^s m$, where s is a non-negative integer and m is an odd number > 1. It follows that $(1/3)(4^n - 1) = (1/3)(2^n - 1)(2^n + 1)$, and $2^n - 1$ contains a factor $2^m - 1$. For example $2^{20} - 1 = (2^{10} + 1)(2^{10} - 1) = (2^{10} + 1)(2^5 + 1)(2^5 - 1)$. Now if m is odd $2^m - 1$ is never divisible by 3, but it is of the form $4t + 3$ for some integer t, and so in its prime factorization must contain an odd number of primes of the form $4k + 3$. It can be shown that if $n = 2^s$, then $N(n)$ contains only prime factors of the form $4k + 1$. Solution's attached. By the way I'm in Year 10.
HelpA Veteran poster Post Number: 1404	Posted on Thursday, 20 December, 2007 - 01:27 pm: 30 minutes is not long in the grand scale of things. Often you can spend 3-4 hours or more on a difficult problem if you're really getting into it. I know what you mean though about not even thinking along the right lines. Often it's



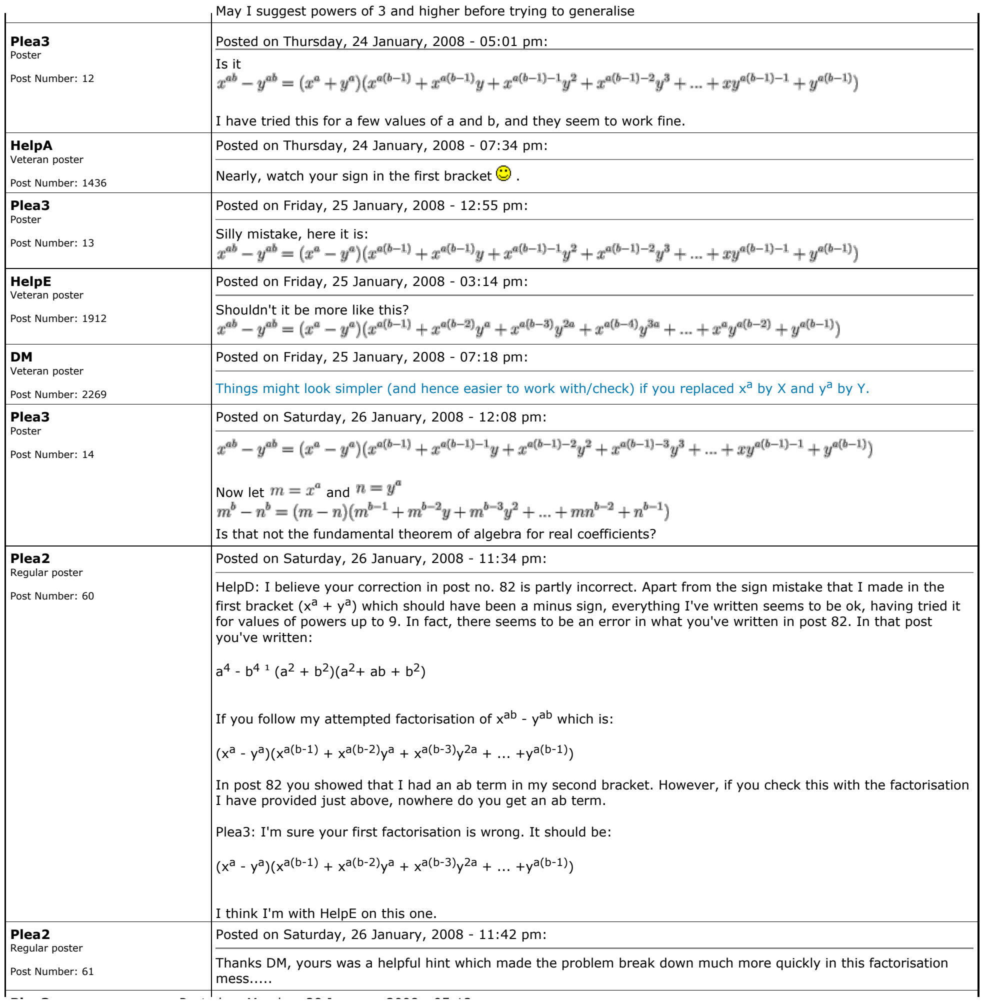
Post Number: 1404	you're really getting into it. I know what you mean though about not even thinking along the right lines. Often it's tempting when faced with a solution to think "Wow, I never would've thought of that", but it's best not to think that way. Instead, make the solution your own! Use the technique in other problems now that you've encountered it ☺ . Always look to improve your problem solving 'toolkit' and to add more tools to it. If it's any concellation, I just spent 20 minutes on a question, approaching it in completely the wrong direction, and at the end I arrived back at the initial problem. Annoying, but, it happens ☺ . I didn't have the required knowledge to solve the problem in fact it turned out. Persistence is key, though once you've bashed away at a problem for a reasonable amount of time, it's not shameful at all to look for hints/solutions ☺ . The more problems you have a good go at, the better you will become, I promise! If you keep at it, in six months time I'm sure a lot of problems you struggle with now will be very easy to you.
Plea2 Regular poster Post Number: 47	Posted on Thursday, 20 December, 2007 - 01:43 pm: Thanks for the motivation [name], I was even contemplating giving up working through the books because I thought the exercises were too hard.
HelpA Veteran poster Post Number: 1406	Posted on Thursday, 20 December, 2007 - 02:33 pm: You're welcome ☺ , never give up!
HelpB Prolific poster Post Number: 294	Posted on Thursday, 20 December, 2007 - 03:21 pm: 30 minutes definitely isn't a long time when attacking a problem. No doubt your used to destroying gcse/alevel problems but i actually think it's more fun tackling a longer question. I remember being disheartened when attempting Step question because i couldn't instantly see the answer which is common in Alevel questions, but now i quite like the fact that i have to rack my brains in order to spot the path. It feels more rewarding when you do actually solve it. I've not done either of these books but if they are stretching you that's always a good thing because unfortunately i doubt Alevel will or does.Maybe parts of further Maths possibly.
Plea2 Regular poster Post Number: 48	Posted on Monday, 24 December, 2007 - 01:12 am: In the commentary that I provided in post 46, are you just meant to 'spot' that 2^m - 1 is of the form 4t+3? I ask this because I would never have thought of doing that....
HelpA Veteran poster Post Number: 1419	Posted on Monday, 24 December, 2007 - 08:32 am: Well, it's not *too* hard to spot if you notice that 2^m is always going to be a multiple of 4 for m > 1. With more experience a lot of things like that will jump out at you quite quickly ☺ .
Plea2 Regular poster Post Number: 53	Posted on Wednesday, 09 January, 2008 - 08:00 pm: Looking back at this proof, can someone justify for me why 2^m- 1 which is of the form 4t+3 where t is an integer, has an odd number of primes of the form 4k+3 in its prime factorisation. Thanks, Plea2
DM Veteran poster Post Number: 2254	Posted on Wednesday, 09 January, 2008 - 08:02 pm: What do you get if you multiply together two numbers of the form 4k+3 (call them 4k+3 and 4l+3)? What form does it take? What if you multiply together four such numbers. Or six? ...
Plea2 Regular poster Post Number: 54	Posted on Wednesday, 09 January, 2008 - 08:35 pm: (4k+3)*(4l+3)= 16kl + 12(k+l) + 9 which can be written (16kl +12(k+l)+8)+ 1. This means that it will leave a remainder of 1 when divided by 4. Tried expanding with odd number of primes of the form 4n+3 and I found that they leave a remainder of 3 when divided by 4. I think I've got it.
DM Veteran poster Post Number: 2255	Posted on Wednesday, 09 January, 2008 - 08:44 pm: Have you come across modular arithmetic? It's a notation that makes this sort of thing much easier; if you know about it, you might like to think about this question using it.



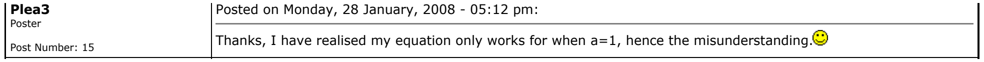
Plea2 Regular poster Post Number: 55	Posted on Wednesday, 09 January, 2008 - 09:02 pm: Thanks a lot
Plea2 Regular poster Post Number: 56	Posted on Wednesday, 16 January, 2008 - 06:02 pm: Can someone show me why $(2^{n}- 1)$ contains a factor $2^{m}- 1$ where n= 2^{s}*m and m is an odd integer. The book just gives an example and no justification. I know that it does, but someone show me how to show it generally (like the one above).
HelpC Veteran poster Post Number: 1089	Posted on Wednesday, 16 January, 2008 - 06:07 pm: Let n=ab, then $2^n - 1 = 2^{ab} - 1 = (2^a - 1)(2^{a(b-1)} + 2^{a(b-2)} + ... + 1)$
DM Veteran poster Post Number: 2263	Posted on Wednesday, 16 January, 2008 - 07:40 pm: (Notice that there's nothing special about 2 here...this is a useful factorisation to remember.)
HelpD Regular poster Post Number: 75	Posted on Thursday, 17 January, 2008 - 04:55 pm: Its not just for 1 either, its a useful factorisation for a^n-b^n (You could also consider whether there is an equivalent one for the sum of a^n and b^n) Hint:
Plea3 Poster Post Number: 9	Posted on Sunday, 20 January, 2008 - 04:52 pm: Plea2 I am doing the same section of the INTI book and I'm in year 9. ☺ 1) Another method then the one given above: to find that 2^m-1 is in the form 4k+3, just factorise it over four, to give: 2^m-1= 4(2^(m-2)- 1)+ 3, since the lowest values of m is 3(odd more than 1): 4k +3 = 4(2^(m-2)- 1)+ 3 ,where k is an interger. However, after all this, I still don't understand how a number N which is not a power of 2, can be witten as N=2^s*m where m is an odd integer more than 1 and s is a non negative integer. Let's say that N=24 , where s is 2 and m is 6 (which is not power of two- BUT IS A SUM OF THE POWERS OF TWO 2^4 + 2^3) Now, if I am not mistaken, there are other value of n which CAN be represented (not a power of two, such as 24) by using even values of m? Or then again, I may be wrong. Can you please justify how the equation N=2^s*m is generated?
HelpD Frequent poster Post Number: 77	Posted on Sunday, 20 January, 2008 - 05:24 pm: I think the confusion is in how it is written. $2^s \times m$ $2^2 \times 6 = 4 \times 6 = 24$ Also, I don't quite follow your logic for in 1)
DM Veteran poster Post Number: 2266	Posted on Sunday, 20 January, 2008 - 07:20 pm: Plea3, the idea is that you take out all of the 2s that divide N, and then whatever's left is odd. So for your example, we could write 24=2x12, and, as you say, 12 is even. But we insist that we have to take *all* of the 2s, so we have to write 24=2^2x3. And 3 is odd. Otherwise we could take out another 2, and so on... Why does m have to be more than 1? Well, if m=1 then N would be a power of 2, and we're not looking at that sort of N.



	of N. Does that help? Do post back if you're still confused. By the way, to get 2^s, for example, type 2\+{s}.
Anon1 Veteran poster Post Number: 1512	Posted on Sunday, 20 January, 2008 - 07:38 pm: $24 = 2^3$x3 🙂
DM Veteran poster Post Number: 2267	Posted on Sunday, 20 January, 2008 - 08:46 pm: Yes, all right, fair enough. Hopefully Plea3 will understand what I was trying to say despite that. (Curiously, I thought that something was a bit odd when I wrote it, but still didn't spot it!)
Plea2 Regular poster Post Number: 57	Posted on Tuesday, 22 January, 2008 - 02:34 pm: HelpD Does x^{ab} - y^{ab} factorise to: $(x^a + y^a)*(x^{a(b-1)} + x^{a(b-2)}y^a + x^{a(b-3)}y^{2a} +...+ y^{a(b-1)})$
HelpD Frequent poster Post Number: 80	Posted on Tuesday, 22 January, 2008 - 05:05 pm: $a^4 - b^4 \neq (a^2 + b^2)(a^2 + b^2)$ You're on the right lines though
HelpD Frequent poster Post Number: 81	Posted on Tuesday, 22 January, 2008 - 05:06 pm: Whoops $a^4 - b^3 \neq (a^2 + b^2)(a^2 + ab + b^2)$
HelpD Frequent poster Post Number: 82	Posted on Tuesday, 22 January, 2008 - 05:07 pm: Whoops $a^4 - b^4 \neq (a^2 + b^2)(a^2 + ab + b^2)$
Plea2 Regular poster Post Number: 58	Posted on Tuesday, 22 January, 2008 - 05:21 pm: I meant $(x^a$- $y^a)$*etc
Plea3 Poster Post Number: 10	Posted on Tuesday, 22 January, 2008 - 05:22 pm: HelpD, here's the correct factorization I hope $2^{m-1} - 1 = 4(2^{m-3} - 1) + 3$ $4(k) + 3 = 4(2^{m-3} - 1) + 3$ since m=>3 (odd more than 1)
Plea2 Regular poster Post Number: 59	Posted on Tuesday, 22 January, 2008 - 05:23 pm: Nah...forget that
Plea3 Poster Post Number: 11	Posted on Tuesday, 22 January, 2008 - 05:26 pm: $x^a - y^a = (x^{a/2} + y^{a/2})(x^{a/2} - y^{a/2})$
HelpD Frequent poster Post Number: 83	Posted on Tuesday, 22 January, 2008 - 06:33 pm: Nope, nothing quite there yet, try doing some simple examples and spot a pattern May I suggest powers of 3 and higher before trying to generalise



	May I suggest powers of 3 and higher before trying to generalise
Plea3 Poster Post Number: 12	Posted on Thursday, 24 January, 2008 - 05:01 pm: Is it $x^{ab}-y^{ab}=(x^a+y^a)(x^{a(b-1)}+x^{a(b-1)}y+x^{a(b-1)-1}y^2+x^{a(b-1)-2}y^3+...+xy^{a(b-1)-1}+y^{a(b-1)})$ I have tried this for a few values of a and b, and they seem to work fine.
HelpA Veteran poster Post Number: 1436	Posted on Thursday, 24 January, 2008 - 07:34 pm: Nearly, watch your sign in the first bracket ☺ .
Plea3 Poster Post Number: 13	Posted on Friday, 25 January, 2008 - 12:55 pm: Silly mistake, here it is: $x^{ab}-y^{ab}=(x^a-y^a)(x^{a(b-1)}+x^{a(b-1)}y+x^{a(b-1)-1}y^2+x^{a(b-1)-2}y^3+...+xy^{a(b-1)-1}+y^{a(b-1)})$
HelpE Veteran poster Post Number: 1912	Posted on Friday, 25 January, 2008 - 03:14 pm: Shouldn't it be more like this? $x^{ab}-y^{ab}=(x^a-y^a)(x^{a(b-1)}+x^{a(b-2)}y^a+x^{a(b-3)}y^{2a}+x^{a(b-4)}y^{3a}+...+x^ay^{a(b-2)}+y^{a(b-1)})$
DM Veteran poster Post Number: 2269	Posted on Friday, 25 January, 2008 - 07:18 pm: Things might look simpler (and hence easier to work with/check) if you replaced x^a by X and y^a by Y.
Plea3 Poster Post Number: 14	Posted on Saturday, 26 January, 2008 - 12:08 pm: $x^{ab}-y^{ab}=(x^a-y^a)(x^{a(b-1)}+x^{a(b-1)-1}y+x^{a(b-1)-2}y^2+x^{a(b-1)-3}y^3+...+xy^{a(b-1)-1}+y^{a(b-1)})$ Now let $m=x^a$ and $n=y^a$ $m^b-n^b=(m-n)(m^{b-1}+m^{b-2}y+m^{b-3}y^2+...+mn^{b-2}+n^{b-1})$ Is that not the fundamental theorem of algebra for real coefficients?
Plea2 Regular poster Post Number: 60	Posted on Saturday, 26 January, 2008 - 11:34 pm: HelpD: I believe your correction in post no. 82 is partly incorrect. Apart from the sign mistake that I made in the first bracket ($x^a + y^a$) which should have been a minus sign, everything I've written seems to be ok, having tried it for values of powers up to 9. In fact, there seems to be an error in what you've written in post 82. In that post you've written: a^4 - b^4 ¹ (a^2 + b^2)(a^2+ ab + b^2) If you follow my attempted factorisation of x^{ab} - y^{ab} which is: (x^a - y^a)($x^{a(b-1)}$ + $x^{a(b-2)}y^a$ + $x^{a(b-3)}y^{2a}$ + ... +$y^{a(b-1)}$) In post 82 you showed that I had an ab term in my second bracket. However, if you check this with the factorisation I have provided just above, nowhere do you get an ab term. Plea3: I'm sure your first factorisation is wrong. It should be: (x^a - y^a)($x^{a(b-1)}$ + $x^{a(b-2)}y^a$ + $x^{a(b-3)}y^{2a}$ + ... +$y^{a(b-1)}$) I think I'm with HelpE on this one.
Plea2 Regular poster Post Number: 61	Posted on Saturday, 26 January, 2008 - 11:42 pm: Thanks DM, yours was a helpful hint which made the problem break down much more quickly in this factorisation mess.....



Plea3 Poster Post Number: 15	Posted on Monday, 28 January, 2008 - 05:12 pm: Thanks, I have realised my equation only works for when a=1, hence the misunderstanding.🙂

On AskNRICH board	Interpretation – first attempt
Friday at 7.29pm Plea1: We are solving simultaneous equations, one linear, one quadratic. I am stuck on two. I know the answers but I can't work out how to get them. Any help is greatly appreciated. [Two questions and answers stated]. I can usually solve them, but these two got me really muddled. Thanks in advance.	The request was made on a Friday evening (with an assumption that there would be no school and thus mathematics teacher contact until Monday). The person making the plea stresses that the answers are known but at the moment they are unable to work through to obtain the values.
8.13pm Help1: The method you want to use here is substitution. Basically find y in terms of x (or vice versa) using the linear equation, and then substitute into the quadratic. You should find that you're left with a quadratic in x (or y), which you can solve as normal to give your solutions. Here's an example [Makes up a similar question and then solves it showing line by line working]. See if you can do it for yours now. If you can't post your working and we can see where you've gone wrong …	The first reply offering help is just 44 minutes after the request is made. (Lines are not manned 24 hours a day so it requires helpers to look for questions coming in, either by logging on or having an e-mail notification). Before Help1 replied, a relevant example has been found, worked through and written up. (Finding an example here is not necessarily trivial as it is preferable to have integer solutions that can only be obtained from a quadratic equation that will factorise, so the initial two equations need to be such that this factorisation can take place). Help1 has also suggested **the** method to use. Someone else might have suggested using the graphical method that would help considerably to visualise the solution, though there is a potential degree of accuracy that could be lost. Plea1's first post does not state what method they are using. Omission of stating the method is more likely to indicate that they are using an algebraic method, further justified that later messages from Plea1 do not indicate that they have been using a graphical method.
8.46pm Plea1: I'm sure I've made a really silly mistake somewhere but I got as far as this [lots of algebraic working] ….. $\frac{4}{3}y^2 + \frac{4}{3}y + 6\frac{5}{9} = 0$ and it didn't factorise… i'm not sure what happened.	Having received this message 'plea' works on the questions and 33 minutes later reports back on question 1 including all the algebraic working and realising that something has gone wrong as it does not factorise. From the working 'plea' has clearly understood the method but has actually made an error in the second line. (This error is subsequently picked up and commented upon by two different people). Not waiting for any further responses from anyone offering more help, 'plea' continues to work on the second of the question and whilst this one does factorise the solution values are not the ones stated with the question. (Another error in multiplying out a bracket in line 2 has caused this, though there will be further complications after this has been corrected). There is a sense of frustration setting in with the use of capital letters for 'THESE AREN'T THE RIGHT ANSWERS'. Again there is a polite 'thanks in advance'.
8.54pm Plea1: and for q 2 [lots of algebraic working] $x = \frac{1}{3}\ or\ x = -\ 0.5$ THESE AREN'T THE RIGHT ANSWERS Thanks in advance.	

On AskNRICH board	Interpretation – first attempt
9.58pm Help2: For 1 you've just made a mistake in expanding the expression, can you see it? For 2 in a few steps you have divided/multiplied by x, which means that you have to check the extra case x=0. Additionally you've made a silly mistake in expanding $5x((1+x)/3)$ Can you solve it now?	A second person [and a first time poster] comes in to help, responding to both messages just four minutes after the second of the two messages is sent. Both errors have been spotted. 'Plea1' is told that for the first question they have made an algebraic error and instead of being told precisely where it is, or indeed what it is, they are asked whether they can find it. However the position of the algebraic error in the second question is given though again no correction is offered. The error sets up a cubic equation for which it looks possible that each term can be divided by x and reduced to a quadratic. The correct equation will allow a similar division and Help2 is 'scaffolding' the common error of doing the division in both these circumstances and forgetting that the equation would also be true if $x=0$. 'Help2' also ends their message with encouragement 'can you solve it now?' and an implicit invite to come back if still unsure. Although Help2's comment 'you've made a silly mistake' could be viewed as a 'put down' and thus a potential dent to 'Plea1's confidence, (a) Plea1 has already used the words 'silly mistake' in the previous message and (b) making 'silly mistakes' is accepted as part of the mathematical processes.
10.07 pm Plea1: For 2 I can see that it goes to $(-5x-5x^2)/3$ which then in the equation and cancelled down goes to $x^2=5x$ is that correct? and if so, what is the algebraic way then, to solve it to make 5, and 0. i can see how the numbers go in but not how to solve it algebraically	Nine minutes later there is a reply from 'Plea1'. They have corrected the algebraic error for question 2 (it may have been easier to do this as help2 had been more explicit where the error lay for this question) and has continued to work through correctly to $x^2=5x$ but is now stuck as it is not quite the usual straightforward quadratic. (It requires the scaffolding ideas that Help2 has given). It is clear from 'plea' that this is new to them, but they appear obviously determined to understand it as they 'see how the numbers go in but not how to solve it'.
10.12pm Moderator: For 1, I suspect it's the sort of blunder you become blind to when going back, because you're too busy checking the steps you did do. So try this: Expand $(y+2/3)^2$ ... NOW look again at $(2/3y)^2$ ….	The forum's moderator joins in at 10.12pm. (An example of how the AskNRICH team contribute 'outside of normal working hours'). The moderator has also picked up the algebraic error in question 1 and suggests to 'plea' that they should try to expand two expressions. The first will be 'Plea1's error whilst the second one given is what 'Plea1' was trying to expand. Again the moderator is scaffolding the situation.

On AskNRICH board	Interpretation – first attempt
10.15pm Help2: For 2 we consider the following cases: 1 $x = 0$, which obviously gives a solution for every y 2 $x \neq 0$, then: [algebraic working] $x = 5$ and it is easy to calculate that $y = 2$ then.	Eight minutes after ‘Plea1’ has posted a message about arriving at $x^2 = 5x$ and being stuck, Help2 responds again. This time ‘Help2’ does give the solution to this unusual case. Giving the solution is against the Posting Protocols and the reason why Help2 chose to do this is not clear cut. It could be due to Help2 being a new poster and thus less familiar with the requirements. Or perhaps they judged that $x = 0$ is new territory for ‘plea’ and there is a need to explain through looking at the solution. There is perhaps nothing wrong with this latter situation if ‘Plea1’ would be able to solve similar problems on their own on later occasions. Such a justification is itself justified by the post ‘Plea1’ makes at 11.33pm, 1 hour and 18 minutes later, that the question is now understood. By now ‘Plea2’ has been working on the questions for some two and three quarter hours - and although it is possible that there had been breaks in between, the postings have been intensive during the time period. In this last post of the evening ‘plea’ adds that question 1 is becoming clearer though ‘taking a little more time’. It would appear that ‘plea’ is still content to work on the problem until its conclusion.
11.33pm Plea: Thank you. I really understand 2 now. Number 1 is coming to me too … taking a little more time.	
Saturday 2.34pm Plea: Thak [sic] you so much. I understand it all now	Without any further postings, ‘Plea1’ sends thanks on Saturday afternoon adding that ‘I understand it all now’. Within and over 24 hours, out of school hours, explanations, hints and one direct solution have been offered to enable ‘Plea1’ to ‘understand’ the problems, though whatever ‘understands’ means cannot be determined by the interpreter).

The person who has posed the original question is denoted as 'Plea1'. People who offer help are designated as Help x with x increasing as each new help person joins. The day and time of the message is indicated that the start of each message.

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<tr><td>1</td><td>Friday at 7.29pm
Plea1: We are solving simultaneous equations, one linear, one quadratic. I am stuck on two. I know the answers but I can't work out how to get them. Any help is greatly appreciated. [Two questions and answers stated]. I can usually solve them, but these two got me really muddled.

Thanks in advance.</td><td></td><td>The person making the plea stresses that the answers are known [though no clue as to how they are known - possibly through given in class or stated in textbook, less likely through trying values out through substituting into the equations until finding the ones that work] but at the moment they are unable to work through to obtain the values.

A polite thank you.</td></tr>
<tr><td colspan="4">The first reply offering help is just 44 minutes after the request is made. Before 'help1' replied, a relevant example has been found, worked through and written up.</td></tr>
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Help1: The method you want to use here is substitution. Basically find y in terms of x (or vice versa) using the linear equation, and then substitute into the quadratic. You should find that you're left with a quadratic in x (or y), which you can solve as normal to give your solutions. Here's an example [Makes up a similar question and then solves it showing line by line working].
See if you can do it for yours now. If you can't post your working and we can see where you've gone wrong …</td><td></td><td>Finding an example here is not necessarily trivial as it is preferable to have integer solutions that can only be obtained from a quadratic equation that will factorise, so the initial two equations need to be such that this factorisation can take place.
Help1 named **the** method to use. An alternative method would be using the graphical method that would help considerably to visualise the solution, though with a potential loss of accuracy. Plea1's first post does not state what method they are using but later messages from Plea1 do not indicate that they had originally been using graphical means.</td></tr>
<tr><td colspan="4">Having received this message 'plea' works on the questions and 33 minutes later reports back (see below) on question 1 including all the algebraic working and realising that something has gone wrong as the equation obtained does not factorise. 'Plea1' sends his message and whilst waiting for any further responses from anyone offering more help, 'Plea1' continues to work on the second of the question. Although the equation now obtained does factorise, the solution values are not the ones stated with the question. 'Plea1' posts another message.</td></tr>
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3	8.46pm Plea1: I'm sure I've made a really silly mistake somewhere but I got as far as this [lots of algebraic working] ….. $\frac{4}{3}y^2+\frac{4}{3}y+6\frac{5}{9}=0$ and it didn't factorise… i'm not sure what happened.		'Plea1's workings show they have clearly understood the method but as they suggest 'I'm sure I've made a silly mistake somewhere' they have indeed made an error in the second line. This error is subsequently picked up and commented upon by two different people.
4	8.54pm Plea1: and for q 2 [lots of algebraic working] $x=\frac{1}{3}$ *or* $x=-0.5$ THESE AREN'T THE RIGHT ANSWERS Thanks in advance.		Another error in multiplying out a bracket in line 2 has caused this, though there will be further complications after this has been corrected There is a sense of frustration setting in with the use of capital letters for 'THESE AREN'T THE RIGHT ANSWERS'. Again there is a polite 'thanks in advance'.
A new person (making their first ever post!) comes in to help, responding to both messages a further one hour and four minutes after the second of the two messages is sent. Both errors have been spotted.			
5	9.58pm Help2: For 1 you've just made a mistake in expanding the expression, can you see it? For 2 in a few steps you have divided/multiplied by x, which means that you have to check the extra case x=0. Additionally you've made a silly mistake in expanding $5x((1+x)/3)$ Can you solve it now?		'Plea1' is told that for the first question they have made an algebraic error and instead of being told precisely where it is, or indeed what it is, they are asked whether they can find it. The position of the algebraic error in the second question is given though again no correction is offered. The error sets up a cubic equation for which it looks possible that each term can be divided by x and reduced to a quadratic. The correct equation will allow a similar division and 'Help2' is anticipating a universal common error of doing the division in both these circumstances and forgetting that the equation would also be true if $x=0$. Help2's comment 'you've made a silly mistake' could be viewed as a 'put down' and thus a potential dent to 'Plea1's confidence. However (a) Plea1 has already used the words 'silly mistake' in the previous message and (b) making 'silly mistakes' is accepted as part of the mathematical processes [seen again in 3Thds]. There is an implicit invite to come back if still unsure. This is the second invitation within the same message as with 'can you see it?' earlier.

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Plea1: For 2 I can see that it goes to $(-5x - 5x^2) / 3$ which then [placed] in the equation [then manipulated] and cancelled down goes to $x^2 = 5x$ is that correct?
and if so, what is the algebraic way then, to solve it to make 5, and 0. i can see how the numbers go in but not how to solve it algebraically</td><td></td><td>‘Plea1’ has continued to work through correctly to $x^2 = 5x$ but is now stuck as it is not quite the usual straightforward quadratic - it requires the anticipated difficulty that ‘Help2’ has given.
It is clear from ‘Plea1’ that this is new or at least the format is unrecognisable to them, but they appear obviously determined to understand it as they ‘see how the numbers go in but not how to solve it’.</td></tr>
<tr><td colspan="4">The forum’s moderator (a trained and experienced teacher) joins in and has also picked up the algebraic error in question 1 and suggests how ‘Plea1’ may be able to find their own error.</td></tr>
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Moderator: For 1, I suspect it’s the sort of blunder you become blind to when going back, because you’re too busy checking the steps you did do. So try this:
Expand $(y + 2 / 3)^2$
NOW look again at $(2 / 3y)^2$ ….</td><td></td><td>The moderator suggests to ‘Plea1’ that they should try to expand two expressions that if expanded correctly will signal the error.
‘Plea1’ workings show that they have expanded $(y + 2 / 3)^2$ when they should have expanded $(2 / 3y)^2$</td></tr>
<tr><td colspan="4">Postings to the forum are beginning to overlap with help arriving from different sources almost simultaneously. Eight minutes after ‘Plea1’ has posted a message about arriving at $x^2 = 5x$ and being stuck, Help2 responds again. This gives the impression that Help2 is on-line and looking out for ‘Plea1’s messages, ready to respond. If this is true then this is similar to teacher and student being together in a classroom.</td></tr>
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Help2: For 2 we consider the following cases:
1 $x = 0$, which obviously gives a solution for every y
2 $x \neq 0$, then: [algebraic working] $x = 5$
and it is easy to calculate that $y = 2$ then.</td><td></td><td>This is the first time that ‘Plea1’ has simply been told the answer as ‘Help2’ gives the solution to this special case. Why Help2 choose to do this is not clear. Help2 as a new poster is inexperienced in using the posting protocols of offering only hints and present the solution. Alternatively ‘Help2’ could have made the judgement that $x = 0$ is new territory for ‘Plea’1 and an explanation is required that can only be given through ‘Plea1’ looking at the worked solution or that. There would be nothing ‘wrong’ with such giving the solution in these circumstances if indeed it enables ‘Plea1’ to be able to a similar problem unaided next time, there is simply no way of knowing whether ‘Plea1’ will be able to do so.</td></tr>
<tr><td colspan="4">The next post made by ‘Plea1’ at 11.33pm – 1 hour and 18 minutes later – indicates that the question is now understood and thus the ‘nothing wrong’ comment above appears justified.</td></tr>
<tr><td>9</td><td>11.33pm
Plea1: Thank you. I really understand 2 now. Number 1 is coming to me too … taking a little more time.</td><td></td><td>By now ‘Plea1’ has been working on the questions for some two and three quarter hours - and although it is possible that been breaks in between, the postings have been intensive during the time period. In this last post of the evening ‘Plea1’ adds that question 1 is becoming clearer though ‘taking a little more time’. It would thus appear that ‘Plea1’ is still content to work on the problem that had been there from the beginning.</td></tr>
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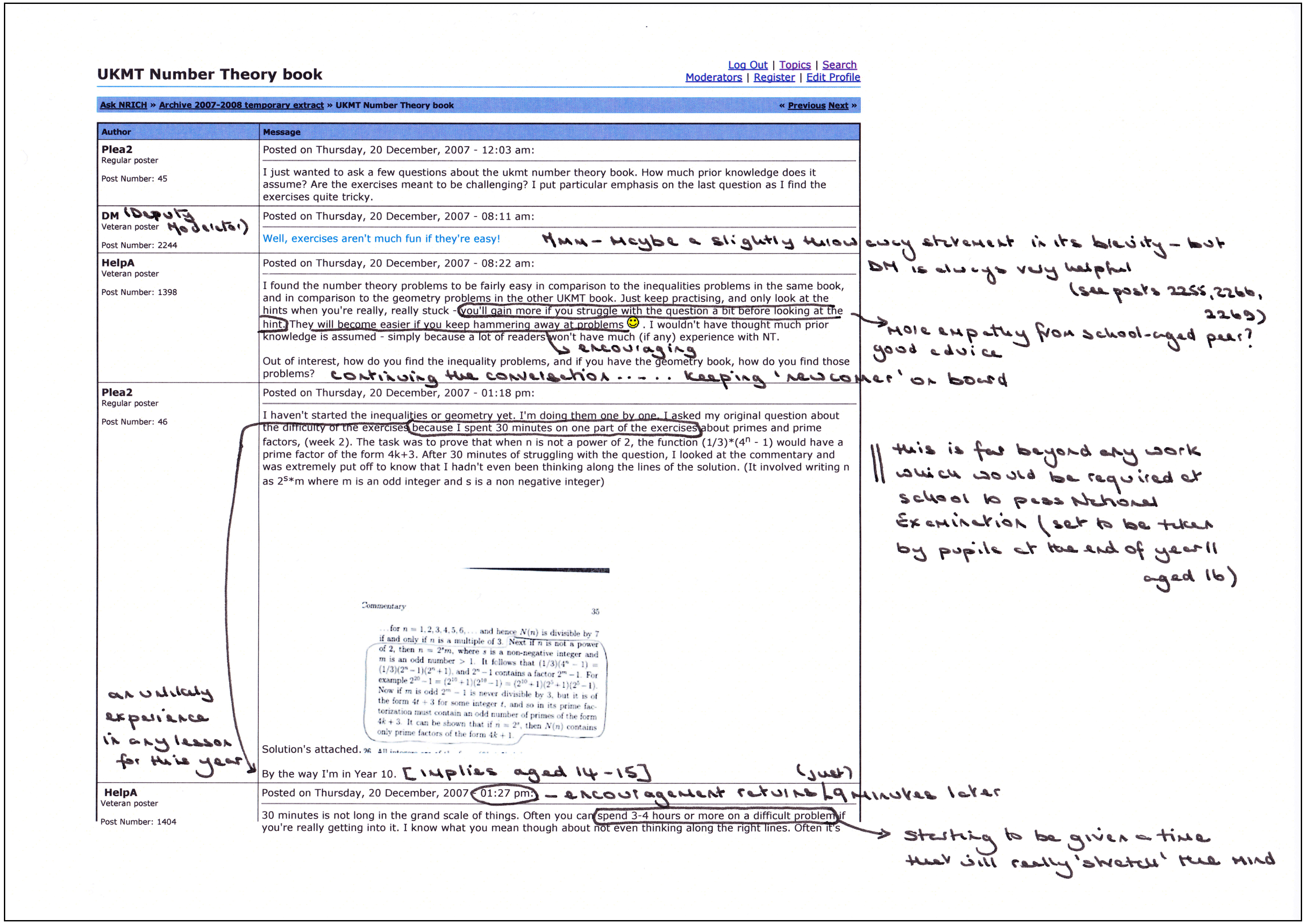
Plea1: Thak [sic] you so much. I understand it all now</td><td></td><td>Within and over 24 hours, out of school hours, explanations, hints and one direct solution have been offered to enable ‘Plea1’ to ‘understand’ the problems. It is not possible to measure the depth of understanding but it is assumed posting such a comment indicates that at least the poster believes they understand. Taking the argument one step further there is the additional inference that they had had a desire to understand and thus was seeking to do so.</td></tr>
</table>

The person who has posed the original question is denoted as ‘Plea2’. People who offer help are designated as Help x with x increasing as each new help person joins. The day and time of the message is indicated that the start of each message.

No	Message		Interpretative Commentary
1	Thursday 12.03am Plea2: I just wanted to ask a few questions about the ukmt number theory book. How much prior knowledge does it assume? Are the exercises meant to be challenging? I put particular emphasis on the last question as I find the exercises quite tricky		The date of this post was five days before Christmas and at a time when schools had broken (or were on the brink of breaking) up for the holidays. The time three minutes after midnight
Deputy moderator posts a brief reply at 8.11am ‘Well exercises aren’t that much fun if they’re easy!. A fuller response by a peer arrives 11 minutes after this.			
2	8.22am Help1: I found the number theory problems to be fairly easy in comparison to the inequalities problems in the same book, and in comparison to the geometry problems in the other UKMT book. Just keep practising, and only look at the hints when you're really, really stuck - you'll gain more if you struggle with the question a bit before looking at the hint. They will become easier if you keep hammering away at problems ☺. I wouldn't have thought much prior knowledge is assumed - simply because a lot of readers won't have much (if any) experience with NT. Out of interest, how do you find the inequality problems, and if you have the geometry book, how do you find those problems?		This post is from an AskNRICH user who has already (though relatively recently) had the experience of undertaking all the studies that Plea is intending to do - a more experienced peer Sincere, genuine, encouraging advice Engaging ‘Plea2’ in conversation
3	1.18pm Plea2: I haven’t started the inequalities or geometry yet. I am doing them one by one. I asked my original question about the difficulty of the exercises because I spent 30 minutes on one part of the exercise about primes and prime factors (week 2). The task was to prove that when n is a power of 2, the function (1/3)*(4^n-1) would have a prime factor of the form 4k+3. After 30 minutes of struggling with the question I looked at the commentary and was extremely put off to know that I had not even been thinking along the lines of the solution. It involved writing n as 2s*m where m is an odd integer and s is a non-negative integer. (Includes scanned image of commentary). By the way I am in year 10		The final sentence provides the evidence that Plea is seeking to increase his own knowledge outside of ‘normal’ school lessons. The work being studied here is far more involved than topics generally studied at GCSE level for passing the examination.. Currently it would appear that ‘plea’ is concerned that 30 minutes to work on a solution without much headway is a serious deficiency on their personal mathematical skills. Almost in passing: ‘Plea2’ reveals he is in year 10 and thus assumed to be 14 to 15 years old.

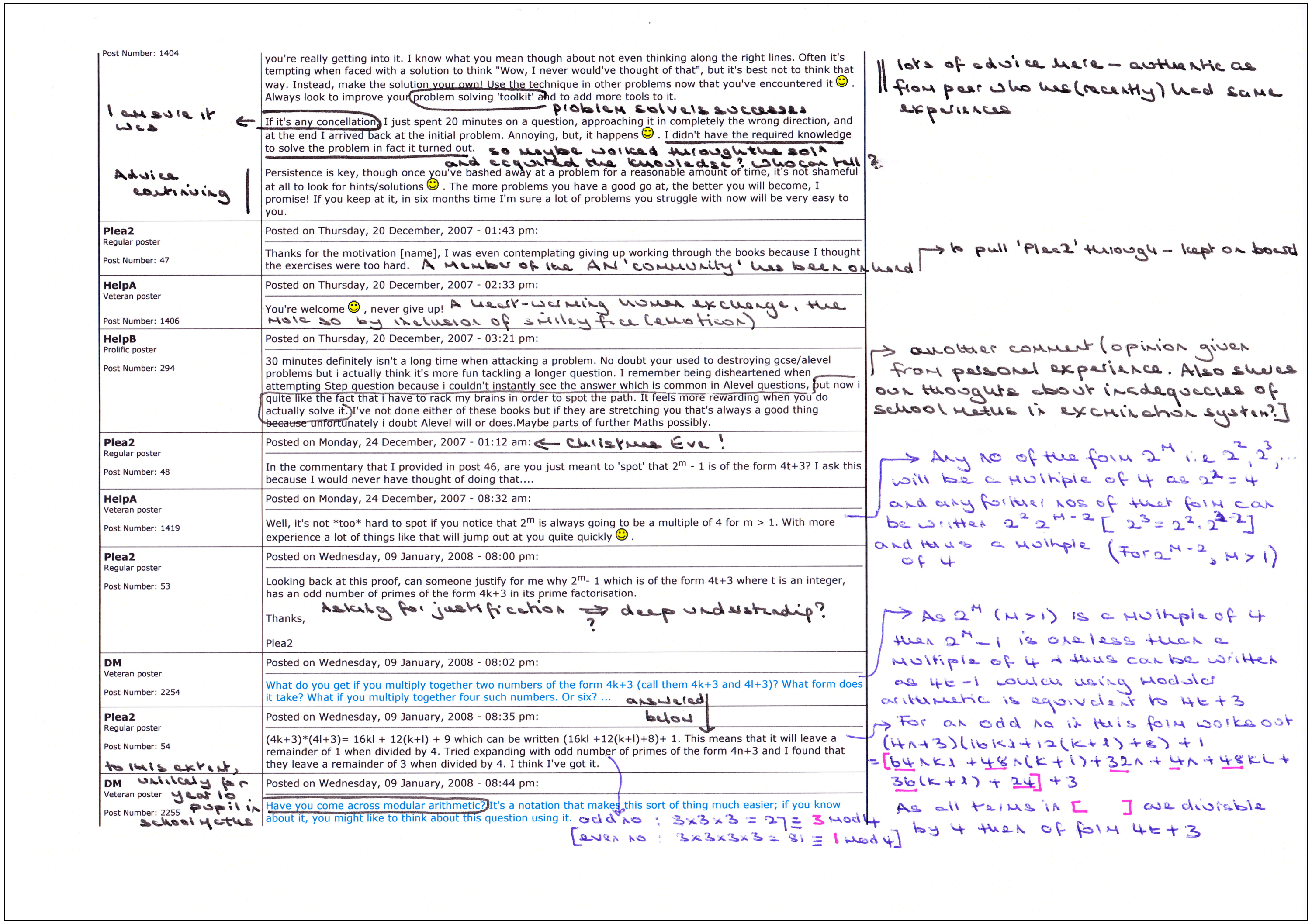
No	Message		Interpretative Commentary
‘Plea’ only needs to wait some 9 minutes before ‘help1’ responds with a reply.			
4	1.27pm Help1: 30 minutes is not long in the grand scale of things. Often you can spend 3-4 hours or more on a difficult problem if you’re really getting into it. I know what you mean though about not even thinking along the right lines. Often it’s tempting when faced with a solution to think “Wow, I never would’ve thought of that”, but it’s best not to think in that way. Instead, make the solution your own! Use the technique in other problems now that you’ve encountered it ☺. Always look to improve your problem solving ‘toolkit’ and to add more tools to it. If it’s any concellation [sic], I just spent 20 minutes on a question, approaching it in completely the wrong direction, and at the end I arrived back at the initial problem. Annoying, but, it happens. [Inserts smiley face]. I didn't have the required knowledge to solve the problem in fact it turned out. Persistence is key, though once you've bashed away at a problem for a reasonable amount of time, it's not shameful at all to look for hints/solutions. ☺. The more problems you have a good go at, the better you will become, I promise! If you keep at it, in six months time I'm sure a lot of problems you struggle with now will be very easy to you.		Help1’ empathising with ‘Plea2’ - being positive, reassuring and supportive Specific advice Highlights his own up-to-the-minute difficulties in attempting a ‘hard’ problem without it turns out sufficient mathematical knowledge to solve it Encouragement –welcoming
5 & 6	1.43pm: Plea2: Thanks for the motivation. I was even contemplating giving up working through the books because I thought the exercises were too hard. 2.33pm: Help1: You’re welcome ☺, never give up!		This short exchange has done much to keep ‘Plea2’ on board – thanks entirely to a fellow user who has been prepared to spend time helping a younger and more inexperienced student wanting to do more mathematics

No	Message		Interpretative Commentary
A little under an hour later Help2 - a second more experienced peer - joins in to support ‘Plea2’ and reiterate some of the things that Help1 has been saying.			
7	3.21pm Help2: 30 minutes definitely isn't a long time when attacking a problem. No doubt your [sic] used to destroying gcse/alevel problems but i actually think it's more fun tackling a longer question. I remember being disheartened when attempting Step question because i couldn't instantly see the answer which is common in Alevel questions, but now i quite like the fact that i have to rack my brains in order to spot the path. It feels more rewarding when you do actually solve it. I've not done either of these books but if they are stretching you that's always a good thing because unfortunately i doubt Alevel will or does. Maybe parts of further Maths possibly.		Recall own feelings when faced with examination questions set for Oxbridge mathematics candidates. The ‘carrot’ is there in the reward of good feeling when the problem is eventually solved. The word ‘fun’ has come into the conversation. Help2 is obviously not a great fan of school mathematics examinations citing ‘destroying’ questions (implying ‘in no time flat’) and doubting whether A level can stretch. Even some parts of Further Maths are implied here as not being too stretching (at least for this particular person).
The exchanges above were all on December 20th. ‘Plea2’s next message comes in the early hours of Christmas Eve. The focus is still on the ‘mystery’ of the solution.			
8	Christmas Eve 1.12am Plea2: In the commentary that I provided in post 46, are you just meant to 'spot' that 2^m- 1 is of the form 4t+3? I ask this because I would never have thought of doing that....		This year 10 pupil appears to be burning the midnight oil?
9	8.32am Help1: Well, it's not *too* hard to spot if you notice that 2^m is always going to be a multiple of 4 for m > 1. With more experience a lot of things like that will jump out at you quite quickly ☺.		Help1 manages to reply around breakfast time of the same day. Clue to 2^m-1 given by actually discussing 2^m which should lead ‘plea’ to 4t+3 senses where ‘plea’ is ‘stuck. Help 1 remains encouraging.
There are no more posts on this until 9th January when ‘Plea2’ begins to ask more questions. Initially, the deputy moderator responds and asks whether ‘plea’ is familiar with using modular arithmetic. Two other users join in until ‘Plea3’ a year 9 pupil (13 to 14 years old) who is trying to do the same problems suggests one solution but then asks for help on the ‘bit’ that they do not understand. By the end of the thread a further 32 posts had been made between 9th and the last on 28th January. The conversation settled down to be between the two ‘young’ users (‘Plea2’ and ‘Plea3’) and the deputy moderator and one helper (whose help was at times seemed slightly confusing). Help1 posted one more statement. The last two posts of the thread came from each of the people trying to do the questions. In summary, two users were asking all the needing-to-know questions, three helpers made a substantial number of posts and four others added one post each.			



UKMT Number Theory book

Author	Message
Plea2 Regular poster Post Number: 45	Posted on Thursday, 20 December, 2007 - 12:03 am: I just wanted to ask a few questions about the ukmt number theory book. How much prior knowledge does it assume? Are the exercises meant to be challenging? I put particular emphasis on the last question as I find the exercises quite tricky.
DM (Deputy Moderator) Veteran poster Post Number: 2244	Posted on Thursday, 20 December, 2007 - 08:11 am: Well, exercises aren't much fun if they're easy! Mmm - maybe a slightly throwaway statement in its brevity - but DM is always very helpful (see posts 2255, 2266, 2269)
HelpA Veteran poster Post Number: 1398	Posted on Thursday, 20 December, 2007 - 08:22 am: I found the number theory problems to be fairly easy in comparison to the inequalities problems in the same book, and in comparison to the geometry problems in the other UKMT book. Just keep practising, and only look at the hints when you're really, really stuck - you'll gain more if you struggle with the question a bit before looking at the hint. They will become easier if you keep hammering away at problems 🙂 . I wouldn't have thought much prior knowledge is assumed - simply because a lot of readers won't have much (if any) experience with NT. encouraging → More empathy from school-aged peer? good advice Out of interest, how do you find the inequality problems, and if you have the geometry book, how do you find those problems? continuing the conversation ..... keeping 'newcomer' on board
Plea2 Regular poster Post Number: 46 an unlikely experience in any lesson for this year	Posted on Thursday, 20 December, 2007 - 01:18 pm: I haven't started the inequalities or geometry yet. I'm doing them one by one. I asked my original question about the difficulty of the exercises because I spent 30 minutes on one part of the exercises about primes and prime factors, (week 2). The task was to prove that when n is not a power of 2, the function (1/3)*(4^n - 1) would have a prime factor of the form 4k+3. After 30 minutes of struggling with the question, I looked at the commentary and was extremely put off to know that I hadn't even been thinking along the lines of the solution. (It involved writing n as 2^s*m where m is an odd integer and s is a non negative integer) ‖ this is far beyond any work which would be required at school to pass National Examination (set to be taken by pupils at the end of year 11 aged 16) *Commentary* 35 ...for $n = 1, 2, 3, 4, 5, 6, \ldots$ and hence $N(n)$ is divisible by 7 if and only if n is a multiple of 3. Next if n is not a power of 2, then $n = 2^s m$, where s is a non-negative integer and m is an odd number > 1. It follows that $(1/3)(4^n - 1) = (1/3)(2^n - 1)(2^n + 1)$, and $2^n - 1$ contains a factor $2^m - 1$. For example $2^{20} - 1 = (2^{10} + 1)(2^{10} - 1) = (2^{10} + 1)(2^5 + 1)(2^5 - 1)$. Now if m is odd $2^m - 1$ is never divisible by 3, but it is of the form $4t + 3$ for some integer t, and so in its prime factorization must contain an odd number of primes of the form $4k + 3$. It can be shown that if $n = 2^s$, then $N(n)$ contains only prime factors of the form $4k + 1$. Solution's attached. By the way I'm in Year 10. [implies aged 14-15]
HelpA Veteran poster Post Number: 1404	Posted on Thursday, 20 December, 2007 - 01:27 pm: - encouragement returns (just) 9 minutes later 30 minutes is not long in the grand scale of things. Often you can spend 3-4 hours or more on a difficult problem if you're really getting into it. I know what you mean though about not even thinking along the right lines. Often it's → starting to be given a time that will really 'stretch' the mind



Post Number: 1404

you're really getting into it. I know what you mean though about not even thinking along the right lines. Often it's tempting when faced with a solution to think "Wow, I never would've thought of that", but it's best not to think that way. Instead, make the solution your own! Use the technique in other problems now that you've encountered it 🙂 . Always look to improve your problem solving 'toolkit' and to add more tools to it.

problem solvers successes

If it's any concellation, I just spent 20 minutes on a question, approaching it in completely the wrong direction, and at the end I arrived back at the initial problem. Annoying, but, it happens 🙂 . I didn't have the required knowledge to solve the problem in fact it turned out. so maybe worked through the sol^n and acquired the knowledge? Who can tell ?

I am sure it was ←

Persistence is key, though once you've bashed away at a problem for a reasonable amount of time, it's not shameful at all to look for hints/solutions 🙂 . The more problems you have a good go at, the better you will become, I promise! If you keep at it, in six months time I'm sure a lot of problems you struggle with now will be very easy to you.

Advice continuing

lots of advice here – authentic as from peer who has (recently) had same experience

Plea2
Regular poster
Post Number: 47

Posted on Thursday, 20 December, 2007 - 01:43 pm:

Thanks for the motivation [name], I was even contemplating giving up working through the books because I thought the exercises were too hard. A member of the AN 'community' has been on hand → to pull 'Plea2' through – kept on board

HelpA
Veteran poster
Post Number: 1406

Posted on Thursday, 20 December, 2007 - 02:33 pm:

You're welcome 🙂 , never give up! A heart-warming human exchange, the more so by inclusion of smiley face (emoticon)

HelpB
Prolific poster
Post Number: 294

Posted on Thursday, 20 December, 2007 - 03:21 pm:

30 minutes definitely isn't a long time when attacking a problem. No doubt your used to destroying gcse/alevel problems but i actually think it's more fun tackling a longer question. I remember being disheartened when attempting Step question because i couldn't instantly see the answer which is common in Alevel questions, but now i quite like the fact that i have to rack my brains in order to spot the path. It feels more rewarding when you do actually solve it. I've not done either of these books but if they are stretching you that's always a good thing because unfortunately i doubt Alevel will or does.Maybe parts of further Maths possibly.

→ another comment (opinion given from personal experience. Also shows our thoughts about inadequacies of school maths in examination system?]

Plea2
Regular poster
Post Number: 48

Posted on Monday, 24 December, 2007 - 01:12 am: ← Christmas Eve!

In the commentary that I provided in post 46, are you just meant to 'spot' that 2^m - 1 is of the form 4t+3? I ask this because I would never have thought of doing that....

HelpA
Veteran poster
Post Number: 1419

Posted on Monday, 24 December, 2007 - 08:32 am:

Well, it's not *too* hard to spot if you notice that 2^m is always going to be a multiple of 4 for m > 1. With more experience a lot of things like that will jump out at you quite quickly 🙂 .

→ Any no of the form 2^m i.e $2^2, 2^3, \ldots$ will be a multiple of 4 as $2^2 = 4$ and any further nos of that form can be written $2^2 2^{m-2}$ $[2^3 = 2^2 . 2^{3-2}]$ and thus a multiple of 4 (for 2^{m-2}, m > 1)

Plea2
Regular poster
Post Number: 53

Posted on Wednesday, 09 January, 2008 - 08:00 pm:

Looking back at this proof, can someone justify for me why 2^m- 1 which is of the form 4t+3 where t is an integer, has an odd number of primes of the form 4k+3 in its prime factorisation.

Thanks, Asking for justification $\overset{?}{\Rightarrow}$ deep understanding?

Plea2

DM
Veteran poster
Post Number: 2254

Posted on Wednesday, 09 January, 2008 - 08:02 pm:

What do you get if you multiply together two numbers of the form 4k+3 (call them 4k+3 and 4l+3)? What form does it take? What if you multiply together four such numbers. Or six? ... answered below

→ As 2^m (m > 1) is a multiple of 4 then $2^m - 1$ is one less than a multiple of 4 & thus can be written as 4t − 1 which using modular arithmetic is equivalent to 4t + 3

Plea2
Regular poster
Post Number: 54

Posted on Wednesday, 09 January, 2008 - 08:35 pm:

(4k+3)*(4l+3)= 16kl + 12(k+l) + 9 which can be written (16kl +12(k+l)+8)+ 1. This means that it will leave a remainder of 1 when divided by 4. Tried expanding with odd number of primes of the form 4n+3 and I found that they leave a remainder of 3 when divided by 4. I think I've got it.

→ For an odd no in this form work out

$(4n+3)(16kl+12(k+l)+8)+1$

$= [\underline{64}nkl + \underline{48}n(k+l) + \underline{32}n + \underline{4}n + \underline{48}kl + \underline{36}(k+l) + \underline{24}] + 3$

As all terms in [ ] are divisible by 4 then of form 4t + 3

to this extent, unlikely for year 10 pupil in school maths

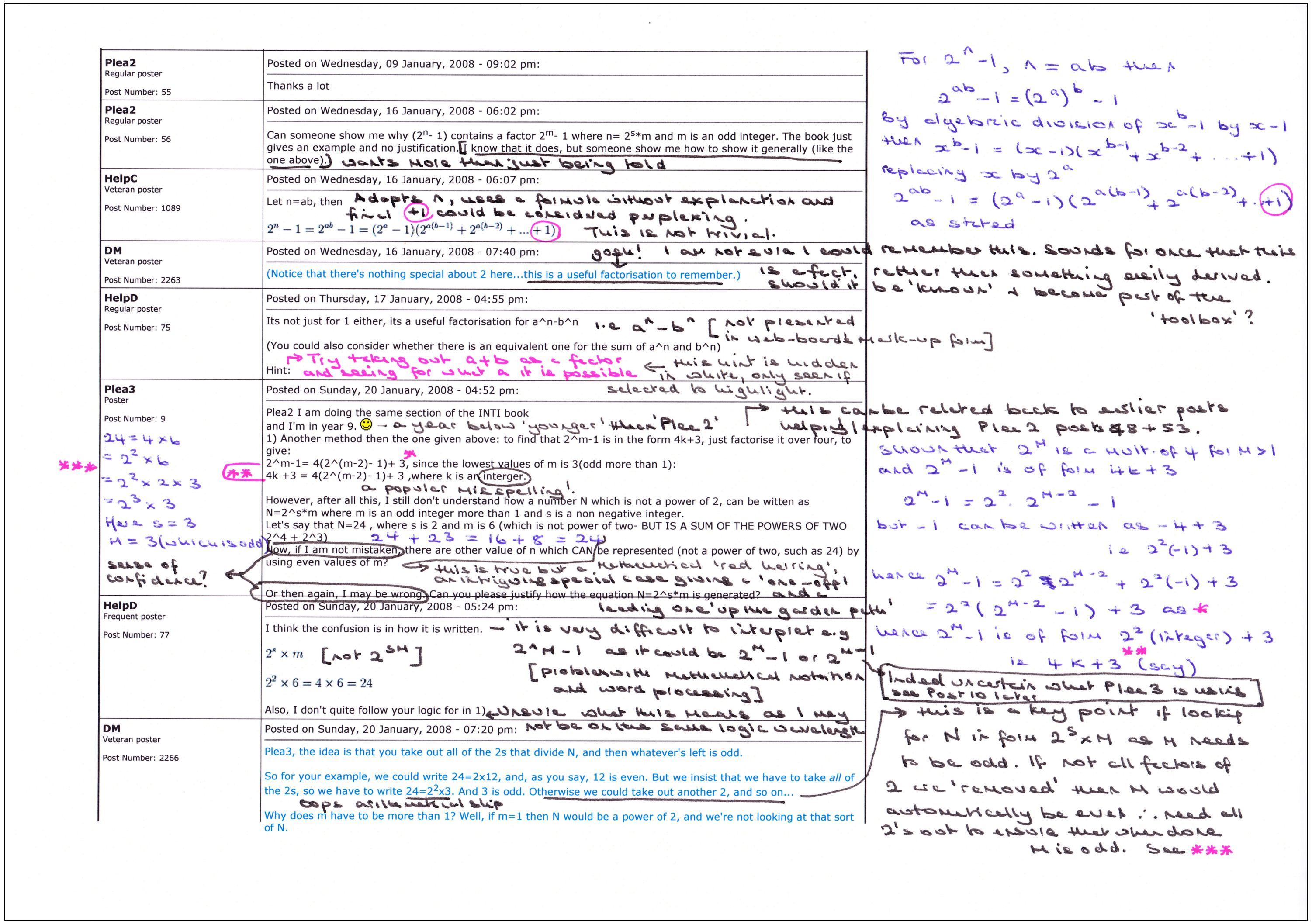
DM
Veteran poster
Post Number: 2255

Posted on Wednesday, 09 January, 2008 - 08:44 pm:

Have you come across modular arithmetic? It's a notation that makes this sort of thing much easier; if you know about it, you might like to think about this question using it.

odd no : $3 \times 3 \times 3 = 27 \equiv 3 \bmod 4$

[even no : $3 \times 3 \times 3 \times 3 = 81 \equiv 1 \bmod 4$]



Plea2
Regular poster
Post Number: 55

Posted on Wednesday, 09 January, 2008 - 09:02 pm:

Thanks a lot

Plea2
Regular poster
Post Number: 56

Posted on Wednesday, 16 January, 2008 - 06:02 pm:

Can someone show me why (2^n- 1) contains a factor 2^m- 1 where n= 2^s*m and m is an odd integer. The book just gives an example and no justification. [I know that it does, but someone show me how to show it generally (like the one above).] wants more than just being told

HelpC
Veteran poster
Post Number: 1089

Posted on Wednesday, 16 January, 2008 - 06:07 pm:

Let n=ab, then Adopts n, uses a formula without explanation and final +1 could be considered perplexing.

$2^n - 1 = 2^{ab} - 1 = (2^a - 1)(2^{a(b-1)} + 2^{a(b-2)} + \ldots + 1)$ This is not trivial.

DM
Veteran poster
Post Number: 2263

Posted on Wednesday, 16 January, 2008 - 07:40 pm: gosh! I am not sure I could remember this. Sounds for once that there is a fact, rather than something easily derived. should it be 'known' & become part of the 'toolbox'?

(Notice that there's nothing special about 2 here...this is a useful factorisation to remember.)

HelpD
Regular poster
Post Number: 75

Posted on Thursday, 17 January, 2008 - 04:55 pm:

Its not just for 1 either, its a useful factorisation for a^n-b^n i.e $a^n - b^n$ [not presented in web-boards mark-up form]

(You could also consider whether there is an equivalent one for the sum of a^n and b^n)

Hint: → Try taking out a+b as a factor and seeing for what n it is possible ← this hint is hidden in white, only seen if selected to highlight.

Plea3
Poster
Post Number: 9

$24 = 4 \times 6$
$= 2^2 \times 6$
*** $= 2^2 \times 2 \times 3$
$= 2^3 \times 3$
Here s = 3
m = 3 (which is odd)

sense of confidence?

Posted on Sunday, 20 January, 2008 - 04:52 pm:

Plea2 I am doing the same section of the INTI book
and I'm in year 9. ☺ → a year below 'younger' than 'Plea 2' → this can be related back to earlier posts helping/explaining Plea2 posts 48 + 53.
1) Another method then the one given above: to find that 2^m-1 is in the form 4k+3, just factorise it over four, to give:
2^m-1= 4(2^(m-2)- 1)+ 3, * since the lowest values of m is 3(odd more than 1):
** 4k +3 = 4(2^(m-2)- 1)+ 3 ,where k is an interger. a popular misspelling!
However, after all this, I still don't understand how a number N which is not a power of 2, can be witten as N=2^s*m where m is an odd integer more than 1 and s is a non negative integer.
Let's say that N=24 , where s is 2 and m is 6 (which is not power of two- BUT IS A SUM OF THE POWERS OF TWO 2^4 + 2^3) $24 + 23 = 16 + 8 = 24$
Now, if I am not mistaken, there are other value of n which CAN be represented (not a power of two, such as 24) by using even values of m? → this is true but a mathematical 'red herring', an intriguing special case giving a 'one-off' and a leading one 'up the garden path'

Or then again, I may be wrong. Can you please justify how the equation N=2^s*m is generated?

HelpD
Frequent poster
Post Number: 77

Posted on Sunday, 20 January, 2008 - 05:24 pm:

I think the confusion is in how it is written. — It is very difficult to interpret e.g 2^m-1 as it could be 2^m-1 or 2^{m-1} [problem with mathematical notation and word processing]

$2^s \times m$ [not 2^{sm}]

$2^2 \times 6 = 4 \times 6 = 24$

Also, I don't quite follow your logic for in 1) ← unsure what this means as I may not be on the same logic wavelength

DM
Veteran poster
Post Number: 2266

Posted on Sunday, 20 January, 2008 - 07:20 pm:

Plea3, the idea is that you take out all of the 2s that divide N, and then whatever's left is odd.

So for your example, we could write 24=2x12, and, as you say, 12 is even. But we insist that we have to take *all* of the 2s, so we have to write 24=2^2x3. And 3 is odd. Otherwise we could take out another 2, and so on...
oops arithmetical slip

Why does m have to be more than 1? Well, if m=1 then N would be a power of 2, and we're not looking at that sort of N.

For $2^n - 1$, $n = ab$ then

$2^{ab} - 1 = (2^a)^b - 1$

By algebraic division of $x^b - 1$ by $x - 1$
then $x^b - 1 = (x-1)(x^{b-1} + x^{b-2} + \ldots + 1)$
replacing x by 2^a
$2^{ab} - 1 = (2^a - 1)(2^{a(b-1)} + 2^{a(b-2)} + \ldots + 1)$
as stated

shown that 2^m is a mult. of 4 for m > 1
and $2^m - 1$ is of form 4k + 3

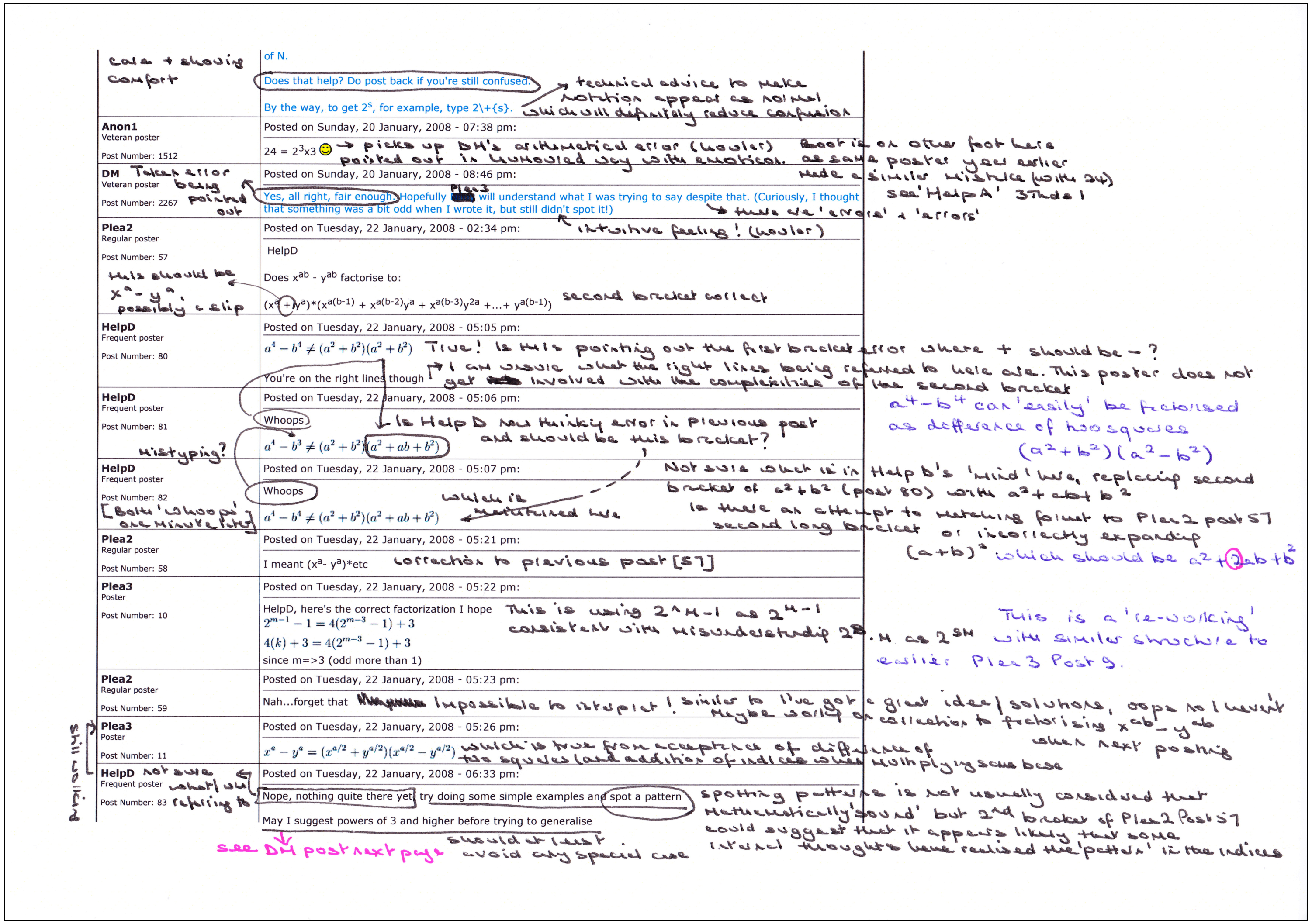
$2^m - 1 = 2^2 . 2^{m-2} - 1$

but −1 can be written as −4 + 3
ie $2^2(-1) + 3$

hence $2^m - 1 = 2^2 \, 2^{m-2} + 2^2(-1) + 3$
$= 2^2(2^{m-2} - 1) + 3$ as *
hence $2^m - 1$ is of form 2^2(integer) + 3
** ie 4k + 3 (say)

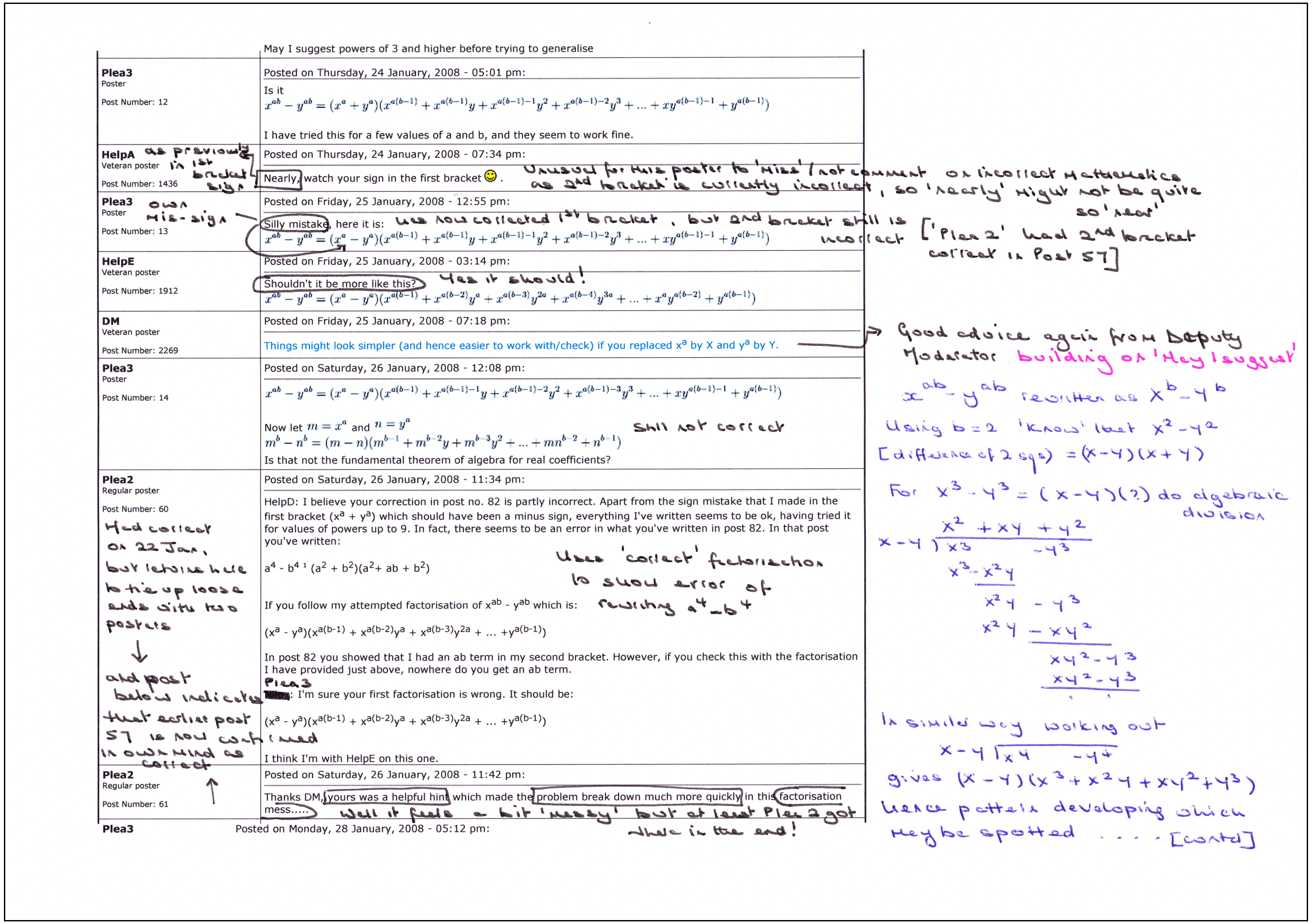
Indeed uncertain what Plea3 is using
see Post 10 later

→ this is a key point if looking for N in form $2^s \times m$ as m needs to be odd. If not all factors of 2 are 'removed' then m would automatically be even ∴ need all 2's out to ensure that when done m is odd. See ***



care + showing comfort	of N. Does that help? Do post back if you're still confused. By the way, to get 2^s, for example, type 2\+{s}. → technical advice to make notation appear as normal which will definitely reduce confusion
Anon1 Veteran poster Post Number: 1512	Posted on Sunday, 20 January, 2008 - 07:38 pm: 24 = 2^3x3 🙂 → picks up DM's arithmetical error (howler) pointed out in humoured way with emoticon. Boot is on other foot here as same poster year earlier made a similar mistake (with 24) see 'HelpA' 3Thds 1
DM Taken error being pointed out Veteran poster Post Number: 2267	Posted on Sunday, 20 January, 2008 - 08:46 pm: Yes, all right, fair enough. Hopefully Plea3 [redacted] will understand what I was trying to say despite that. (Curiously, I thought that something was a bit odd when I wrote it, but still didn't spot it!) → there are 'errors' + 'arrors' intuitive feeling! (howler)
Plea2 Regular poster Post Number: 57 this should be $x^a - y^a$, possibly a slip	Posted on Tuesday, 22 January, 2008 - 02:34 pm: HelpD Does x^{ab} - y^{ab} factorise to: $(x^a+y^a)*(x^{a(b-1)} + x^{a(b-2)}y^a + x^{a(b-3)}y^{2a} +...+ y^{a(b-1)})$ second bracket correct
HelpD Frequent poster Post Number: 80	Posted on Tuesday, 22 January, 2008 - 05:05 pm: $a^4 - b^4 \neq (a^2+b^2)(a^2+b^2)$ True! Is this pointing out the first bracket error where + should be − ? You're on the right lines though ↑ I am unsure what the right lines being referred to here are. This poster does not get involved with the complexities of the second bracket a^4-b^4 can 'easily' be factorised as difference of two squares $(a^2+b^2)(a^2-b^2)$
HelpD Frequent poster Post Number: 81 mistyping?	Posted on Tuesday, 22 January, 2008 - 05:06 pm: Whoops ← Is HelpD now thinking error in previous post and should be this bracket? $a^4 - b^3 \neq (a^2+b^2)\boxed{(a^2+ab+b^2)}$
HelpD Frequent poster Post Number: 82 [Both 'whoops' one minute later]	Posted on Tuesday, 22 January, 2008 - 05:07 pm: Whoops $a^4 - b^4 \neq (a^2+b^2)(a^2+ab+b^2)$ ← which is maintained here Not sure what is in HelpD's 'mind' here, replacing second bracket of a^2+b^2 (post 80) with a^2+ab+b^2. Is there an attempt to matching first to Plea2 post 57 second long bracket or incorrectly expanding $(a+b)^2$ which should be $a^2+2ab+b^2$
Plea2 Regular poster Post Number: 58	Posted on Tuesday, 22 January, 2008 - 05:21 pm: I meant (x^a- y^a)*etc correction to previous post [57]
Plea3 Poster Post Number: 10	Posted on Tuesday, 22 January, 2008 - 05:22 pm: HelpD, here's the correct factorization I hope $2^{m-1}-1=4(2^{m-3}-1)+3$ $4(k)+3=4(2^{m-3}-1)+3$ since m=>3 (odd more than 1) This is using 2^M-1 as 2^{M-1} consistent with misunderstanding $2^S.M$ as 2^{SM} This is a 're-working' with similar structure to earlier Plea3 Post 9.
Plea2 Regular poster Post Number: 59	Posted on Tuesday, 22 January, 2008 - 05:23 pm: Nah...forget that Impossible to interpret! similar to I've got a great idea/solutions, oops no I haven't. Maybe worry on collection to factorising $x^{ab}-y^{ab}$ when next posting
still working **Plea3** Poster Post Number: 11	Posted on Tuesday, 22 January, 2008 - 05:26 pm: $x^a - y^a = (x^{a/2}+y^{a/2})(x^{a/2}-y^{a/2})$ which is true from acceptance of difference of two squares (and addition of indices when multiplying same base
HelpD not sure what/who referring to Frequent poster Post Number: 83	Posted on Tuesday, 22 January, 2008 - 06:33 pm: Nope, nothing quite there yet, try doing some simple examples and spot a pattern May I suggest powers of 3 and higher before trying to generalise should at least avoid any special case. spotting patterns is not usually considered that Mathematically 'sound' but 2nd bracket of Plea2 Post 57 could suggest that it appears likely that some internal thoughts have realised the 'pattern' in the indices

see DM post next page



May I suggest powers of 3 and higher before trying to generalise

Plea3
Poster

Post Number: 12

Posted on Thursday, 24 January, 2008 - 05:01 pm:

Is it
$x^{ab}-y^{ab}=(x^a+y^a)(x^{a(b-1)}+x^{a(b-1)}y+x^{a(b-1)-1}y^2+x^{a(b-1)-2}y^3+...+xy^{a(b-1)-1}+y^{a(b-1)})$

I have tried this for a few values of a and b, and they seem to work fine.

HelpA
Veteran poster

Post Number: 1436

as previously in 1st bracket sign

Posted on Thursday, 24 January, 2008 - 07:34 pm:

Nearly, watch your sign in the first bracket ☺ .

Unusual for this poster to 'miss'/not comment on incorrect mathematics as 2nd bracket is currently incorrect, so 'nearly' might not be quite so 'near'

Plea3
Poster

Post Number: 13

own mis-sign

Posted on Friday, 25 January, 2008 - 12:55 pm:

Silly mistake, here it is: has now corrected 1st bracket, but 2nd bracket still is incorrect ['Plea2' had 2nd bracket correct in Post 57]
$x^{ab}-y^{ab}=(x^a-y^a)(x^{a(b-1)}+x^{a(b-1)}y+x^{a(b-1)-1}y^2+x^{a(b-1)-2}y^3+...+xy^{a(b-1)-1}+y^{a(b-1)})$

HelpE
Veteran poster

Post Number: 1912

Posted on Friday, 25 January, 2008 - 03:14 pm:

Shouldn't it be more like this? Yes it should!
$x^{ab}-y^{ab}=(x^a-y^a)(x^{a(b-1)}+x^{a(b-2)}y^a+x^{a(b-3)}y^{2a}+x^{a(b-4)}y^{3a}+...+x^ay^{a(b-2)}+y^{a(b-1)})$

DM
Veteran poster

Post Number: 2269

Posted on Friday, 25 January, 2008 - 07:18 pm:

Things might look simpler (and hence easier to work with/check) if you replaced x^a by X and y^a by Y.

Good advice again from Deputy Moderator building on 'May I suggest'

$x^{ab}-y^{ab}$ rewritten as X^b-Y^b

Using b=2 'know' that X^2-Y^2

[difference of 2 sqs) $=(X-Y)(X+Y)$

For $X^3-Y^3=(X-Y)(?)$ do algebraic division

X^2+XY+Y^2
$X-Y\,)\,X^3\quad -Y^3$
X^3-X^2Y
X^2Y-Y^3
X^2Y-XY^2
XY^2-Y^3
XY^2-Y^3
$\cdot\quad\cdot$

In similar way working out

$X-Y\,)\,X^4\quad -Y^4$

gives $(X-Y)(X^3+X^2Y+XY^2+Y^3)$

Hence pattern developing which may be spotted ....[contd]

Plea3
Poster

Post Number: 14

Posted on Saturday, 26 January, 2008 - 12:08 pm:

$x^{ab}-y^{ab}=(x^a-y^a)(x^{a(b-1)}+x^{a(b-1)-1}y+x^{a(b-1)-2}y^2+x^{a(b-1)-3}y^3+...+xy^{a(b-1)-1}+y^{a(b-1)})$

Now let $m=x^a$ and $n=y^a$
$m^b-n^b=(m-n)(m^{b-1}+m^{b-2}y+m^{b-3}y^2+...+mn^{b-2}+n^{b-1})$ Still not correct
Is that not the fundamental theorem of algebra for real coefficients?

Plea2
Regular poster

Post Number: 60

Had correct on 22 Jan, but returns here to tie up loose ends with two posters
↓
and post below indicates that earlier post 57 is now confirmed in own mind as correct

Posted on Saturday, 26 January, 2008 - 11:34 pm:

HelpD: I believe your correction in post no. 82 is partly incorrect. Apart from the sign mistake that I made in the first bracket (x^a + y^a) which should have been a minus sign, everything I've written seems to be ok, having tried it for values of powers up to 9. In fact, there seems to be an error in what you've written in post 82. In that post you've written:

a^4 - b$^{4\ 1}$ (a^2 + b^2)(a^2+ ab + b^2)

Uses 'correct' factorisation to show error of rewriting a^4-b^4

If you follow my attempted factorisation of x^{ab} - y^{ab} which is:

(x^a - y^a)(x$^{a(b-1)}$ + x$^{a(b-2)}$y^a + x$^{a(b-3)}$y^{2a} + ... +y$^{a(b-1)}$)

In post 82 you showed that I had an ab term in my second bracket. However, if you check this with the factorisation I have provided just above, nowhere do you get an ab term.

Plea3
[redacted]: I'm sure your first factorisation is wrong. It should be:

(x^a - y^a)(x$^{a(b-1)}$ + x$^{a(b-2)}$y^a + x$^{a(b-3)}$y^{2a} + ... +y$^{a(b-1)}$)

I think I'm with HelpE on this one.

Plea2
Regular poster

Post Number: 61

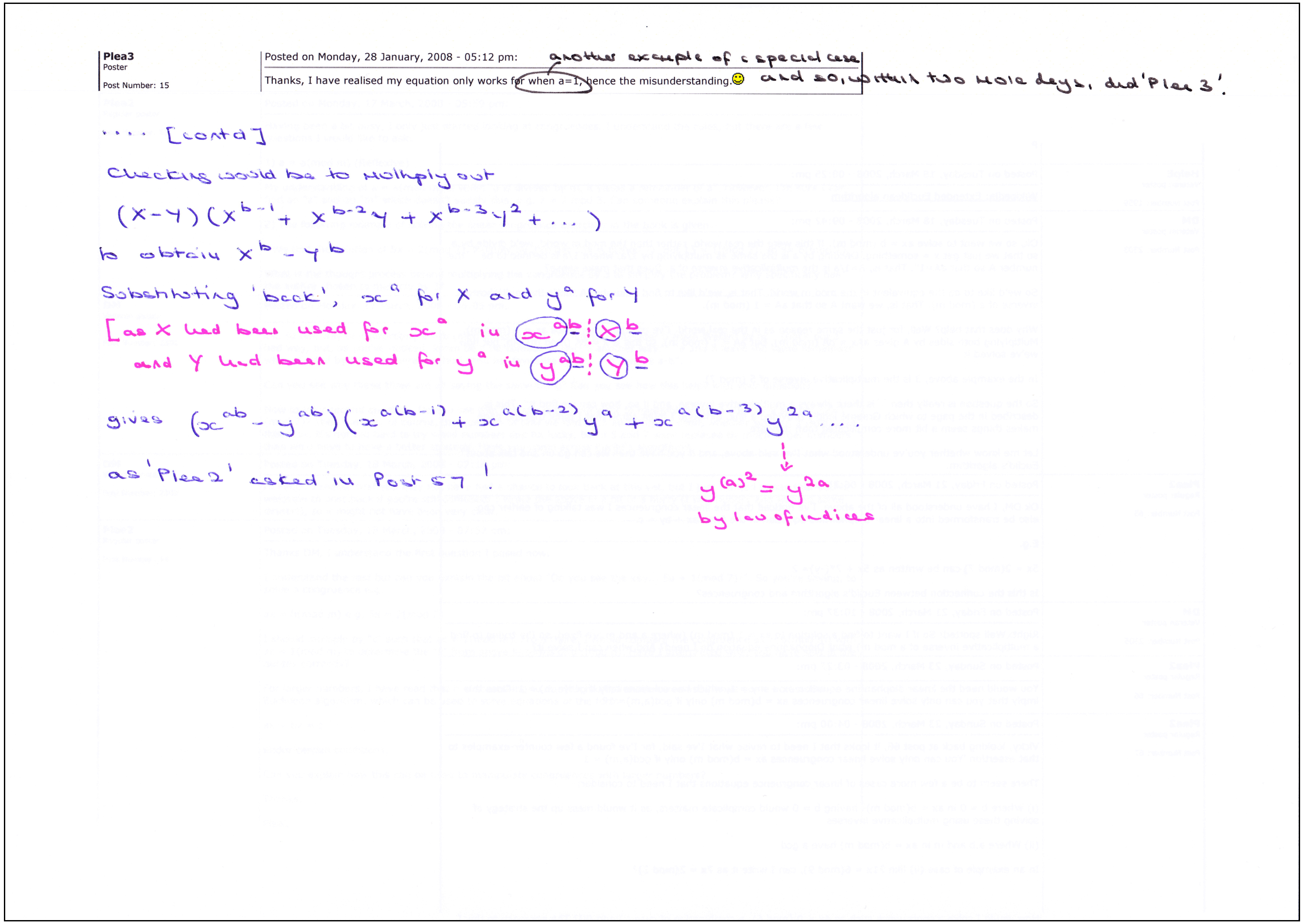
↑

Posted on Saturday, 26 January, 2008 - 11:42 pm:

Thanks DM, yours was a helpful hint which made the problem break down much more quickly in this factorisation mess.....

Well it feels a bit 'messy' but at least Plea2 got there in the end!

Plea3 Posted on Monday, 28 January, 2008 - 05:12 pm:



Plea3
Poster
Post Number: 15

Posted on Monday, 28 January, 2008 - 05:12 pm: another example of a special case

Thanks, I have realised my equation only works for when a=1, hence the misunderstanding.☺ and so, within two more days, did 'Plea 3'.

.... [contd]

Checking would be to multiply out

$(X-Y)(X^{b-1} + X^{b-2}Y + X^{b-3}Y^2 + \ldots)$

to obtain $X^b - Y^b$

Substituting 'back', x^a for X and y^a for Y

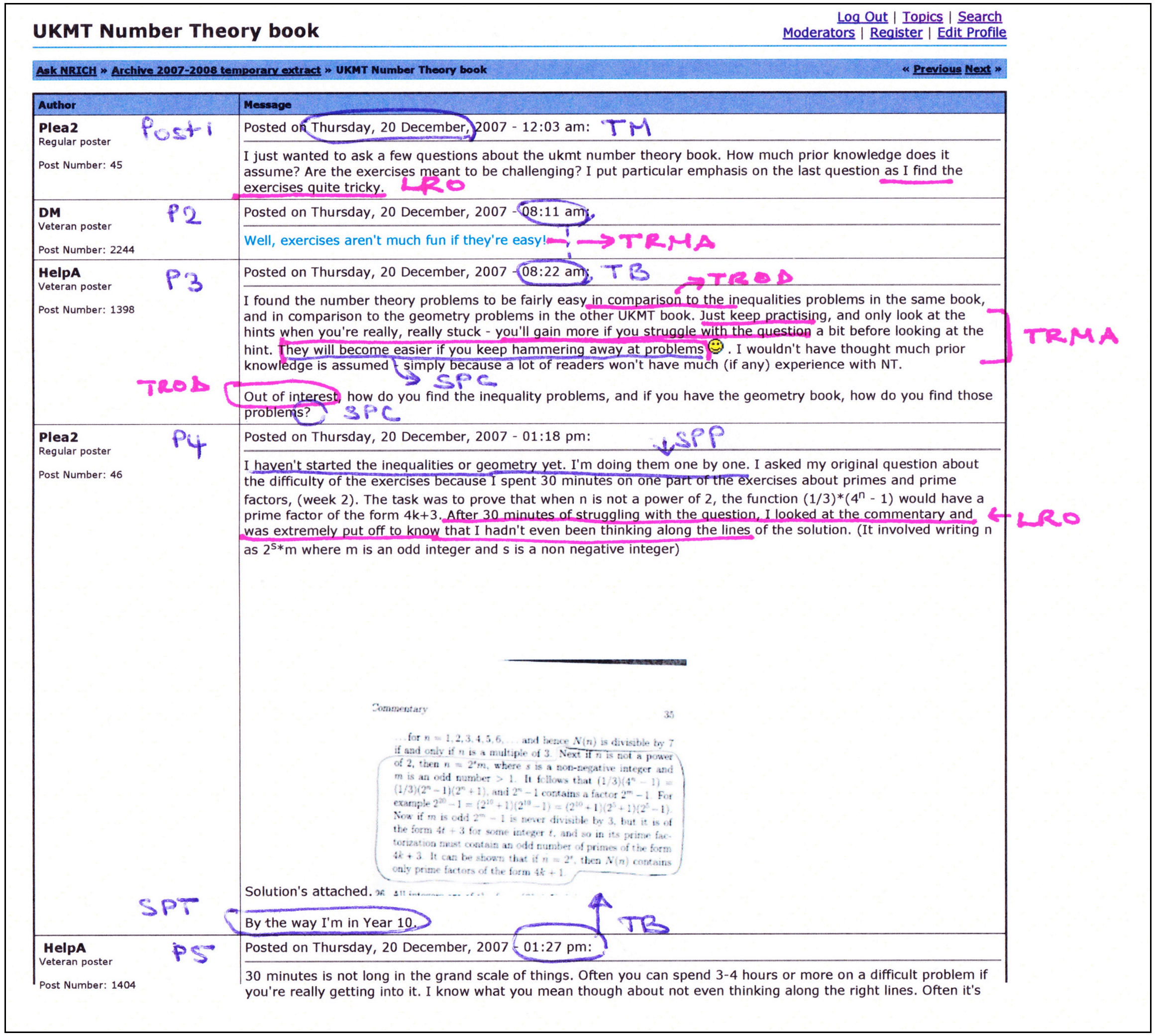
[as X had been used for x^a in $\boxed{x}^{ab} \vdots \boxed{X}^{b}$

and Y had been used for y^a in $\boxed{y}^{ab} \vdots \boxed{Y}^{b}$

gives $(x^{ab} - y^{ab})(x^{a(b-1)} + x^{a(b-2)}y^a + x^{a(b-3)}y^{2a} \ldots$

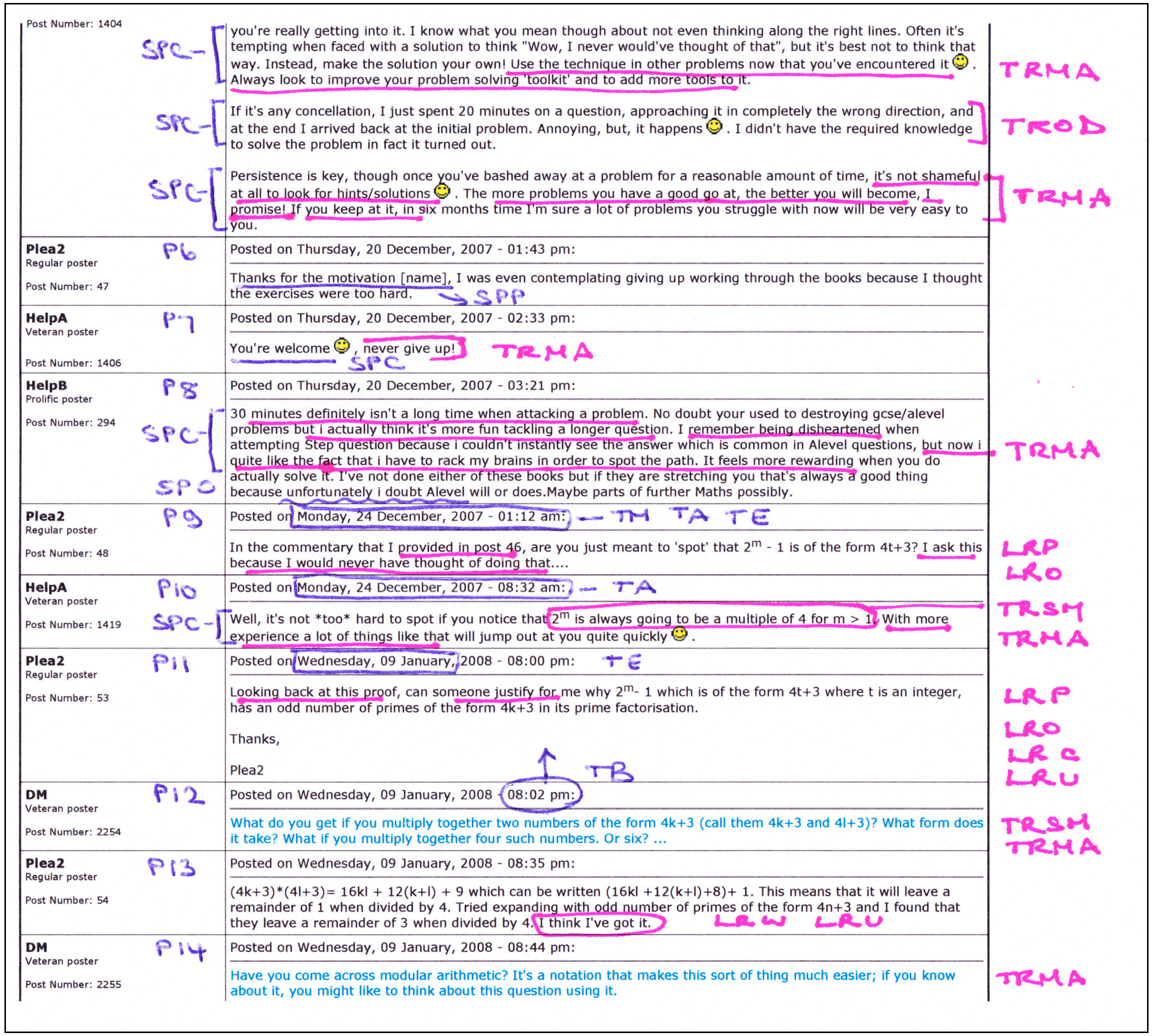
as 'Plea2' asked in Post 57!

$y^{(a)^2} = y^{2a}$
by law of indices



UKMT Number Theory book

Author	Message
Plea2 Regular poster Post Number: 45 Post 1	Posted on Thursday, 20 December, 2007 - 12:03 am: TM I just wanted to ask a few questions about the ukmt number theory book. How much prior knowledge does it assume? Are the exercises meant to be challenging? I put particular emphasis on the last question as I find the exercises quite tricky. LRO
DM Veteran poster Post Number: 2244 P2	Posted on Thursday, 20 December, 2007 - 08:11 am: Well, exercises aren't much fun if they're easy! → TRMA
HelpA Veteran poster Post Number: 1398 P3	Posted on Thursday, 20 December, 2007 - 08:22 am: TB → TROD I found the number theory problems to be fairly easy in comparison to the inequalities problems in the same book, and in comparison to the geometry problems in the other UKMT book. Just keep practising, and only look at the hints when you're really, really stuck - you'll gain more if you struggle with the question a bit before looking at the hint. They will become easier if you keep hammering away at problems 😊. I wouldn't have thought much prior knowledge is assumed - simply because a lot of readers won't have much (if any) experience with NT. TRMA SPC TROD Out of interest, how do you find the inequality problems, and if you have the geometry book, how do you find those problems? SPC
Plea2 Regular poster Post Number: 46 P4	Posted on Thursday, 20 December, 2007 - 01:18 pm: SPP I haven't started the inequalities or geometry yet. I'm doing them one by one. I asked my original question about the difficulty of the exercises because I spent 30 minutes on one part of the exercises about primes and prime factors, (week 2). The task was to prove that when n is not a power of 2, the function (1/3)*(4^n - 1) would have a prime factor of the form 4k+3. After 30 minutes of struggling with the question, I looked at the commentary and was extremely put off to know that I hadn't even been thinking along the lines of the solution. (It involved writing n as 2^s*m where m is an odd integer and s is a non negative integer) ← LRO Commentary 35 …for $n = 1, 2, 3, 4, 5, 6, \ldots$ and hence $N(n)$ is divisible by 7 if and only if n is a multiple of 3. Next if n is not a power of 2, then $n = 2^s m$, where s is a non-negative integer and m is an odd number > 1. It follows that $(1/3)(4^n - 1) = (1/3)(2^n - 1)(2^n + 1)$, and $2^n - 1$ contains a factor $2^m - 1$. For example $2^{20} - 1 = (2^{10} + 1)(2^{10} - 1) = (2^{10} + 1)(2^5 + 1)(2^5 - 1)$. Now if m is odd $2^m - 1$ is never divisible by 3, but it is of the form $4t + 3$ for some integer t, and so in its prime factorization must contain an odd number of primes of the form $4k + 3$. It can be shown that if $n = 2^s$, then $N(n)$ contains only prime factors of the form $4k + 1$. Solution's attached. SPT By the way I'm in Year 10. TB
HelpA Veteran poster Post Number: 1404 P5	Posted on Thursday, 20 December, 2007 - 01:27 pm: 30 minutes is not long in the grand scale of things. Often you can spend 3-4 hours or more on a difficult problem if you're really getting into it. I know what you mean though about not even thinking along the right lines. Often it's



Post Number: 1404

SPC- you're really getting into it. I know what you mean though about not even thinking along the right lines. Often it's tempting when faced with a solution to think "Wow, I never would've thought of that", but it's best not to think that way. Instead, make the solution your own! Use the technique in other problems now that you've encountered it ☺. Always look to improve your problem solving 'toolkit' and to add more tools to it. TRMA

SPC- If it's any concellation, I just spent 20 minutes on a question, approaching it in completely the wrong direction, and at the end I arrived back at the initial problem. Annoying, but, it happens ☺. I didn't have the required knowledge to solve the problem in fact it turned out. TROD

SPC- Persistence is key, though once you've bashed away at a problem for a reasonable amount of time, it's not shameful at all to look for hints/solutions ☺. The more problems you have a good go at, the better you will become, I promise! If you keep at it, in six months time I'm sure a lot of problems you struggle with now will be very easy to you. TRMA

Plea2 P6
Regular poster
Post Number: 47

Posted on Thursday, 20 December, 2007 - 01:43 pm:

Thanks for the motivation [name], I was even contemplating giving up working through the books because I thought the exercises were too hard. → SPP

HelpA P7
Veteran poster
Post Number: 1406

Posted on Thursday, 20 December, 2007 - 02:33 pm:

You're welcome ☺, never give up! TRMA
SPC

HelpB P8
Prolific poster
Post Number: 294

Posted on Thursday, 20 December, 2007 - 03:21 pm:

SPC- 30 minutes definitely isn't a long time when attacking a problem. No doubt your used to destroying gcse/alevel problems but i actually think it's more fun tackling a longer question. I remember being disheartened when attempting Step question because i couldn't instantly see the answer which is common in Alevel questions, but now i quite like the fact that i have to rack my brains in order to spot the path. It feels more rewarding when you do actually solve it. I've not done either of these books but if they are stretching you that's always a good thing because unfortunately i doubt Alevel will or does.Maybe parts of further Maths possibly. TRMA
SPO

Plea2 P9
Regular poster
Post Number: 48

Posted on Monday, 24 December, 2007 - 01:12 am: — TM TA TE

In the commentary that I provided in post 46, are you just meant to 'spot' that $2^m - 1$ is of the form 4t+3? I ask this because I would never have thought of doing that.... LRP LRO

HelpA P10
Veteran poster
Post Number: 1419

Posted on Monday, 24 December, 2007 - 08:32 am: — TA

SPC- Well, it's not *too* hard to spot if you notice that 2^m is always going to be a multiple of 4 for m > 1. With more experience a lot of things like that will jump out at you quite quickly ☺. TRSM TRMA

Plea2 P11
Regular poster
Post Number: 53

Posted on Wednesday, 09 January, 2008 - 08:00 pm: TE

Looking back at this proof, can someone justify for me why $2^m - 1$ which is of the form 4t+3 where t is an integer, has an odd number of primes of the form 4k+3 in its prime factorisation. LRP LRO LRC LRU

Thanks,

Plea2 TB

DM P12
Veteran poster
Post Number: 2254

Posted on Wednesday, 09 January, 2008 - 08:02 pm:

What do you get if you multiply together two numbers of the form 4k+3 (call them 4k+3 and 4l+3)? What form does it take? What if you multiply together four such numbers. Or six? ... TRSM TRMA

Plea2 P13
Regular poster
Post Number: 54

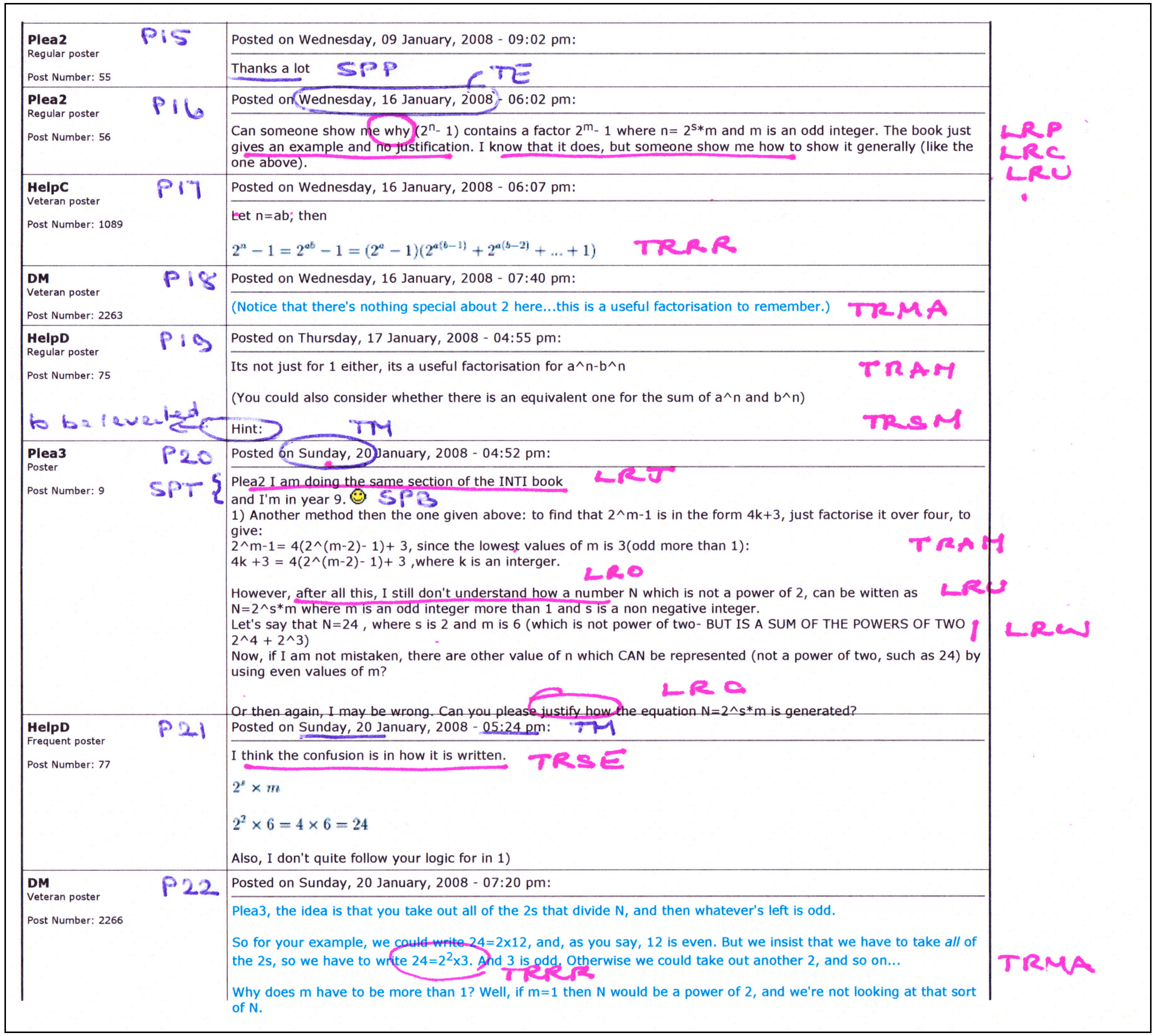
Posted on Wednesday, 09 January, 2008 - 08:35 pm:

(4k+3)*(4l+3)= 16kl + 12(k+l) + 9 which can be written (16kl +12(k+l)+8)+ 1. This means that it will leave a remainder of 1 when divided by 4. Tried expanding with odd number of primes of the form 4n+3 and I found that they leave a remainder of 3 when divided by 4. I think I've got it. LRW LRU

DM P14
Veteran poster
Post Number: 2255

Posted on Wednesday, 09 January, 2008 - 08:44 pm:

Have you come across modular arithmetic? It's a notation that makes this sort of thing much easier; if you know about it, you might like to think about this question using it. TRMA



Plea2 P15
Regular poster
Post Number: 55

Posted on Wednesday, 09 January, 2008 - 09:02 pm:

Thanks a lot SPP

TE

Plea2 P16
Regular poster
Post Number: 56

Posted on Wednesday, 16 January, 2008 - 06:02 pm:

Can someone show me why (2^{n}- 1) contains a factor 2^{m}- 1 where n= 2^{s}*m and m is an odd integer. The book just gives an example and no justification. I know that it does, but someone show me how to show it generally (like the one above).

LRP
LRC
LRU

HelpC P17
Veteran poster
Post Number: 1089

Posted on Wednesday, 16 January, 2008 - 06:07 pm:

Let n=ab; then

$$2^n - 1 = 2^{ab} - 1 = (2^a - 1)(2^{a(b-1)} + 2^{a(b-2)} + ... + 1)$$ TRRR

DM P18
Veteran poster
Post Number: 2263

Posted on Wednesday, 16 January, 2008 - 07:40 pm:

(Notice that there's nothing special about 2 here...this is a useful factorisation to remember.) TRMA

HelpD P19
Regular poster
Post Number: 75

Posted on Thursday, 17 January, 2008 - 04:55 pm:

Its not just for 1 either, its a useful factorisation for a^n-b^n TRAM

(You could also consider whether there is an equivalent one for the sum of a^n and b^n)

to be revealed

Hint: TM TRSM

Plea3 P20
Poster
Post Number: 9
SPT

Posted on Sunday, 20 January, 2008 - 04:52 pm:

Plea2 I am doing the same section of the INTI book LRJ
and I'm in year 9. 🙂 SPB
1) Another method then the one given above: to find that 2^m-1 is in the form 4k+3, just factorise it over four, to give:
2^m-1= 4(2^(m-2)- 1)+ 3, since the lowest values of m is 3(odd more than 1): TRAM
4k +3 = 4(2^(m-2)- 1)+ 3 ,where k is an interger.

LRO

However, after all this, I still don't understand how a number N which is not a power of 2, can be witten as N=2^s*m where m is an odd integer more than 1 and s is a non negative integer. LRU
Let's say that N=24 , where s is 2 and m is 6 (which is not power of two- BUT IS A SUM OF THE POWERS OF TWO 2^4 + 2^3) LRW
Now, if I am not mistaken, there are other value of n which CAN be represented (not a power of two, such as 24) by using even values of m?

LRQ

Or then again, I may be wrong. Can you please justify how the equation N=2^s*m is generated?

HelpD P21
Frequent poster
Post Number: 77

Posted on Sunday, 20 January, 2008 - 05:24 pm: TM

I think the confusion is in how it is written. TRSE

$$2^s \times m$$

$$2^2 \times 6 = 4 \times 6 = 24$$

Also, I don't quite follow your logic for in 1)

DM P22
Veteran poster
Post Number: 2266

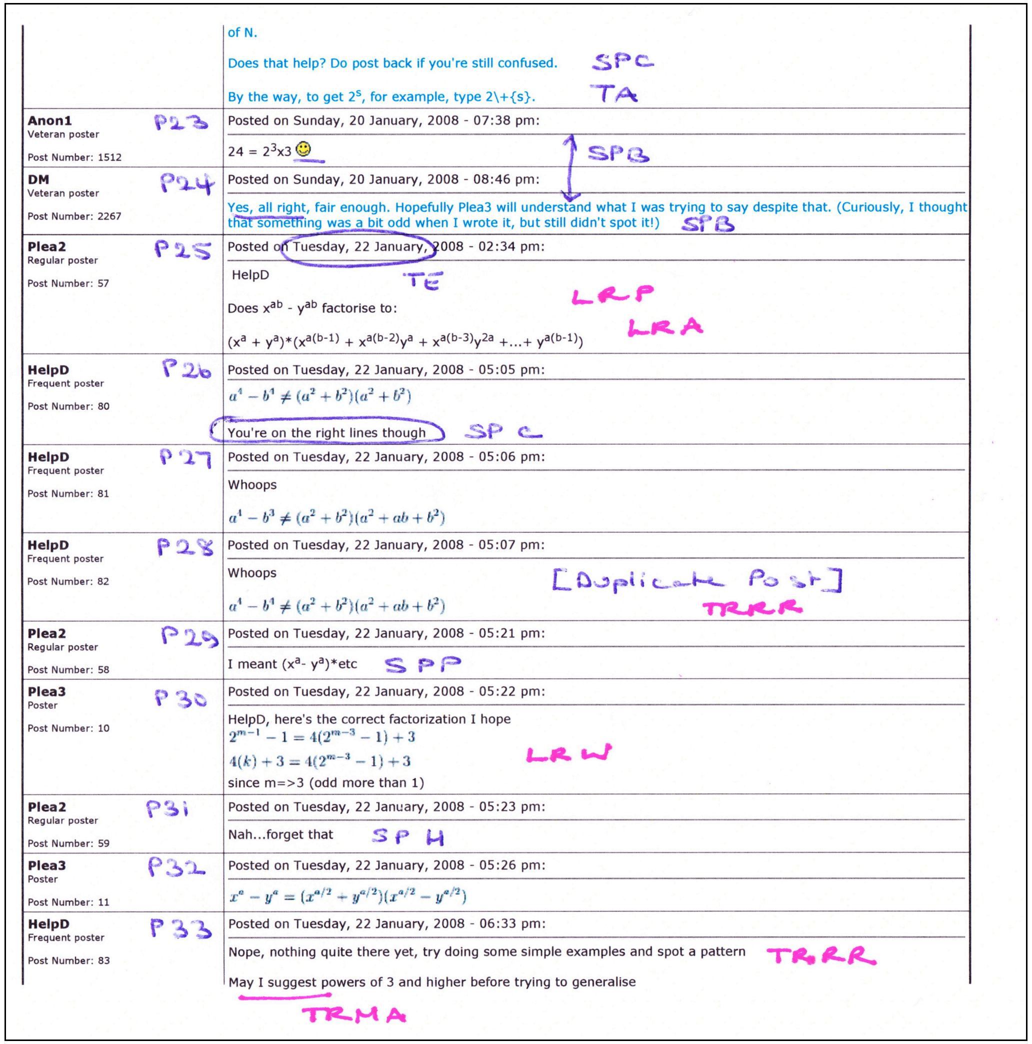
Posted on Sunday, 20 January, 2008 - 07:20 pm:

Plea3, the idea is that you take out all of the 2s that divide N, and then whatever's left is odd.

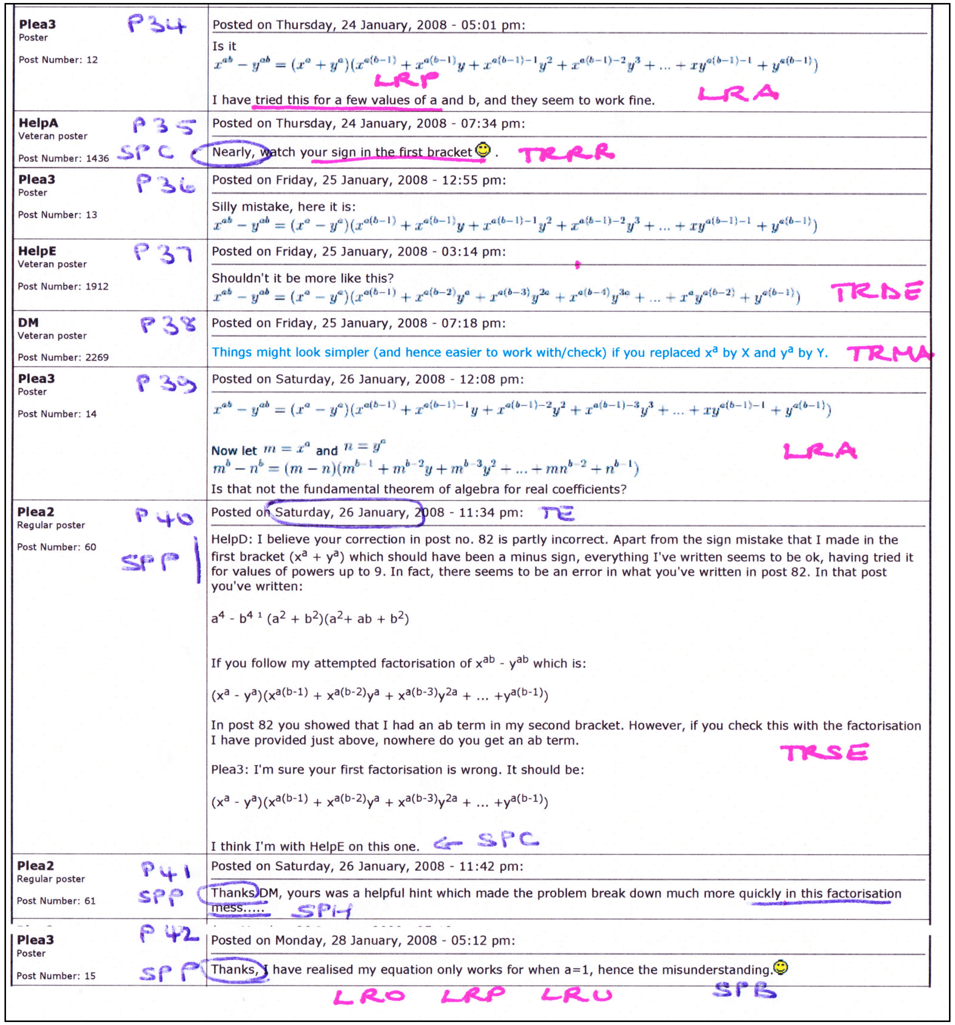
So for your example, we could write 24=2x12, and, as you say, 12 is even. But we insist that we have to take *all* of the 2s, so we have to write 24=2^{2}x3. And 3 is odd. Otherwise we could take out another 2, and so on... TRMA

TRRR

Why does m have to be more than 1? Well, if m=1 then N would be a power of 2, and we're not looking at that sort of N.



	of N. Does that help? Do post back if you're still confused. SPC By the way, to get 2^5, for example, type 2\+{s}. TA
Anon1 P23 Veteran poster Post Number: 1512	Posted on Sunday, 20 January, 2008 - 07:38 pm: 24 = 2^3x3 ☺ SPB
DM P24 Veteran poster Post Number: 2267	Posted on Sunday, 20 January, 2008 - 08:46 pm: Yes, all right, fair enough. Hopefully Plea3 will understand what I was trying to say despite that. (Curiously, I thought that something was a bit odd when I wrote it, but still didn't spot it!) SPB
Plea2 P25 Regular poster Post Number: 57	Posted on Tuesday, 22 January, 2008 - 02:34 pm: HelpD TE Does x^{ab} - y^{ab} factorise to: LRP $(x^a + y^a)*(x^{a(b-1)} + x^{a(b-2)}y^a + x^{a(b-3)}y^{2a} +...+ y^{a(b-1)})$ LRA
HelpD P26 Frequent poster Post Number: 80	Posted on Tuesday, 22 January, 2008 - 05:05 pm: $a^4 - b^4 \neq (a^2 + b^2)(a^2 + b^2)$ You're on the right lines though SP C
HelpD P27 Frequent poster Post Number: 81	Posted on Tuesday, 22 January, 2008 - 05:06 pm: Whoops $a^4 - b^3 \neq (a^2 + b^2)(a^2 + ab + b^2)$
HelpD P28 Frequent poster Post Number: 82	Posted on Tuesday, 22 January, 2008 - 05:07 pm: Whoops [Duplicate Post] $a^4 - b^4 \neq (a^2 + b^2)(a^2 + ab + b^2)$ TRRR
Plea2 P29 Regular poster Post Number: 58	Posted on Tuesday, 22 January, 2008 - 05:21 pm: I meant $(x^a- y^a)$*etc SPP
Plea3 P30 Poster Post Number: 10	Posted on Tuesday, 22 January, 2008 - 05:22 pm: HelpD, here's the correct factorization I hope $2^{m-1} - 1 = 4(2^{m-3} - 1) + 3$ $4(k) + 3 = 4(2^{m-3} - 1) + 3$ LRW since m=>3 (odd more than 1)
Plea2 P31 Regular poster Post Number: 59	Posted on Tuesday, 22 January, 2008 - 05:23 pm: Nah...forget that SP H
Plea3 P32 Poster Post Number: 11	Posted on Tuesday, 22 January, 2008 - 05:26 pm: $x^a - y^a = (x^{a/2} + y^{a/2})(x^{a/2} - y^{a/2})$
HelpD P33 Frequent poster Post Number: 83	Posted on Tuesday, 22 January, 2008 - 06:33 pm: Nope, nothing quite there yet, try doing some simple examples and spot a pattern TRRR May I suggest powers of 3 and higher before trying to generalise TRMA



Plea3
Poster
Post Number: 12
P34

Posted on Thursday, 24 January, 2008 - 05:01 pm:

Is it
$x^{ab} - y^{ab} = (x^a + y^a)(x^{a(b-1)} + x^{a(b-1)}y + x^{a(b-1)-1}y^2 + x^{a(b-1)-2}y^3 + \ldots + xy^{a(b-1)-1} + y^{a(b-1)})$
LRP

I have tried this for a few values of a and b, and they seem to work fine. LRA

HelpA
Veteran poster
Post Number: 1436
P35 SPC

Posted on Thursday, 24 January, 2008 - 07:34 pm:

Nearly, watch your sign in the first bracket☺ . TRRR

Plea3
Poster
Post Number: 13
P36

Posted on Friday, 25 January, 2008 - 12:55 pm:

Silly mistake, here it is:
$x^{ab} - y^{ab} = (x^a - y^a)(x^{a(b-1)} + x^{a(b-1)}y + x^{a(b-1)-1}y^2 + x^{a(b-1)-2}y^3 + \ldots + xy^{a(b-1)-1} + y^{a(b-1)})$

HelpE
Veteran poster
Post Number: 1912
P37

Posted on Friday, 25 January, 2008 - 03:14 pm:

Shouldn't it be more like this?
$x^{ab} - y^{ab} = (x^a - y^a)(x^{a(b-1)} + x^{a(b-2)}y^a + x^{a(b-3)}y^{2a} + x^{a(b-4)}y^{3a} + \ldots + x^a y^{a(b-2)} + y^{a(b-1)})$ TRDE

DM
Veteran poster
Post Number: 2269
P38

Posted on Friday, 25 January, 2008 - 07:18 pm:

Things might look simpler (and hence easier to work with/check) if you replaced x^a by X and y^a by Y. TRMA

Plea3
Poster
Post Number: 14
P39

Posted on Saturday, 26 January, 2008 - 12:08 pm:

$x^{ab} - y^{ab} = (x^a - y^a)(x^{a(b-1)} + x^{a(b-1)-1}y + x^{a(b-1)-2}y^2 + x^{a(b-1)-3}y^3 + \ldots + xy^{a(b-1)-1} + y^{a(b-1)})$

Now let $m = x^a$ and $n = y^a$
$m^b - n^b = (m - n)(m^{b-1} + m^{b-2}y + m^{b-3}y^2 + \ldots + mn^{b-2} + n^{b-1})$
Is that not the fundamental theorem of algebra for real coefficients? LRA

Plea2
Regular poster
Post Number: 60
P40 SPP

Posted on Saturday, 26 January, 2008 - 11:34 pm: TE

HelpD: I believe your correction in post no. 82 is partly incorrect. Apart from the sign mistake that I made in the first bracket ($x^a + y^a$) which should have been a minus sign, everything I've written seems to be ok, having tried it for values of powers up to 9. In fact, there seems to be an error in what you've written in post 82. In that post you've written:

$a^4 - b^{4\ 1} (a^2 + b^2)(a^2 + ab + b^2)$

If you follow my attempted factorisation of $x^{ab} - y^{ab}$ which is:

$(x^a - y^a)(x^{a(b-1)} + x^{a(b-2)}y^a + x^{a(b-3)}y^{2a} + \ldots + y^{a(b-1)})$

In post 82 you showed that I had an ab term in my second bracket. However, if you check this with the factorisation I have provided just above, nowhere do you get an ab term. TRSE

Plea3: I'm sure your first factorisation is wrong. It should be:

$(x^a - y^a)(x^{a(b-1)} + x^{a(b-2)}y^a + x^{a(b-3)}y^{2a} + \ldots + y^{a(b-1)})$

I think I'm with HelpE on this one. ← SPC

Plea2
Regular poster
Post Number: 61
P41 SPP

Posted on Saturday, 26 January, 2008 - 11:42 pm:

Thanks DM, yours was a helpful hint which made the problem break down much more quickly in this factorisation mess..... SPH

Plea3
Poster
Post Number: 15
P42 SPP

Posted on Monday, 28 January, 2008 - 05:12 pm:

Thanks, I have realised my equation only works for when a=1, hence the misunderstanding.☺

LRO LRP LRU SPB

Date: Thu, 15 Nov 2007 14:29:19 +0000
From: ecj20@cam.ac.uk
To: [Peter]
Subject: Ask NRICH

Dear [Peter],

Actually should I call you [real name] or [posting name]? Thanks for responding to [AskNRICH Moderator] e-mail to say that I could speak to you. I'm sure you will know I phoned your Dad and he said it was OK (as you said [to AskNRICH Moderator] he would).

I was looking a while ago at AskNRICH and saw that you had been involved in posting during the last year or so - it was not only your name that did it, but also what you were writing or asking.

For my research I am wanting to talk (well through e-mail) to people who obviously do lots of mathematics for themselves (as well as in school) as I thinking that now we have the Internet available there are more opportunities for people like you to find extra people to communicate with than just teachers. (I should say at this point that if I write about you in any of my research you would have a new anonymous identity and not be referred to as either [real name] or [posting name].

Can I ask you a few questions to start with and then if I think of others from your replies, maybe I could maintain a conversation for a short while. (You are free to choose not to answer particular questions if you don't want to). Here goes:

1. Could you give a brief description about yourself, your school, your maths lessons and what you are doing for maths away from the classroom.

2. If you haven't covered it in q1, can you say how you learn your maths for yourself? Do you teach yourself new material and if so how do you do this?

3. Do you feel that the Internet has given you more opportunities than perhaps in the past when it wasn't around? (I'm sure you can't remember a time when it wasn't, but I can!)

4. Do you (did you) do any of the NRICH maths problems on the main site? If you know about them are they different from what you get in school? What type of maths do you like doing the most and why?

5. What made you become involved with AskNRICH (at least 123 times I noticed at the beginning of this year). How do you see things like this forum contributing to maths teaching and learning.

Do only reply when you have the time to spare. I am grateful that you said I could ask you some questions.

Last January to celebrate NRICH being 10 years old, I chose the problem Group Photo for my favourite NRICH puzzle. If you go to Jan 07 and look in the notes for the problem I say why I chose it. I only mention this in case you want to find out a little more about me.

All my teacher training students call me by my first name, Libby, so if you would feel comfortable with calling me Libby (or Libby Jared) that would be quite OK.

with best wishes,
Libby

Thread Name: as appearing on web-board
Thread Level: Please Explain (PE) Onwards and Upwards (OU) Higher Dimension (HD)
Private: Posts made within the private posting of the board and not available to the general public
Post number
Day & Time, (+ number) indicates for non-private number of further days Peter is involved
Type of Post: Initiating the query (Q1), Asking own question within another's thread (Q2)
Offering help (H),
P(number) indicates same thread as first indicated by that posting number
* the same question being asked on different occasions and used for Three Threads Chapter

Thread and Level (PE OU HD) or Private	Post No	No of Peter's Post	Date	Day & Time	Type of post
	1-8	8			
Primes PE	9-11	3	25.11.06	Sat 1.39pm	Q1
LCM PE	12-19	8	26.11. 06	?Sun 10.20am (+1)	Q1
Private	20	1	25.11.06	Sat 9.53pm	Profile
No Theory PE	21-23	3	28.11.06	Tues 5.59pm	Q1
	24,25	2			
BMO question OU	26-28	3	29.11.06	Wed 4.47pm	H
Divisibility OU	29-32	4	30.11.06	Thurs 6.40pm	H
Formula PE	33-36	4	2.12.06	Sat 8.29pm	H
Zero OU	37-42	6	3.12.06	Sun 4.47pm	H Q2
	43	1			
Probability OU	44-46	3	4.12.06	Mon 5.30pm	Q1
Induction OU	47,48,50-65	18	4.12.06	Mon 8.12pm (+1)	Q1
Multiples OU	49	1	5.12.06	Tues 6.47pm	H
Surds PE	66-68	3	9.12.06	Sat 3.08pm	H
	69	1			
Summation HD	70,74-78	6	10.12.06	Sun 4.13pm	H
AP OU	71,72	2	10.12.06	Sun 4.30pm	Q1
	73	1			
Puzzle OU	79	1	10.12.06	Sun 10.26	n/a
Private	80	1	11.12.06	Mon 4.02pm	
Private	81	1	11.12.06	Mon 4.16pm	
	82	1			
GP/AP OU	83-86, 88-90	7	11.12.06	Mon 7.13pm	Q1
	87	1			
Chemistry q PE	91	1	13.12.06	Wed 4.13pm	H
Algebraic Nos OU	92-99	8	14.12.06	Thurs 8.56pm	Q1
Private	100,101	2	15.12.06	Fri 4.13pm	
Private	102, 104,105	3	16.12.06	Sat 2.47pm	P81
	103	1			
No Theory	106	1	4.01.07	Thurs 1.43pm	H
Powers OU	107,108,110,111, 113-121	13	6.01.07	Sat 8.04pm	Q1
Distance OU	109	1	6.01.07	Sat 8.42pm	H
	112	1			
Private	122	1	7.01.07	Sun 10.06pm	
	123	1			
Numbers OU	124-130	7	16.01.07	Tues 8.11pm	Q1
No Theory* OU	131–134	4	20.01.07	Sat 11.13am	Q1
Triangles PE	135-140	6	21.01.07	Sun 3.43pm	Q1
Private	141-144	4	26.01.07	Fri 9.47pm	

Thread and Level (PE OU HD) or Private	Post No	No of Peter's Post	Date	Day & Time	Type of post
Private	145, 147-149	4	27.01.07	Sat 9.28am	
	146	1			
	150	1			
Private	151-153	2	27.01.07	Sat 8.13pm	P141
0th root HD	154	1	28.01.07	Sun 9.23am	H
Game Strategy HD	155	1	28.01.07	Sun 12.45pm	H
Private	156-159	4	28.01.07	Sun 5.26pm	P145
Induction OU	160-164	5	29.01.07	Mon 8.13pm	Q1
Digit sum HD	165	1	30.01.07	Tues 5.33pm	H
Private	166–168	3	31.01.07	Wed 8.12pm	P141
Chess Board PE	169, 171,172	3	3.02.07	Sat 10.54pm	Q1
Private	170	1	3.02.07	Sat 10.57pm	P141
Induction OU	173	1	5.02.07	Mon 5.49pm	H
Private	174	1	5.02.07	Mon 8.47pm	P145
Growth rates HD	175	1	6.02.07	Tues 8.26am	H
Private	176	1	6.02.07	Tues 6.50pm	P145
Powers HD	177,179	2	7.02.07	Wed 8.27am	H
Private	178	1	7.02.07	Wed 4.43pm	P145
Four colours PE	180,181	2	7.02.07	Wed 5.39pm	H Q2
Private	182	1	9.02.07	Fri 3.31pm	P145
Sequence OU	183,185,187	3	10.02.07	Sat 8.37am	Q1
Private	184,186	2	10.02.07	Sat 9.41am	P145
Private	188	1	11.02.07	Sun 10.37pm	P141
Private	189	1	11.02.07	Sun 2.37pm	P145
Primes HD	190-193	4	11.02.07	Sun 6.36pm	Q2
Quadratic OU	194,195	2	12.02.07	Mon 4.02pm	H
Private	196,197	2	12.02.07	Mon 8.27pm	P145
Proof OU	198-200	3	14.02.07	Wed 12.43pm	Q1
Private	201-203	4	14.02.07	Wed 2.33pm	P145
Inequality OU	204–208, 211	6	16.02.07	Fri 11.41am	Q1
Private	209, 210, 212-224	15	16.02.07	Fri 1.16pm	P145
Cubic Seq PE	225,226	2	18.02.07	Sun 7.49pm	H
Private	227-231	5	18.02.07	Sun 9.05pm	P145
Private	232	1	19.02.07	Mon 6.17pm	P141
Private	233-235	3	19.02.07	Mon 6.50pm	P145
Cubic Seq PE	236-238	3	22.02.07	Thurs 6.31pm	H
Inequality OU	239,240	2	22.03.07	Thurs 9.47pm	H
Median PE	241	1	23.02.07	Fri 4.22pm	H
Bodmas PE	242	1	23.02.07	Fri 5.15pm	H
Sequence OU	243,245,253	3	25.02.07	Sun 9.49am (+1)	H
Induction OU	244	1	25.02.07	Sun 10.20am	H
C1 past q OU	246,250,252	3	25.02.07	Sun 4.08pm	H
Private	247,248,251	3	25.02.07	Sun 4.13pm	P145
Function OU	249,256	2	25.02.07	Sun 5.03pm	H
Private	254	1	26.02.07	Mon 5.22pm	P145
Induction OU	255,259,260	3	26.02.07	Mon 9.15pm	H
	257,258	2			
Private	261	1	1.03.07	Thurs 10.21pm	P145
Decimals PE	262	1	2.03.07	Fri 5.22pm	H
C2 Proof OU	263-265	3	4.03.07	Sun 9.45am	Q1

Thread and Level (PE OU HD) or Private	Post No	No of Peter's Post	Date	Day & Time	Type of post
Egyptian FractionsPE	266	1	4.03.07	Sun 1.58pm	H
BMO q* OU	267-271	5	5.03.07	Mon 7.05pm (+1)	H
Private	272-277	5	6.03.07	Tues 8.15pm	P145
	278	1			
BMO q OU	279-282	4	8.03.07	Thurs 8.26pm	H
Private	283,284	2	9.03.07	Fri 12.21pm	P145
Co-efficients OU	285,286	2	10.03.07	Sat 1.54pm	H
Algebra OU	287,289,291,292	4	10.03.07	Sat 5.56pm	H
BMO q OU	288,290	2	10.03.07	Sat 6.42pm	H
	293,295	2			
Private	294,296	2	10.03.07	Sat 9.01pm	P145
Triples PE	297	1	12.03.07	Mon 8.37pm	H
Abs value OU	298	1	12.03.07	Mon 8.39pm	Q2
Private	299,300,302-305	5	13.03.07	Tues 1.42pm	P145
Geometry OU	301	1	14.03.07	Wed 5.57pm	H
Function Rule OU	306-310	5	15.03.07	Thurs 7.58pm	H
Slab Invest'n PE	311	1	16.03.07	Fri 5.02pm	H
	312,313	2			
No Theory OU	314,315	2	18.03.07	Sun 2.25pm	Q1
GCSE q PE	316	1	20.03.07	Tues 6.05pm	H
Private	317,319	2	24.03.07	Sat 4.57pm	P145
BMO q OU	318,320,325	3	25.03.07	Sun 12.10pm	Q1
Inequality OU	321	1	25.03.07	Sun 4.16pm	H
Private	322,324	2	25.03.07	Sun 4.36pm	P145
	323	1			
Private	326,327, 329-332	6	26.03.07	Mon 4.19pm	P145
	328	1			
Sine Rule OU	333	1	4.04.07	Wed 9.02am	H
Private	334-339, 341,342	8	4.04.07	Wed 8.02pm	P145
	340	1			
	343	1			
Quartic roots OU	344,345	2	18.04.07	Wed 7.16am	H Q2
Investigation	346-348	3	18.04.07	Wed 6.03pm	H
Private	349,350	2	19.04.07	Thurs 8.21am	P145
Coursework	351	1	19.04.07	Thurs 9.12pm	H
Heron's Formula OU	352	1	20.04.07	Fri 4.03pm	Q2
Investigation	353	1	21.04.07	Sat 10.17am	H
Geometry OU	354-355	12	21.04.07	Sat 12.26pm	Q1
Mod arith OU	356	1	21.04.07	Sat 6.34pm	H
BMO q OU	357,358	2	22.04.07	Sun 1.05pm	H
Indices	359	1	22.04.07	Sun 8.43pm	H
Nth term	360	1	24.04.07	Tues 7.10pm	H
Private	361,362, 364-370	9	25.04.07	Wed 7.57pm	P145
BMO q OU	363,371	2	29.04.07	Sun 11.07am	Q1
Trig OU	372,373	2	29.04.07	Sun 6.49pm	H
Simult Eqns	374	1	29.04.07	Sun 9.02pm	H
Geometry OU	375,376	2	1.05.07	Tues 7.39pm	Q1
Private	377	1	2.05.07	Wed 4.25pm	P145
Quadratic OU	378,379	2	3.05.07	Thurs 7.07pm	H
Geometry OU	380,381	2	5.05.07	Sat 4.37pm	Q1

Thread and Level (PE OU HD) or Private	Post No	No of Peter's Post	Date	Day & Time	Type of post
Private	382,383	2	5.05.07	Sat 6.43pm	P145
Fractions PE	384	1	11.05.07	Fri 10.14pm	H
Trig OU	385-388	4	12.05.07	Sat 4.25pm	Q1
Private	389,394	2	16.05.07	Wed 8.18pm	P145
Closure OU	390-392	3	18.05.07	Fri 9.18pm	H
Quadratic graph OU	393	1	19.05.07	Sat 4.31pm	H
Convergence HD	395,396	2	21.05.07	Mon 5.51pm	H
Private	397	1	22.05.07	Tues 4.21pm	
Simult Eqns	398	1	23.05.07	Wed 5.06pm	
Trig identity OU	399-405	7	26.05.07	Sat 11.20am	Q1
Trig again! OU	406-409	4	26.05.07	Sat 9.02pm	Q1
Pascal's Triangle PE	410,411	2	27.05.07	Sun 10.04am	H
Lucas Number PE	412,413	2	27.05.07	Sun 11.44am	Q2
Number Theory OU	414-416	3	28.05.07	Mon 1.13pm	Q1
BMO q OU	417-419	3	30.05.07	Wed 5.32pm	Q1
GSCE q PE	420-422	3	10.06.07	Sun 7.50pm	Q1
Private	423	1	14.06.07	Thurs 6.14pm	
Private	424	1	21.06.07	Thurs 5.47pm	
Private	425	1	26.06.07	Thurs 5.22pm	
Inflexion pts OU	426-428	3	8.07.07	Sun 11.24am	Q1
Differentiation OU	429,430	2	11.07.07	Wed 6.39pm	H
Trig Eqn OU	431,432	2	27.07.07	Fri 7.20pm	H
Expressions PE	433	1	4.08.07	Sat 11.12pm	H
Incremental Rates PE	434	1	6.08.07	Mon 10.18am	H
Combinations OU	435	1	14.08.07	Tues 9.22am	H
Private	436,437	2	15.08.07	Wed 10.56pm	P145
Proof OU	438	1	28.08.07	Tues 3.56pm	H
Trig OU	439,440	2	29.08.07	Wed 11.35am	H
Vectors OU	441-443	3	23.09.07	Sun 9.33pm	H
Private	444	1	23.09.07	Sun 10.16pm	Profile
Private	445	1	27 09.07	Thurs 4.30pm	P145
Combinations OU	446,447	2	29.09.07	Sat 12.30pm	H
GP OU	448	1	29.09.07	Sat 12.38pm	H
General term PE	449,450	2	30.09.07	Sun 1.18pm	H
Science OU	451	1	7.10.07	Sun 1.18pm	H
SMC q OU	452-455	4	10.10.07	Wed 6.50pm	H
Logs OU	456	1	14.10.07	Sun 12.36pm	H
Private	457,458	2	17.10.07	Wed 7.44pm	
Natural logs OU	459	1	22.10.07	Mon 8.26am	H
Indices OU	460	1	23.10.07	Tues 9.33am	H
SMC q OU	461-464	4	2.11.07	Fri 5.23pm	Q1
IMO q OU	465,466	2	12.11.07	Mon 5.07pm	H
Primes OU	467	1	13.12.07	Tues 9.35pm	H
	468	1			
BMO q OU	469-473	5	28 11 07	Wed 4.37pm	H
*BMO q OU	474	1	29.11.07	Thurs 4.43pm	H
BMO q OU	475	1	3.12.07	Mon 8.49pm	H
Logs OU	476,477	2	5.12.07	Wed 9.05pm	H
Complex Nos OU	478-481	4	8.12.07	Sat 12.02pm	Q1
Private	482,484	2	16.12.07	Sun 10.16am	

Thread and Level (PE OU HD) or Private	Post No	No of Peter's Post	Date	Day & Time	Type of post
Private	483,485	2	16.12.07	Sun 10.18am	
Private	486,487	2	17.12.07	Mon 7.03pm	P482
SMCq OU	488	1	24.12.07	Mon 3.03pm	H
	489	1			
Probability OU	490	1	2.01.08	Wed 1.01pm	H
Private	491	1	18.01.08	Fri 6.20pm	
Trig	492-494	3	4.02.08	Mon 6.53pm	Q1
Parametric Eqns OU	495,496	2	12.04.08	Sat 9.55am	Q1
Multiplication PE	497,499	2	13.04.08	Sun 8.52pm	H
Complex NOs OU	498,500	2	13.04.08	Sun 9.06pm	H
Induction	501	1	1.05.08	Thurs 9.44pm	H

* The Three Threads used in Chapter Eleven

Message Thread Saturday November 2006	Interpretative Commentary
Peter Saturday 1.39pm hi ive just began to read an introduction to number theory and inequalities and i seem to have fallen at the first hurdle. when defining a prime number the writer says "we define a prime number p to be a positive interger bigger than 1 such that given intergers m and n, if p/mn, then either p/m or p/n." later he proves that this also means that p only divides by ±1 and ±p but i am still confused. surely this means that 6 is a prime as 6/42 and 6/6 but not 7. clearly i have over looked something. please explain. thank you.	Peter cannot see why 6 does not fit the some written formal definition although he knows that 6 is not a prime number. He realises that he somehow has a misunderstanding as there cannot be a contradiction.
HelpA. 1.43 pm Yes, but if we choose m and n to be 14 and 3, 6 goes into neither 14 nor 3, but goes into 14*3 = 42	Reply comes 4 minutes later with example that shows using the definition that 6 is not prime. Providing a numerical example requires Peter to consider where his misconception lay – 'scaffolding' Peter's learning
Peter 1.46pm Thanks. i new that it would be something basic like that i had overlooked. i thought it meant for any choice of multiples in which case all numbers would be prime. i get it now. cheers.	3 minutes later Peter replies. The content shows that in that short space of time he has understood ('I get it now') the underlying reason (if one pair could be found which 6 did not both divide into) made only implicitly behind the numerical example that HelpA gave. Peter also comments that his original misunderstanding would have implied that 'all numbers would be prime'.
Deputy Moderator 1.47pm The important thing about primes is that this property holds for *any* m and n. That is, given any m and n, if p divides mn then p divides m or p divides n. (Of course, this is `or' in the mathematical sense, which means that both can happen. For example, 2 is prime, and 2 divides 8=2*4.) In your example, we know that 6 divides 21*2 (that is, 42), but 6 doesn't divide 21 and 6 doesn't divide 2, so it can't be prime. If you stare at it for long enough, you should be able to see why this definition coincides with how you normally think of what a prime is. Somehow the reason that 6 can divide 21*2 but not 21 or 2 is that 6 can be broken down into 2*3, and 2 divides the 2 bit and 3 divides the 21 bit so 6 divides 42. Does that help? Do post back if not!	8 minutes after Peter posts his query the moderator has written a very full explanation as to the reason why the definition does not imply that 6 is a prime number. There is additional example where p can divide both m and n and not just one and not the other which further illustrates the mathematical sense of 'or' which can imply 'both'. 'Normally think what a prime is' – the most common definition used in school seems to be that a prime number can only be divided by itself and one. Invitation to ask for further explanation if needed.

Message Thread Saturday November 2006	Interpretative Commentary
By the way, it's *integer*, not *interger*. I don't want to be picky, but it's sometimes useful to know the correct spelling as it makes looking things up easier! This definition may seem a bit awkward at first, compared with the one that says that p is a prime if its only factors are 1 and itself. However, I think that in just about any application, the thing that you want to know about a prime is that it has the property above about dividing m and n. So either you take this as your definition (and check that it coincides with what you thought was the definition), or you take as the definition the thing about factors and then immediately prove that this means it has the property above.	A gentle comment (reprimand?) about the correct spelling of the word 'integer'. A difficult balance for needing to correct an error but not to dent the confidence Peter who is still a newcomer. The post returns to consider more about the definition 'p is a prime if its only factors are 1 and itself'. [Another definition used in school (which avoids the difficulty of considering the number 1) is a number that has two and only two different factors.]
Deputy Moderator 1.52pm Oops, it took me so long to write that that HelpA got there first!	5 minutes later acknowledges a faster reply by another general user who instantaneously graciously acknowledges the depth of the moderator's response to Peter which links the common definition to the tighter mathematical one given in the text.
HelpA 1.52pm Better to get a full explanation after 10 minutes than a one-line sentence after 5!	
Peter 3.14pm thanks to both of you. i get it now. just out of interest, others who have read the book how did you find it. ive never looked at number theory before but im finding it a bit harder to understand than other maths i have looked at. thanks to all	Peter completes the thread with a thank you to both HelpA. Peter also seems content to comment on his own mathematical ability and asks whether others are having problems too. (Unfortunately) no-one responds to this open query here).
Peter starts another thread at 10.20am the next day, which is based on another section of Number Theory: find the intergers s and t …. And one minute later posts a second message: sorry i meant integer. please pardon my spelling.	

<table>
<tr><th>No</th><th>Message within thread</th><th></th><th>Interpretative Commentary</th></tr>
<tr><td colspan="4">This is Peter’s 385th post – and around the 20th question that he has initiated. Peter is still in year 10 (14 to 15 years old) at school and will sit his GCSE examinations early next month (one year ahead than the expected age). The level of mathematics here is part of the A level course (some way in).</td></tr>
<tr><td>1</td><td>Peter Sat 4.25pm
Can some one help me with this problem. This is the first trigonometrical equation I have done so please take it slowly and drop me a few subtle hints.
Prove that:
tan(45'+A/2)=(1+sinA)/cosA = cosA/(1-sinA)
where 45' means 45 degrees
Sorry for the lack of formatting but I tried to put it in latex and it didn’t work.
Thanks for any help</td><td></td><td>Peter has posted several questions by now and as with all others he asks only for hints. He makes it clear that this is a new area for his studies and would therefore like people to start at the beginning rather than assuming too much.
Here the formulae text is written using only a standard keyboard that can be open to confusion. However Peter is careful to bracket each part that essentially forms the top and bottom of a division.
The AskNRICH board has instructions on how to use a mathematical text (LaTeX) for clarity but Peter has not yet been able to master this.</td></tr>
<tr><td colspan="4"></td></tr>
<tr><td>2</td><td>Help 1 Team Member A Sat 4.49pm
Hi Peter
For the first equality, do you know the formula for tan (x +y)? Do you need help with the second equality?
To write in LaTeX, start your line with \[, end with \], and write maths in the middle! (There's a slightly more comprehensive guide here.)</td><td></td><td>The first subtle hint arrives with a simple question as to whether Peter knows the standard formula for expanding tan (x+y).
Help also concentrates on dealing with the first part which is to show that tan(45^0+A/2)=(1+sinA)/cosA.
Help leaves the second part to show that tan(45^0+A/2)= cosA/(1-sinA) for now to give Peter the opportunity for himself but by asking whether Peter would like help gives Peter the clue that he is not being abandoned.
Peter is also given some advice on how to deal with marking up mathematical formulae using LaTeX and provided with the link on AskNRICH where there are full instructions.</td></tr>
<tr><td colspan="4"></td></tr>
</table>

No	Message within thread		Interpretative Commentary
3	Peter Sat 5.12pm If I sort the first equality then I'll give the second a go. Yes I do know the formula to expand tan(X+Y) I have tried doing this and meant to post my workings here but forgot. ☺ $\tan(45+\frac{A}{2}) = \frac{\tan 45 + \tan\frac{A}{2}}{1 - \tan\frac{A}{2}\tan 45}$ Expanding since tan45=1 the above equality becomes $\tan(45+\frac{A}{2}) = \frac{1 + \tan\frac{A}{2}}{1 - \tan\frac{A}{2}}$ From here I have tried a variety of things but each one has failed, quite possibly because of a lack of competence on my part. Can you nudge me from here please.		Peter responds directly (in reverse order) to both of Help 1's questions and is quite clear that he will try to do the first one now with the hint and if successful he will try the second one before asking for further help. Peter also remembers that he did not post the work that he had done before he became stuck, providing a smiley face to convey friendliness. All Peter's working is correct but he has not been able to think what to do next and conveys that he thinks that this is his own lack of competence – though it might be less derogatory and more accurate to describe it as a lack of experience. Again he asks for just a small hint – a nudge.
4	Help 2 5.20pm Might help if you write sinA and cosA in terms of tan(A/2).		This helper (still at school) tends to post very succinct (though extremely useful) hints. Peter has to make the connection that sine A and cosine A can be written in terms of tan(A/2) a standard result that is 'available' from a formulae book.

No	Message within thread		Interpretative Commentary
5	Peter Sat 5.50pm As I guessed I failed because of a lack of competence on my part when trying the correct option. I did this before but I think I must have gone wrong short of the mark. I'll put it down to experience. If anyone is interested I did the following: where t=tanA/2 $1+\sin A=2t/(1+t^2)+1=t^2+2t+1/(1+t^2)$ $\cos A=1-t^2/(1+t^2)$ $(1+\sin A)/\cos A=(t^2+1)/(1-t^2)*((t+1)^2)/(1+t^2)$ $=(t+1)/(t-1)$ Thanks Help 2 and Help 1. I'll post back if I can't get the second one		Half an hour later Peter has (he thinks) got the correct answer. He had obviously been trying things in the right sort of area but either didn't choose the appropriate formulae or made an error. This he is going to put down to experience, indicating that he will be better placed to deal with similar questions next time. Having asked for help Peter is willing to share his solution with anyone else who may be looking at AskNRICH. There is a small error in the very last line where t-1 should be 1-t. Subsumed in the working is that t^2+2t+1 is $(t+1)^2$ and /cos A has been replaced by reciprocal and post multiplied by 1+sinA. All of this is correct and acceptable – a rapid - in the head - way of working. Peter spells Help 1's name incorrectly. Peter indicates that he is going to try the next one now and will ask for help later if necessary.
6	Help 1 6.10pm Almost - have another look at your very last line. Great stuff otherwise! Help 1 6.13pm Also, it helps to write "\tan" instead of "tan" in LaTeX, as then it comes out as tan A instead of *tan*A (in the latter, LaTeX thinks you're multiplying the variables t, a, n and A together). Same goes for other 'standard' functions: sin, arccos, cosec, log, ln etc.		Help 1 spots Peter's small error in the 'very last line' but gives much praise and encouragement in 'great stuff otherwise'. Three minutes after this, Help 1 offers further advice on using LaTeX getting into quite a technical discussion how italicised letters in mathematical processing is used for variables and if values were given would multiply the values together.

No	Message within thread		Interpretative Commentary
7	Peter 6.37pm I put t-1 not 1-t like it should be and ... spelt your name wrong. Now I've got the first one I'm motoring through the exercises. Who would have thought trigonometry could be this much fun. Thanks again		There would appear to be two very last lines! Peter corrects his error in his last line of mathematical text but… … in the very last line of Help 1's post, Peter had spelt his name incorrectly! These two sentences convey Peter's pleasure at doing this work – the warmth help to make Peter appear as a human being letting others share what appears to be his enthusiastic personality. He shows his appreciation.
8	Help 1 6.56pm Lol, I was referring to the 1-t, but that too! Good luck with the rest of the problems.		Help 1 is laughing out loud (Lol) when he realises that Peter has looked to see that he had spelt his name incorrectly, probably in Help 1's mind rather more insignificant than getting the mathematics incorrect. Wishing Peter luck with the rest adds to the impression of friendliness and 'real people'.

Table A – Coding letters used

Letter	Representation
C	Friendly communication
D	Development in work – often through some degree of explanation
E	Etiquette/politeness in posting
M	Mathematics undertaken – working through the problem
O	Openly shares limitations – of knowledge, experience
T	Thinking
U	Understanding
W	Ways of working (mathematically)

Table B – First retrievable thread where Peter has requested help

Message Thread 26 11 2006 [Additional commentary relevant to analysis]	Interpretative Commentary	
P. Saturday 1.39pm hi ive[1] just began to read an introduction to number theory and inequalities and i seem to have fallen at the first hurdle. [Book defines a prime number] "later [author] proves that this also means that p only divides by ±1 and ±p but i am still confused. surely this means that 6 is a prime as 6/42 and 6/6 but not 7. clearly i have over looked something. please explain. thank you.	Friendly opening, light hearted description of his current difficulty P has considered the definition carefully as he has come up with, for the moment, a contradiction P cannot see why 6 does not fit the some written formal definition although he knows that 6 is not a prime number. He openly admits that he somehow has a misunderstanding.	C E T M O
Help A. 1.43 pm [4 minutes later] Yes, but if we choose m and n to be 14 and 3, 6 goes into neither 14 nor 3, but goes into 14*3 = 42. [i.e. a numerical example that shows using the definition that 6 is not prime.]	Example requires P to consider where his misconception lay –P's learning will develop through thinking things through.	D
P. 1.46pm [4 minutes later] Thanks. i new that it would be something basic like that i had overlooked. i thought it meant for any choice of multiples in which case all numbers would be prime. i get it now. cheers.	In a short space of time Peter has understood ('I get it now') the generalising the underlying reason made only implicitly by being given a specific numerical example that Help A gave. P shows further understanding by commenting that his original misunderstanding would have implied that 'all numbers would be prime'.	U
Deputy Moderator 1.47pm [Only 8 minutes after P has posted his query] [Writes a very full explanation as to the reason why the definition does not imply that 6 is a prime number].[Extends the examples to illustrate the mathematical sense of 'or' which can imply 'both']. Does that help? Do post back if not!	'Normally think what a prime is' – the most common definition used in school seems to be that a prime number can only be divided by itself and one. P has his general mathematical knowledge extended beyond the problem being considered. Invitation to ask for further explanation if needed.	M E

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[1] Apart from interesting spellings, P does not use capital letters, which in a modern-day texting life is common and normal for some people. P's spelling and punctuation will be normally left as it appeared in the posts

Message Thread 26 11 2006 [Additional commentary relevant to analysis]	Interpretative Commentary	
By the way, it's *integer*, not *interger*. I don't want to be picky, but it's sometimes useful to know the correct spelling as it makes looking things up easier! [Continues to discuss the formal, rigorous definition in the book in relation to school definitions of prime numbers].	As a newcomer to the site P's confidence is not dented by this comment – P. starts another thread at 10.20am the next day and one minute later posts a second message: 'sorry i meant integer. please pardon my spelling'[2] Extending P's knowledge to a pure argument/definition rather than a less rigorous one used normally in school. Peter is being introduced to rigorous mathematics.	E C M
Deputy Moderator 1.52pm Oops, it took me so long to write that that Help A got there first!	This exchange between two helpers, introduces P to the range of quick to measured solutions – he is party to seeing two different ways of working.	W
Help A 1.52pm Better to get a full explanation after 10 minutes than a one-line sentence after 5!		
P. 3.14pm thanks to both of you. i get it now. just out of interest, others who have read the book how did you find it. ive never looked at number theory before but im finding it a bit harder to understand than other maths i have looked at. thanks to all	P's completes the thread with a thank you. He also seems content to comment on his own mathematical ability - he is starting out on studying number theory and asks whether others are having problems too[3]. (Unfortunately) no-one responds to this open query here).	E O

Table C Representative Thread – when established as a regular contributor
Interpretative Commentary on Trigonometrical Problem

Message Thread 12 05 2007 [Additional commentary relevant to analysis]	Interpretative Commentary	
Peter Sat 4.25pm Can some one help me with this problem. This is the first trigonometrical equation I have done so please take it slowly and drop me a few subtle hints. Prove that: tan(45'+A/2)=(1+sinA)/cosA = cosA/(1-sinA) where 45' means 45 degrees Sorry for the lack of formatting but I tried to put it in latex and it didn't work. Thanks for any help. [Here formulae text written using only standard keyboard that can be open to confusion The AskNRICH board has instructions on how to use a mathematical text (LaTeX).	Still asks only for hints. Makes it clear that this is a new area for his studies and would therefore like people to start at the beginning rather than assuming too much. Unusually, no sharing of his workings and thoughts so far. Peter has carefully bracketed each part to help clarity and is thus presenting the question as clearly as he is currently able to.	E O C
Help 1 Team Member A Sat 4.49pm Hi Peter: For the first equality, do you know the formula for tan (x +y)? Do you need help with the second equality?	Help 1 is 'testing the water' to see how much Peter knows and how much help he would like. Concentrates on first part of equality leaving Peter for now to try out second part (if he can do the first), but	E M

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[2] In later posts, Peter generally reverts to his original spelling of interger!

[3] (Unfortunately) no-one responds to this open query here).

Message Thread 12 05 2007 [Additional commentary relevant to analysis]	Interpretative Commentary	
To write in LaTeX, start your line with \[, end with \], and write maths in the middle! (There's a slightly more comprehensive guide here.)	making clear help is available if necessary. Instruction for marking up mathematical formulae relevant to AskNRICH	D
Peter Sat 5.12pm If I sort the first equality then I'll give the second a go. Yes I do know the formula to expand tan(X+Y) I have tried doing this and meant to post my workings here but forgot. ☺. [Provides workings – all correct]. From here I have tried a variety of things but each one has failed, quite possibly because of a lack of competence on my part. Can you nudge me from here please.	Peter responds directly (in reverse order) to both of Help 1's questions. Redresses his forgetfulness in not having posted his workings. Friendly smiley face (☺) conveying 'oops sorry'. Suggests it is his fault that he cannot do more, rather than a lack of experience. Asks again for just a small hint – a nudge.	M E C O T
Help 2 5.20pm Might help if you write sinA and cosA in terms of tan(A/2). [A succinct but key hint]	Hint given leaves Peter to make the connection for himself and using the standard 're-written' form given in any formulae or relevant text book.	M
Peter Sat 5.50pm As I guessed I failed because of a lack of competence on my part when trying the correct option. I did this before but I think I must have gone wrong short of the mark. I'll put it down to experience. If anyone is interested I did the following: [Shares solution although there is a small error writing 1-t not t-1in the final line]. Thanks Help 2 and Help 1. [Misspells latter name]. I'll post back if I can't get the second one	Half an hour later Peter has (he thinks) got the correct answer. Continues to comment on his own incompetence and inexperience, though a possible implicit indication that he will be better placed in future. Fostering community membership by offering his solution to others. Sharing his progess. Solution is succinct with some necessary working not painstakingly written out – similar ability would be able to make the links. A rapid, 'in the head' way of working.	M O C W
Help 1 6.10pm Almost - have another look at your very last line. Great stuff otherwise! Help 1 6.13pm [Additional technical advice on even better use in marking-up mathematical text distinguishing between ordinary text and italicized script for variables]	Peter's small error in the 'very last line' spotted but praise and encouragement given. Technical advice useful for further studies which involve writing and reading mathematics	C D
Peter 6.37pm I put t-1 not 1-t like it should be … and ... spelt your name wrong. Now I've got the first one I'm motoring through the exercises. Who would have thought trigonometry could be this much fun. Thanks again	There would appear to be two very last lines – one for the mathematics, the other for the post. Peter corrects both errors. Conveys Peter's pleasure at doing this work. Peter appear as a human being letting others share what appears to be his enthusiastic personality. He shows his appreciation.	M E C
Help 1 6.56pm Lol, I was referring to the 1-t, but that too! Good luck with the rest of the problems.	Help 1 is laughing out loud (Lol) when only now he realises the 'very last line' confusion. Friendly banter.	C

Table A – List of key themes with representative letter used in coding

Code	Key Theme
C	Community Characteristics
M	Mathematics*
T	Thinking
U	Understanding*
W	Ways of Working (Mathematically)

* to represent - mathematical subject knowledge, or mathematical facts or the specific mathematics required for the problem

Table B – First retrievable thread where Peter has requested help

Message Thread 26 11 2006 [Additional commentary relevant to analysis]	Interpretative Commentary – initial analysis	Code
Peter. Post 9 Saturday 1.39pm hi ive[1] just began to read an introduction to number theory and inequalities and i seem to have fallen at the first hurdle. [Book defines a prime number] "later [the author] proves that this also means that p only divides by ±1 and ±p but i am still confused. surely this means that 6 is a prime as 6/42 and 6/6 but not 7. clearly i have over looked something. please explain. thank you.	Friendly opening, light hearted description of his current difficulty Peter has considered the definition carefully as he has come up with, for the moment, a contradiction Peter cannot see why 6 does not fit the some written formal definition although he knows that 6 is not a prime number. He openly admits that he somehow has a misunderstanding.	C T M C
Help A. 1.43 pm [4 minutes later] Yes, but if we choose m and n to be 14 and 3, 6 goes into neither 14 nor 3, but goes into 14*3 = 42. [i.e. a numerical example that shows using the definition that 6 is not prime.]	Example requires Peter to consider where his misconception lay – Peter's learning will develop through thinking things through.	W
Peter. Post 10 1.46pm [4 minutes later] Thanks. i new that it would be something basic like that i had overlooked. i thought it meant for any choice of multiples in which case all numbers would be prime. i get it now. cheers.	In a short space of time Peter has understood ('I get it now') the underlying reason made only implicitly by Help A giving a specific numerical example. Peter shows further thinking (and understanding) by commenting that his original misunderstanding would have implied that 'all numbers would be prime'.	U T
Deputy Moderator 1.47pm [Only 8 minutes after Peter has posted his query] [Writes a very full explanation as to the reason why the definition does not imply that 6 is a prime number].[Extends the examples to illustrate the mathematical sense of 'or' which can imply 'both']. Does that help? Do post back if not! By the way, it's *integer*, not *interger*. I don't want to be picky, but it's sometimes useful to know the correct spelling as it makes looking things up easier!	'Normally think what a prime is' – the most common definition used in school seems to be that a prime number can only be divided by itself and one. (See footnote 13). Peter has his general mathematical knowledge extended beyond the problem being considered. Invitation to ask for further explanation if needed.	M M C

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[1] Apart from interesting spellings, Peter does not use capital letters, which in a modern-day texting life is common and normal for some people. Peter's spelling and punctuation will be normally left as it appeared in the posts

Message Thread 26 11 2006 [Additional commentary relevant to analysis]	Interpretative Commentary – initial analysis	Code
[Continues to discuss the formal, rigorous definition in the book in relation to school definitions of prime numbers].	As a newcomer to the site Peter’s confidence is not dented by this comment. Peter starts another thread at 10.20am the next day and one minute later posts a second message: ‘sorry i meant integer. please pardon my spelling’[2] By extending Peter’s knowledge to a pure argument/definition rather than a less rigorous one used normally in school. Peter is being introduced to rigorous mathematics.	C W
Deputy Moderator 1.52pm Oops, it took me so long to write that that Help A got there first!	This exchange between two helpers, introduces Peter to the range of quick to measured solutions – he is party to seeing two different ways of working on the same problem and the merits of each.	W
Help A 1.52pm Better to get a full explanation after 10 minutes than a one-line sentence after 5!		
Peter. Post 11 3.14pm thanks to both of you. i get it now. just out of interest, others who have read the book how did you find it. ive never looked at number theory before but im finding it a bit harder to understand than other maths i have looked at. thanks to all	Peter completes the thread with a thank you. He also seems content to comment on his own mathematical ability - he is starting out on studying number theory and asks whether others are having problems too[3].	C

Table C: Representative Thread – when Peter established as a regular contributor
Interpretative Commentary on Trigonometrical Problem

Message Thread 12 05 2007 [Additional commentary relevant to analysis]	Interpretative Commentary – initial analysis	Code
Peter Post 385 Sat 4.25pm can some one help me with this problem. this is the first trigonometrical equation i have done so please take it slowly and drop me a few subtle hints. prove that: tan(45'+A/2)=(1+sinA)/cosA = cosA/(1-sinA) where 45' means 45 degrees sorry for the lack of formatting but i tried to put it in latex and it didn’t work. thanks for any help. [Here formulae text written using only standard keyboard that can be open to confusion The AskNRICH board has instructions on how to use a mathematical text (LaTeX).	Still asks only for hints. Makes it clear that this is a new area for his studies and would therefore like people to start at the beginning rather than assuming too much. Unusually, no sharing of his workings and thoughts so far – though later redressed Peter has carefully bracketed each part to help clarity and is thus presenting the question as clearly as he is currently able to.	T C C M
Help 1 Team Member A Sat 4.49pm Hi Peter: For the first equality, do you know the formula for tan (x +y)? Do you need help with the second equality? To write in LaTeX, start your line with \[, end with \], and write maths in the middle! (There's a slightly more comprehensive guide here.)	Help 1 is ‘testing the water’ to see how much Peter knows and how much help he would like. Concentrates on first part of equality leaving Peter, for now, to try out second part (if he can do the first), but making clear help is available if necessary. Instruction for marking up mathematical formulae relevant to AskNRICH	C M C W

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[2] In later posts, Peter generally reverts to his original spelling of interger! [Post 13].

[3] (Unfortunately) no-one responds to this open query here.

Message Thread 12 05 2007 [Additional commentary relevant to analysis]	Interpretative Commentary – initial analysis	Code
Peter Post 386 Sat 5.12pm if i sort the first equality then ill give the second a go. Yes I do know the formula to expand tan(X+Y) I have tried doing this and meant to post my workings here but forgot. ☺. [Provides workings – all correct]. from here i tried a variety of things but each one has failed, quite possibly because of a lack of competence on my part. can you nudge me from here please.	Peter responds directly (in reverse order) to both of Help 1's questions. Redresses his forgetfulness in not having posted his workings. Friendly smiley face (☺) conveying 'oops sorry'. Suggests it is his fault that he cannot do more, rather than a lack of experience. Asks again for just a small hint – a nudge.	M C T
Help 2 5.20pm Might help if you write sinA and cosA in terms of tan(A/2). [A succinct but key hint]	Hint given leaves Peter to make the connection on his own and using the standard 're-written' form given in any formulae or relevant text book.	M
Peter Post 387 Sat 5.50pm as i guessed i failed because of a lack of competence on my part when trying the correct option. i did this before but I think I must have gone wrong short of the mark. ill put it down to experience. if anyone is interested i did the following: [Shares solution although there is a small error writing 1-t not t-1in the final line]. thanks Help 2 and Help 1. [Misspells latter name]. ill post back if i can't get the second one	Half an hour later Peter has (he thinks) got the correct answer. Continues to comment on his own incompetence and inexperience, though a possible implicit indication that he will be better placed in future. Fostering community membership by offering his solution to others and sharing his progress. Solution is succinct with some necessary working not painstakingly written out – similar ability would be able to make the links. A rapid, 'in the head' way of working. Content to continue to try the second alone first but knows that he can call on others to help if necessary.	M C W C
Help 1 6.10pm Almost - have another look at your very last line. Great stuff otherwise! Help 1 6.13pm [Additional technical advice on even better use in marking-up mathematical text distinguishing between ordinary text and italicized script for variables]	Peter's small error in the 'very last line' spotted but praise and encouragement given. Technical advice useful for further studies which involve writing and reading mathematics	C M
Peter Post 388 6.37pm i put t-1 not 1-t like it should be … and ... spelt your name wrong. Now ive got the first one im motoring through the exercises. who would have thought trigonometry could be this much fun. thanks again	There would appear to be two very last lines – one for the mathematics, the other for the post. Peter corrects both errors. This sections conveys Peter's pleasure at doing this work. Peter appears as a human being letting others share what appears to be his enthusiastic personality. He also shows his appreciation.	M U C
Help 1 6.56pm Lol, I was referring to the 1-t, but that too! Good luck with the rest of the problems.	Help 1 is laughing out loud (Lol) when only now he realises the 'very last line' confusion. Friendly banter.	C

Post	Post text
Peter Mon 8.12pm	I do appologise for keeping on asking all these questions you but I would be grateful for any help. I am looking at mathematical induction and have been shown this proof "we show that f(n)=4^(3n-2)+2(3n-2)+1 is divisible by 7 for all positive intergers n. call this proposition p(n). Now f(1)=4+2+1+7 so p(1) is true. If p(k) is true then there exists an interger m such that f(k)=4^(3k-2)+2^(3k-2)+1=7m. multiplying by 64 we get 4^(3k+1)+2^(3k+4)+64=448m and so f(k+1)=4^(3k+1)+2^(3k+1)+1=448m+2^(3k+1)-2^(3k+4)-63=448m-(2^(3k+1)(8-1))-63=7(64m-2^3k+1)" --- i understand all before and after the induction of f(k+1). i cannot see why f(k+1)=4^(3k+1)+2^(3k+1)+1 when f(k)=4^(3k-2)+2^(3k-2)+1 thanks in advance.
Help 1 8.16pm	substitute in k+1 for k in the formula f(k)=4^(3k-2)+2^(3k-2)+1
Peter 8.22pm	GRRRRRR I've just spent about 1 1/2 hours staring at it in this book and couldnt see what I was missing and all it took was to see that it was 3k not k. just goes to show how easy it is to miss the silliest things. thanks anyway, I'll sleep tonight now.
Peter 8.50pm	Could I please ask two further questions. 1. Can anybody reccomend an indtroductory website to induction with some basic questions to "break me in to it". 2. I'm having some trouble with applying it in a proof im constructing myself though I can follow the proof when I see it. Is this normal when learning induction as a new subject?
Nick (Help 2) 8.58pm	If you're at a college or a sixth form, try to get hold of an FP3/P6 book (it's FP3/P6 in Edexcel, not sure about other exam boards). I imagine you'll be able to get the hang of it by doing some of the questions in there ...
Peter 9.12pm	im currently doing my gcse's. im reading a book on number theory and am trying to get the hang of induction. would you recommend that book any way? if so would it be available readily from libaries or good book shops? also is it good for covering other simlair topics?
Nick 9.19pm	Hmm. I guess a good idea in that case then would be to get your maths teacher to spend a little time aside with you to explain the general principle, and to give you some questions to have a go at. If he or she explains it to you then it could 'click', and you'll be fine from then on. Here's a pdf I found after a quick Google search: http://www.maths.uwa.edu.au/~gregg/Academy/1995/inductionprobs.pdf Questions 1 and 2 look approachable, you should start with those ... Also, there are solutions with the questions ^_^ ...
Peter 9.29pm	thanks for that. its not that i dont understand the principle of how it works it just i struggle to construct any proves myself. thanks for the questions. they should help me understand how to construct them. has anybody else on here had simlar problems? thanks once again.

Post	Post text
Nick 9.38pm	Once you get the hang of it I'm sure you'll be fine. I remember my first induction proof hehe. "Prove that the sum of the first 'n' natural numbers is equal to n(n + 1)/2." Try that ^_^ , post back if you need help with it.
Peter 9.55pm	can you tell me if my method is correct please. 1+2+3...+n=f(n)=(n(n+1))/2 so f(n+1)=(n(n+1)/2)+n+1 =(n^2+3n+2)/2 if n=even, f(n+1)=even+even+even=even and if n=odd f(n+1)=odd+odd+even=even so f(n+1)/2 is this correct? thanks
Help 3 10.04pm	You should make sure you have a base case. Just put f(1)=1=1*2/2 so it is true for n=1. Once you have your $(n^2+3n+2)/2$ you wnat to show this is the same as f(n+1) (i.e what you get by substituting n+1 into f(n)) which isn't too hard if you factorise it.
Peter 10.13pm	thanks, i forgot f(1). actually i sort of went off the point here i just realised, i ve been trying to prove that sequences equal interger values tonight so i was in that mind set. lol.
Peter 10.21pm	i've ran myself into knots. i apprieciate i have gone down totally the wrong path. could i have some hints please? just so that i know the way in which i need to approach the question. thanks.
Peter 10:32pm	i can prove this by pairing of the 1st and last and 2nd and 1 from last numbers but cannot do it through induction. i think that i may need to fill in gaps in my knowledge. thanks for all the help you have offered.
Help 4 10.33pm	1. Show it is true for some integer (usually 1). 2. Assume it is true for n=k. 3. Show that it is true for n=k+1. In this case, just see what you get when you factorise (n^2 + 3n + 2)/2.
Moderator 11.16pm	There's an NRICH article (given as a hyperlink) with an introduction and some questions which may be of interest.
Next day	
Peter 8.37am	when factorised it equals (n+1)(n+2) and this is the same as n(n+1))/2 with n=n+1, is this correct? thanks to everyone for your help.
Help 5 4.25pm	Yes, that is correct. You pretty much had it first time until you went off on a tangent about odd and even numbers! ;) Now try the n^2 one, that $1^2 + 2^2 + 3^2 + ... + n^2 = (2n^3 + 3n^2 + n)/6$, that is n(n + 1)(2n + 1)/6.
Peter 4.42pm	when n=1 (2n^3+3n^2+n)/6=1 now assume it is true for n now induce (is induce the correct word) n+1 (2n^3+3n^2+n)/6+(n+1)^2=1^2 + 2^2 + 3^2 + ... + n^2+(n+1) =(2n^3+3n^2+n)/6+n^2+1+2n =(2n^3+3n^2+n)+6n^2+12n+6/6 =2n^3+9n^2+13n+6/6 =(2n^3+3n^2+n)/6 when n=n+1 so by induction this is true yay!!! can you check this and tell me whether it is correct. Thanks

Post	Post text
Help 6 4.48pm	quote: it's FP3/P6 in Edexcel, not sure about other exam boards It's fp1 on OCR MEI (and one of my favourite sections from the module) and I had the impression that it's fp1 on most other exam boards too, though I don't actually know for certain for anything other than OCR. quote: 2n^3+9n^2+13n+6/6 =(2n^3+3n^2+n)/6 This stage doesn't seem to make any sense. Plus you appear to have ended up with what you started with instead of what you started with subed in. I suggest working with everything factorised - it allows you to see what you are going to end up with when you've proved it much easier. The statement n=n+1 doesn't make much sense either. Edit: Having reconsidered I think you may actually be correct (though I still can't really tell). It's just the use of n=n and n=n+1 (which is why you are supposed to stick to n=k and n=k+1) and the means that I have trouble seeing what you are saying.
Help 7 4.59pm	=2n^3+9n^2+13n+6/6 =(2n^3+3n^2+n)/6 when n=n+1 I see what you're trying to say, but you need to be much clearer than this!
Help 8 5.08pm	In particular, you need to explicitly write it out in terms of (n+1), so you get ... $= (2n^3 + 9n^2 + 13n + 6)/6$ $= (2(n+1)^3 + 3(n+1)^2 + (n+1))/6$
Peter 7.16pm	yeh i see that it would be helpful to use another symbol, it does look confusing. thanks to everyone who has helped me with this, you have realy improved my understanding of this topic.
Peter 7.40pm	would anybody mind explaining the meaning of xxxxx. thanks because i have encountered it in the induction article for the first time and can not decifer its meaning. Thanks.
Nick 7.47pm	That means the sum of all the integral values of 'n', from n = 1 to n = infinity. This is known as an 'infinite sum' I think.
Deputy Moderator (DM) 7.53pm	Nick has explained what you've written, and he's quite right, it's an infinite sum. I just wanted to point out that none of the sums in the article is infinite. They're all things like xxx which is the sum from *i*=1 to *i*=*n* of *i*, i.e., the sum of the integers from 1 to *n* inclusive.
Help 9 7.53pm	1+2+3+4+5+...
Peter 8.11pm	yes i used the infinite sum above because i had trouble formatting. how would one go about reading such an expression? is the top number the upper limit, the bottom number the lower limit and the middle number the way in which it adds, so if it is xxx then it would increase by cubes? i appolagise for all these questions but am trying to learn a totally new subject. thanks to all.
Help 6 8.20pm	$=1^3+2^3...+n^3$
DM 8.36pm	I'd read what you've written as ``the sum from i=1 to n of i cubed", which is what raoulh has written out in symbols. Please don't apologise for asking questions: we're here to try to answer them! (And asking how to read maths is always a good idea, because books and articles very seldom tell you.)
Peter 8.39pm	thanks, your all really helpful.

Post	Post text
Next day	
Peter 8.23pm	xxxx = (r^n)-1 when r =/= 1 -------- r-1 btw r(i-1) is r^(i-1) this question was in the article on nrich on induction im reading and i guess that it means for all values of r. am i correct, im new to this notation as yoou may have guessed. Thanks
DM 8.28pm	You mean xxx, I think. (I'm putting that there so that you can click on it to find out how to get it in LaTeX. The important thing is that if you want more than one thing in a superscript (or subscript) then you have to include it in curly brackets.) Yes, this is for any *r* (except 1, of course, because then we'd be dividing by 0, which isn't allowed). You might like to write out what this means without a big sigma sign, to get some practice at decoding. This thing is called a geometric series, by the way; I think they come up in A level maths. I hope that the article is starting to make sense!
Peter 8.41pm	yes thank you i have found the article enjoyable and informative even though the questions after the first are hard for me to understand but at least its a challenge am i correct that = 1 + r^1 + r^2 .. + r^n?
DM 8.42pm	Very, very close. But you might want to check exactly where the sum stops.
Peter 10.13pm	would it be r^(n-1) because of the i-1 rather than r? thanks
DM 10.19pm	Spot on well done!

Tables A, B and C present the three different threads [**3Thds**] that appeared on the conference board all dealing with the same problem with an interpretative commentary focused on the mathematics. The problem comes from the BMO 2005 paper and which is now available for would-be aspiring BMO candidates (or others) to use as a practice question.

For the first thread the originator is Peter (the AskNRICH participant used as the case study) who is ‘stuck’ on the second part of the question. For the second thread, appearing some six weeks later, a different AskNRICH member, R, is ‘stuck’ on the first part of the question and this time it is Peter who offers substantial help over several messages. The third thread appearing some nine months after the second and in the next academic year, is posted by S and focuses (initially) on the first part. Peter is again the first person to offer help, though this time only once. One of the other contributors [Help C] returns to respond to a comment that is using the same value as Help C gave in the first thread.

Table A Thread One [**3Thd1**]

Post Number Day/time	Message extract	Commentary on Mathematics
Peter (the AskNRICH participant used as the case study) aged 14 to 15 begins this thread. (He has now posted over 100 messages).		
Thd1 P1 Saturday 11.13am	Peter: Let n be an integer greater than 6. Prove that if n - 1 and n + 1 are both prime, then $n^2(n^2 +16)$ is divisible by 720. Is the converse true? i have managed to prove the first part of the question using the fact that all primes are of the form 6n-1 and 6n+1. when i tried to prove the converse i cant do it. i know that 2 and 3 divide n and n is of the form 2mod5 3mod5 or mod5. from here where do i go? thanks	This is the question Will see later in Threads Two and Three that this line is essential in finding the solution to the first part (and see P4 below to see how it will help in solving the second part). [It would be less confusing if written as 6k-1 and 6k+1 as in this problem n is representing specific integers. [See Thd1 P5] For converse to be true then whenever $n^2(n^2 +16)$ is divisible by 720 then n-1 and n+1 are both prime (numbers) Peter’s mod5 should strictly be 0mod5
Thd1 P2 Saturday 11.14am	Help A: Do you think the converse is true?	The converse is not true – as will be seen later
Thd1 P3 Saturday 11.37am	Peter: i presume that it isn't but im not very sure	No indication from Peter as why he would presume that it was not true, but it is a reasonable assumption that the wording of the question wording leads one to favour the converse not being true. If the converse were true it would be natural as a question setter perhaps to simply ask for a proof, or in fact most likely to set the whole question up as an ‘if and only if’

Post Number Day/time	Message extract	Commentary on Mathematics
Thd1 P4 Saturday 11.39am	Help B: If you look back over your proof, you used the fact that ALL primes are 6n-1 and 6n+1. However, is the converse of *this* true? Are all 6n-1 and 6n+1 prime? Using this, you can construct a counterexample.	Finding a counterexample is one type of proof – i.e. if one example can be found that does not work then such an example is sufficient to show that in general the statement will not be true. A useful strategy to start with if it is suspected (as in the case here) that it will be found not to be true
Thd1 P5 Saturday 11.58am	Peter: thanks i ve got it now, for anyone who's interested one counter example is 48.	If n is 48 then 2304 x 2320 is divisible by 720. Although n-1 (47) is prime, n+1 (49) is not prime hence n-1 and n+1 are not both prime numbers and the converse is not true [Returning to prime numbers being of the form 6k+1 and 6k-1 (generally written as 6k±1) then for example 49 can be written as 6x8 + 1 [k being 8 here] and as such is in the form 6k+1 but 49 is not a prime number. Likewise 35 can be written as 6x6 – 1[k being 6 here] and as such is in the form of 6n-1 but is not prime. Hence numbers of the form 6n+1 and 6n-1 are not necessarily prime]
Thd1 P6 Saturday 12.02pm	Help A: or 24 ☺	This is not actually a counterexample as will be seen later in [P9 below]. Is there some implication being made here that a lower value i.e. a smaller number for the counterexample is (even) 'easier' to find?
Thd1 P7 Saturday 12.07pm	Help C: Or if you really want to do no work whatsoever when it comes to multiplication just use 720	If n = 720 is used then n+1 is 721 which is not prime (divisible by seven). Given the original $n^2 (n^2 +16)$ divisible by 720, then for n = 720 n^2 must be divisible by 720 and thus $n^2 (n^2 +16)$ is also divisible by 720
Thd1 P8 Saturday 12.10pm	Peter: lol i totally missed that	
Thd1 P9 Saturday 6.59pm	Help D: Not to be a spoil sport, but I don't think 24 quite cuts it as a counterexample ☺	[See P6 above]. For n = 24, n+1 (25) is indeed not prime and of the form 6n+1 with n = 4. However $n^2 (n^2 +16)$ is not divisible by 720
Thd1 P10 Saturday 7.11pm	Help A: Sorry haha, I was thinking that all numbers 0 (mod 6) worked. Good job i didn't make that mistake when I took the paper last year!	

Table B Thread Two [**3Thd2**]

One and a half months later, a second thread on same question (though this time the problem is with the first part of the question which Peter had been able to do). R is in Year 9 (13 to 14 years old), one year younger than Peter. This is only R's 12th post

Post Number Day/time	Message extract	Commentary on Mathematics
Thd2 P1 Monday 7.05 pm	R: Let n be an integer greater than 6. Prove that if n-1 and n+1 are both prime, then $n^2(n^2+16)$ is divisible by 720. How would I start this question? I thought of trying to split 720 up into its prime factors, 2,2,2,2,3,3,5. However, this didn't get me anywhere. Can anybody give me any hints?	The prime product of 720 is 2x2x2x2x3x3x5 (2^4x3^2x5) and indicates the powers of the primes needed Prime factors will underpin S's solution (later) in Thread Three [Thd3 P4]
Thd2 P2 Monday 7.07 pm	[Help from] Peter: what form do primes greater than 6 take. as soon as you see what to do this is very simple so i shall leave the hint at that	See Peter's very first message in the first thread [Thd1 P1] where he has stated that he proved that all primes are of the form 6n-1 and 6n+1
Thd2 P3 Monday 7.07 pm	Help E: If n-1 and n+1 are prime, what can you say about n (nothing too deep - just divisibility sorts of things)?	See R's next message in response to this question
Thd2 P4 Monday 7.14pm	R: My thoughts: • n must be even, because primes above 6 are all odd, so (n-1) must be odd, so n is even. • n must also be divisible by 3, because (n+1) and (n-1) are both **not** divisible by 3. • I think that there must be some way to work out the last digit of n by knowing that (n-1) and (n+1) are not divisible by 5, but either n^2 or n^2+16 must be divisible by 5 (so the product is divisible by 720). I can't work out how though.	2 is the only even prime number so all even numbers greater than two are not prime (as they can be divided by two) hence for n-1 to be a prime number (greater than six as given in the question) then n-1 must be odd. As n is the whole number next to n-1 (which is odd), then n is even. The same argument would follow for n+1 being a prime number In any set of the three consecutive integers (whole numbers) one of the set must be divisible by 3. n-1, n and n+1 are three consecutive numbers and thus one of them must be divisible by 3. As the question states that both n-1 and n+1 are prime numbers then neither of these can be divisible by three and thus n remains the only possibility and must be the integer that is divisible by 3 [See Thd2 P1above about the necessity for divisibility by 5] As later help messages will show R is on the 'right' lines by considering '(n-1) and (n+1) are not divisible by 5' but R appears, as yet, unable to go further

Post Number Day/time	Message extract	Commentary on Mathematics
Thd2 P5 Monday 7.14pm	Help F: The first two points suffice. n is even, and n is divisible by 3, can you make a slightly stronger statement about n and use this as the base of your argument?	For n to be even **and** divisible by 3, it must be divisible by 2 and 3 and thus is divisible by 6 (2x3). Thus for any six consecutive numbers starting with a multiple of 6 (i.e. 6k) then the second (6k+2) and fourth (6k+4) are divisible by 2 (as even numbers) and the third (6k+3) is divisible by 3. This leaves only the first (6k+1) and the fifth (6k+5) as possible candidates to be prime numbers. 6k+5 is often represented by 6k-1 as both imply the integer immediately before a multiple of 6 Thus as Peter first remarked (using n instead of k) in Thd1 P1 all primes are either of the form 6k-1 or 6k+1
Thd2 P6 & P7 Monday 7.32pm &	R: I think n must end in 2 or 8. This is because n^2+16 must divide by 10, so n^2 must end in 4, so n must end in 2 or 8	R is aiming for 5 to be a factor of n^2+16 (in that 2, as n is even is already a factor). It might however be the case that n is a multiple of 5 in which case n^2+16 would not be needed) If n^2+16 is indeed divisible by 10 then n^2+16 must total a number ending in zero and thus any number that 16 is added to that can result in a number ending in zero, must itself end in 4 [4 +16, 14+16, 24+16 etc]. As n^2 ends in a 4 then only numbers ending in a 2 or 8 when squared would end in a 4
7.34pm	R: n must divide by 6, and end in 2 or 8, so n can be expressed as (some multiple of 30)+(12 or 18). I'm not sure how to write this algebraically though	Numbers that are multiples of 6 that end with either a 2 or an 8 are e.g. 12, 18, 42, 48, 72, 78, etc. i.e. the numbers come in pairs, 30 apart. 30 apart leads to a topic known as remainder or modular [mod] arithmetic – i.e. the remainder is written down division by, in this case, 30. 12,42,72 are all equivalent to 12 mod 30 and 18, 48, 78 are all equivalent to 18 mod 30
Thd2 P8 & P9 Monday 7.38pm	Peter: if you know modular arithmetic try putting multiples of 6 where 6kmod30+1 and 6kmod30-1 are both possibly prime as n. i think that there is a nicer way but this is still nice and simple and im tired at the moment. hope this helps. if you dont know modular arithmetic mod 30 is just a multiple of 30 and 6kmod30 is 6k + mod30	Link attempted here of three established facts: from Peter's knowledge that 'primes can only be of the form 6n+1 and 6n-1' and from R's messages: 'n must divide by 6' and 'some multiple of 30'
7.39pm	sorry ive realised you can write this as mod5	Given that 5x6=30, multiples of 6 modulo 30 may as well be written modulo 5

Post Number Day/time	Message extract	Commentary on Mathematics
Thd2 P10 Tuesday 5.07pm	R: I don't really understand this Peter. Sorry. Does anyone know a different way to give a hint/explain Peter's hint?	
Thd2 P11 5.27pm	Help C: If n-1 and n+1 are prime, what are the possible remainders when you divide n by 5?	
Thd2 P12 5.38pm	Peter: since i don think that you understand modular arithmetic (dont worry about this) i shall write in a basic algerbraic form 6k - cannot be prime because 6 divides it 6k+1 - maybe prime	
	6k+2 - cannot be prime because 2 divides it 6k+3 - cannot be prime because 3 divides it 6k+4 - cannot be prime because 2 divides it 6k+5 - maybe prime where k > 0	This is the 'same' explanation given in this column under Thd2 P5
	so for x to be prime x is in the form 6k-1 or 6k-1.	Peter has made a typing error repeating as the first 6k-1 should be 6k+1, (This is never picked up)
	so if n-1 and n+1 are both prime n = 6k so then this proves your statement earlier that 6 divides n.	
	so now you must show that n^2 or n^2+16 is divisible by 5	See R's message [Thd2 P15] where this is reasoned
	now n is in the form 5k, 5k+1, 5k+2, 5k+3, 5k+4 for which of these is it true that n-1 and n+1 is prime?	If n = 5k (and therefore divisible by 5) then n - 1=5k-1 and as this is not divisible by 5 it could be prime. Similarly for n+1. Hence for 5k it could be true that both n-1 and n+1 are prime numbers. The same argument would apply for n = 5k+2 and n = 5k+3. However for n = 5k+1, n-1=5k and 5k is not prime (divisible by 5 at the very least) though n+1 = 5k+2 could be prime. But importantly not both are prime. This is also the case with n = 5k+4 as n+1= 5k+5 and this is not prime.
	now square n and sub in $n^2 = 5k + m$ where you know m and then prove what that n^2+16 or n^2 are divisible by 5 i hope this makes sense.	i.e square the cases for n=5k+m (As it stands it is not clear as n itself will be of the form 5k+m. Actually Peter means square 5k+m for known values of m)

Post Number Day/time	Message extract	Commentary on Mathematics
Thd2 P13 5.48pm	R: OK. 5k+1, because n-1 would divide by 5 and 5k+4, because n+1 would divide by 5. Is there a way to narrow it down further? Or can m have multiple values.	As explained immediately above! [Thd2 P12] However this is the opposite (converse) to what will be needed for further work as the forms required are for when both n-1 and n+1 could be prime i.e. for n=5k, or 5k+2 or 5K+3 (and not divisible by 5). This relates to Peter's 'm' in message immediately above being 0 or 2 or 3 [Thd2 P12] Currently has more than one possibility – enquiring if there should be only one. 0 or 2 or 3 (as given in Thd2 P12) are the multiple values that R's question seeks
Thd2 P14 5.57pm	R: Assuming m can have multiple values: For m=0, n^2 will equal $25k^2$, so it will be divisible by 5. For m=2, n^2 will equal $25k^2+20k+4$. Therefore, $n^2+16=25k^2+20k+20$, so it will be divisible by 5. For m=3, n^2 will equal $25k^2+30k+9$. Therefore, $n^2+16=25k^2+30k+25$, so it will be divisible by 5. Yes, so n can be expressed as 5k+m, where k is a positive non-zero integer, and m equals 0, 2, or 3.	[Has gone on to answer own question posed immediately above]
Thd2 P15 6.02pm	R: Got it! n is divisible by 6, so n^2 is divisible by *36*. n^2 is divisible by 4, so n^2+16 **must** also be divisible by *4*. Either n^2 or n^2+16 must be divisible by *5*, from the last post. *36*5*4*=**720**	See Thd2 P12 for the instruction/statement to show this
Thd2 P16 6.05pm	Peter: yes well done this completes the proof. i remember fondly this question. this was my first bmo question i completed. arrr memories ... yes anyway. well done.	

Table C Thread Three [**3Thd3**]

The same question is started on a third thread some 9 months later (and a different academic year). S is in year 12. Help G (who first appears in post 3 is in year 11, the same year as Peter).

Post Number Day/time	Message extract	Commentary on Mathematics
Thd3 P1 Thursday 4.33pm	S: n be an integer greater than 6 where n - 1 and n + 1 are both prime. Prove that $n^2(n^2 +16)$ is divisible by 720. I've written 720 as prime factors, and I've proved that the expression is divisible by all of the prime factors except 5. Can anyone give me a hint?	
Thd3 P2 Thursday 4.43pm	[Help from] Peter: consider n mod5, what can it/cant it be?	
Thd3 P3 Thursday 4.45pm	Help G: Write n as 5k+a and consider which numbers a can be given n-1 and n+1 are both prime (so not divisible by 5) and substitute these possible expressions for n into $n^2(n^2+16)$. (Alternatively/equivalently if you know about modular arithmetic consider possible values of n mod 5) You said you showed it is divisible by the other prime factors of 720 - did you make sure you showed that it was divisible by them to the right power (ie that 3^2 and 2^4)?	a here is m in Thd2 P12 9x16=144 leading to 144 x 5 =720 (5 being the missing prime factor)
Thd3 P4 & P5 Thursday 5.03pm 5.08pm	S: sorry about the formatting, I am putting the backslashes in, but they don't seem to be working [To Help G] This is what I've done - we know that n must be divisible by 2 and 3 for the numbers on either side of it to be prime. $n^2(n^2+16) = n^4+ 16n$. n^4 is divisible by 2^4 and 3^4 (so is also divisible by 3^2). $16n^2$ is divisible by 2^4x2^2 (so is also divisible by 2^4). Therefore, $n^2(n^2+16)$ must be divisible by 3^2 and 2^4 n-1 and n+1 can't be 0 mod 5, therefore n can't be 1 or 4 mod 5. This means it must be 2 or 3 mod five, so it's square must be 4 mod5. Therefore $n^2 + 16$ is 4+1 mod 5 = 0 mod 5. Thanks everyone. I can't believe I didn't see that... Sorry again, I've just realized that you need to put + in ….	As n is divisible by 6 (see e.g. Thd2 P7) then n is divisible by 2 and 3 Fails to include the step that as n^2 is divisible by 3^2 then $16n^2$ is divisible by 3^2 [maybe not helped by multiplying out to $n^4+ 16n$] which is (strictly) needed to state therefore … Now looking for divisibility by 5 for $n^2(n^2+16)$ As n-1 and n+1 are prime numbers that cannot be divisible by 5 (or ano) Has omitted the case of 0 mod 5 which is picked up in P6 below square 2: 2x2=4 which is 4 mod 5 and square 3: 3x3=9 which is also 4 mod 5 hence S's comment here that '2 or 3 mod five, it's square must be 4 mod5' [To be able to format powers for normal text on screen]

Post Number Day/time	Message extract	Commentary on Mathematics
Thd3 P6 Thursday 5.08pm	Help G: One small thing you've missed - n can be 0 mod 5, but that gives you n^2 is 0 mod 5 so you're still fine. Have you got the second bit about the converse as well?	i.e. this time it is n^2 that is divisible by 5 (instead of showing that $n^2 + 16$ is divisible by 5 conveyed in Thd3 P4 above)
Thd3 P7 Thursday 6.58pm	S: The converse is false because 720 is divisible by 720, but 720+1=721 has a factor of 7	The same counterexample as suggested by Help C in Thd1 P7
At this stage the problem has been fully resolved (or so it appears). Two other AskNRICH posters (ANP(1) and ANP(2)) join in the 'conversation'.		
Thd3 P8 & P9 Thursday 7.41pm 7.44pm	ANP(1): If the converse were true, then it would be a really, really fast way to find big prime numbers! ANP(2): Not to mention being a proof of the twin prime conjecture!	The twin prime conjecture is that there are infinitely many primes p such that p+2 is also prime. n-1 and n+1 both being prime and also two apart are therefore twin primes. The proof is still being debated
Thd3 P10 Thursday 8.46pm	S: I've just realised that my counter example is exceedingly wrong as while 720 is divisible by 720, $720^2(720^2+16)$ isn't. So is seems that converse is false, but finding a counter example might be hard. Maybe by trying to solve $n^4 + 16n^2 - 720n=0$, but I feel there should be something like a proof by contradiction. (By the way, for it to be a proof of the twin prime conjecture you'd also have to prove that there are infinitely many multiples of 720 that can by written as the product of a square number and that number +16.)	It is difficult to know why S has suddenly decided that this incorrect (as it isn't!)
Thd3 P11 Thursday 8.56pm	Help C: [copies S's post from above] **quote:** I've just realised that my counter example is exceedingly wrong as while 720 is divisible by 720, $720^2(720^2+16)$ isn't. Yes it is! $720^2(720^2+16)/720 = 720(720^2+16)$.	See P3 Thd7 above 'Yes it is' to relate to the original counterexample, rather than the error is exceedingly wrong!
Thd3 P12 Friday 5.05pm	S [To Help C]: For some reason I was thinking of $720^2 + (720^2+16)$. Lets just hope I didn't make a mistake like that on the BMO today...	Indeed!

Tables A, B and C present the three different threads [**3Thds**] with an interpretative commentary dealing all with the same problem as its focus. The problem comes from the BMO 2005 paper and which is now available for would-be aspiring BMO candidates (or others) to use as a practice question.

For the first thread [Thd1] the originator is Peter (the AskNRICH participant used as the case study) who is 'stuck' on the second part of the question. For the second thread [Thd2], which appears around six weeks later, a different AskNRICH member R is 'stuck' on the first part of the question and this time it is Peter who offers substantial help over several messages. The third thread [Thd3] arrived some nine months later and in the next academic year is posted by S and focuses (initially) on the first part. Peter is again the first person to offer help, though this time only once. One of the other contributors [Help C] returns to respond to a comment that is using the same value as Help C gave in the first thread. [Thdx Px represents Thread number, Post number].

Table A: Thread One

Post Number Day/time	Message extract	Commentary on Social Actions
Peter (the AskNRICH participant used as the case study) aged 14 to 15 begins this thread. (He has now posted over 100 messages).		
Thd1 P1 Saturday 11.13am	Peter: Let n be an integer greater than 6. Prove that if n - 1 and n + 1 are both prime, then $n^2(n^2 +16)$ is divisible by 720. Is the converse true? i have managed to prove the first part of the question using the fact that all primes are of the form 6n-1 and 6n+1. when i tried to prove the converse i cant do it. i know that 2 and 3 divide n and n is of the form 2mod5 3mod5 or mod5. from here where do i go? thanks	Posted during a weekend, out of school time Provides information that first part has been successful and specified the fact used Offers current working and thoughts Offers thanks in advance
Thd1 P2 Saturday 11.14am	Help A: Do you think the converse is true?	Just one minute later, a reply arrives and a suggestion of an Intuitive Approach Good intervention as a key question is offered to help Peter to move forward: a personal feeling will lead to different proof strategies depending if felt to be or not to be true. In addition it is a useful strategy to acknowledge (be aware of) own feelings to pursue a reasoned path to the solution, even if eventually the feeling turns out to be incorrect
Thd1 P3 Saturday 11.37am	Peter: i presume that it isn't but im not very sure	It is not possible to know whether the time gap of 23 minutes was used to think about Help A's question Help A's question appears not to have been sufficient to 'nudge' Peter to be able to solve the problem

Post Number Day/time	Message extract	Commentary on Social Actions
Thd1 P4 Saturday 11.39am	Help B: If you look back over your proof, you used the fact that ALL primes are 6n-1 and 6n+1. However, is the converse of *this* true? Are all 6n-1 and 6n+1 prime? Using this, you can construct a counterexample.	Two minutes after Peter's Post about being unsure a second helper joins in reiterating what Help A was asking but providing greater elaboration on how to consider the converse by posing … … this key question 'Are all 6n-1 … ' If Peter succeeds with a counterexample proof then he has had exposure to a useful strategy when he attempts to conduct further proofs on other mathematical problems. He will have increased his problem solving toolbox (as referred to in Exemplar Thread Two, Chapter Nine)
Thd1 P5 Saturday 11.58am	Peter: thanks i ve got it now, for anyone who's interested one counter example is 48.	19 minutes later Peter has used Help B's suggestion and found a counterexample so solution is completed, and after 45 minutes of Peter's first post Offers/shares his counterexample to other readers of the thread implicitly sensing a group of like-minded peers or 'club'
Thd1 P6 Saturday 12.02pm	Help A: or 24 ☺	Although solution has been posted Help A remains involved with this thread and offers a second lower number to be another counterexample. Including the emoticon suggests that the number being lower is thus even easier to find. 24 is not a counterexample and in P9 below Help A gets their 'comeuppance'
Thd1 P7 Saturday 12.07pm	Help C: Or if you really want to do no work whatsoever when it comes to multiplication just use 720	Help C also joins in after the after the problem has been solved but widens the mathematical conversation between participants of this discussion thread Once 720 has been suggested, this telling, 'very clever' counterexample becomes blindingly obvious! Help C will return to this counterexample during Thread 3 [Thd3 P7, P10 - 12]
Thd1 P8 Saturday 12.10pm	Peter: lol i totally missed that	Peter 'laughs out loud' - akin to kicking one's self for not noticing the blindingly obvious (once it has been pointed out). Implies admiration for thinking of the 'easy' value and further evidence of a friendly group atmosphere
Thd1 P9 Saturday 6.59pm	Help D: Not to be a spoil sport, but I don't think 24 quite cuts it as a counterexample ☺	Some seven hours later a new helper (who has obviously read the thread through) responds to suggest to Help A mistake made in suggesting 24 as a counterexample. This is done in a polite/good humoured way tempered further with a repeat of the emoticon

Post Number Day/time	Message extract	Commentary on Social Actions
Thd1 P10 Saturday 6.59pm	Help A: Sorry haha, I was thinking that all numbers 0 (mod 6) worked. Good job i didn't make that mistake when I took the paper last year!	Help A acknowledges his error (laughing at himself) and shares his incorrect assumption P6-10 all convey a sense of 'jolliness' amongst the participants in this thread

Table B: Thread Two

One and a half months later, a second thread on same question (though this time the problem is with the first part of the question which Peter had been able to do). R is in Year 9 (13 to 14 years old), one year younger than Peter. This is only R's 12th post

Post Number Day/time	Message extract	Commentary on Social Actions
Thd2 P1 Monday 7.05 pm	R: Let n be an integer greater than 6. Prove that if n-1 and n+1 are both prime, then $n^2(n^2+16)$ is divisible by 720. How would I start this question? I thought of trying to split 720 up into its prime factors, 2,2,2,2,3,3,5. However, this didn't get me anywhere. Can anybody give me any hints?	School day R is making only their 12th post but already is prepared to share their thinking and is requesting only hints R is on the right track but appears unaware of this
Thd2 P2 Monday 7.07 pm	[Help from] Peter: what form do primes greater than 6 take. as soon as you see what to do this is very simple so i shall leave the hint at that	First of two simultaneous help messages that arrive only two minutes after R has asked for help. Peter had been able to do this part successfully and had used the hint he is giving here in his own solution (Thd1 P1 above). Hence Peter is likely to feel that R should be able to approach this in the same way (i.e. Peter is leading R). It may not be as 'very simple' as Peter suggests Only a hint is given to allow R to try things out for themselves
Thd2 P3 Monday 7.07 pm	Help E: If n-1 and n+1 are prime, what can you say about n (nothing too deep - just divisibility sorts of things)?	Help E's Post arrives simultaneously with Peter's This appears a fairly minimal hint, (though possibly easier than Peter's) that leaves R to think things through

Post Number Day/time	Message extract	Commentary on Social Actions
Thd2 P4 Monday 7.14pm	R: My thoughts: • n must be even, because primes above 6 are all odd, so (n-1) must be odd, so n is even. • n must also be divisible by 3, because (n+1) and (n-1) are both **not** divisible by 3. • I think that there must be some way to work out the last digit of n by knowing that (n-1) and (n+1) are not divisible by 5, but either n^2 or n^2+16 must be divisible by 5 (so the product is divisible by 720). I can't work out how though.	Seven minutes later R shares his 'new' thoughts. The response illustrates R's current knowledge of primes and further thoughts on working towards a solution. Some of this may be new as a result of the help that has been offered by Peter and Help E but (a) he appears however not yet to have realised the form 6k±1 and (b) he has written a lot (succinctly and clearly) in, on the surface at least, seven minutes
Thd2 P5 Monday 7.31pm	Help F: The first two points suffice. n is even, and n is divisible by 3, can you make a slightly stronger statement about n and use this as the base of your argument?	Here Help F is attempting to bring R round to realising that primes are of the form 6k±1 [even if R is not used to this notation] and would use other words to describe it Help F's use of the words 'slightly stronger' provides a good example of scaffolding R's learning
Thd2 P6 & P7 Monday 7.32pm & 7.34pm	R: I think n must end in 2 or 8. This is because n^2+16 must divide by 10, so n^2 must end in 4, so n must end in 2 or 8 R: N must divide by 6, and end in 2 or 8, so n can be expressed as (some multiple of 30)+(12 or 18). I'm not sure how to write this algebraically though	R is still working on the problem but appears to have moved his focus onto considering the type of numbers that are divisible by 720 The second of these two posts may be in response to Help F's question (post 5) which arrived respectively 1 and 3 minutes earlier to these two posts by R

Post Number Day/time	Message extract	Commentary on Social Actions
Thd2 P8 & P9 Monday 7.38pm 7.39pm	Peter: if you know modular arithmetic try putting multiples of 6 where 6kmod30+1 and 6kmod30-1 are both possibly prime as n. i think that there is a nicer way but this is still nice and simple and im tired at the moment. hope this helps. if you dont know modular arithmetic mod 30 is just a multiple of 30 and 6kmod30 is 6k + mod30 sorry ive realised you can write this as mod5	Peter makes a second response 30 minutes after the first and 8 minutes after Peter’s latest thoughts, to help R make further progress He is enquiring whether R is familiar with modular arithmetic and tries to explain what modular arithmetic is, though this is partial to the reader. He adds a personal detail that he is feeling tired (at 7.38pm in the evening?), but one minute later sends a second Post to tell R that it need not be mod 30 but mod 5. Mod 5 is a link to R’s previous thoughts in Thd2 P4 that ‘(n-1) and (n+1) are not divisible by 5’ Peter’s post is not clear about what R should do and would suggest that Peter needs to gain further experience in assembling written instructions and clear explanations. However it needs to be remembered that Peter is only about 15 years old and not an experienced teacher
Thd2 P10 Tuesday 5.07pm	R: I don't really understand this Peter. Sorry. Does anyone know a different way to give a hint/explain Peter's hint?	R has not stated whether or not he is familiar with modular arithmetic but politely apologies to Peter for not understanding his explanation and asks others (unknown) for further clarification. It would likely be easier for R to push Peter further to explain and work things through together face-to-face (f2f). On the one hand a limitation here of the medium but on the other hand R can choose not to be helped by Peter and continue to ask others
Thd2 P11 5.27pm	Help C: If n-1 and n+1 are prime, what are the possible remainders when you divide n by 5?	Help C contributed to the first thread C’s help here continues Peter’s ‘sorry, I’ve just realised’ hint about mod 5,in Thd2 P7. Even though R has stated in Thd2 P4 that ‘(n-1) and (n+1) are not divisible by 5’ this could be beyond R’s experience of modular arithmetic and current help on offer is insufficient to move R forward

Post Number Day/time	Message extract	Commentary on Social Actions
Thd2 P12 5.38pm	Peter: since i don think that you understand modular arithmetic (dont worry about this) i shall write in a basic algerbraic form	Peter has taken 31 minutes from R's post 8 (it is impossible to know at what precise time Peter read R's post) to express the facts algebraically and appears not to have been daunted by R's comment that he doesn't understand what Peter has been doing Peter is kind to reassure R that they should not worry if they do not understand modular arithmetic. [An alternative interpretation would be that the tone is patronising, but I reject this as I feel from looking at all of Peter's posts this would not have been the intention
	6k - cannot be prime because 6 divides it 6k+1 - maybe prime 6k+2 - cannot be prime because 2 divides it 6k+3 - cannot be prime because 3 divides it 6k+4 - cannot be prime because 2 divides it 6k+5 - maybe prime where k > 0	Although Help C 11 minutes earlier (Thd2 P11) was suggesting that R should focus on mod 5, and Peter's previous Post also mentioned mod 5, Peter, has returned to the very first question he posed in Thd2 P2 to show what form prime numbers can take
	so for x to be prime x is in the form 6k-1 or 6k-1.	(Mistype here as should be 6k+1 or 6k-1. This is never picked up) There is no explanation that 6k+5 mod 6 can be written as 6k-1 mod 6. Again f2f in the classroom would allow R to immediately come back to this or Peter to explain more fully
	so if n-1 and n+1 are both prime n = 6k so then this proves your statement earlier that 6 divides n.	This is the first time that R is **told** that all primes are of the form 6k+1 or 6k-1 (k and n appearing synonymous here). Peter was **hinting** at in his first help Post, Thd2 P2, but no link back is made explicit
	so now you must show that n^2 or n^2+16 is divisible by 5	Here Peter does make a specific reference to a fact that R established, Thd2 P7
	[continued on next page]	[continued on next page]

Post Number Day/time	Message extract	Commentary on Social Actions
[contd]	now n is in the form 5k, 5k+1, 5k+2, 5k+3, 5k+4 for which of these is it true that n-1 and n+1 is prime? now square n and sub in $n^2 = 5k + m$ where you know m and then prove what that n^2+16 or n^2 are divisible by 5 i hope this makes sense.	This is formalising Help C's post Thd2 P11 'If n-1 and n+1 are prime, what are the possible remainders when you divide n by 5?' and this, if understood, would complete the solution It is likely that this will need greater clarity as is the case in Thd2 P13 from R below It is clear that Peter is trying very hard to help but there is a possibility that his understandable inexperience prevents him from presenting it in unambiguous detail
Thd2 P13 5.48pm	R: OK. 5k+1, because n-1 would divide by 5 and 5k+4, because n+1 would divide by 5. Is there a way to narrow it down further? Or can m have multiple values.	R also now appears to have gained familiarity with modular reasoning though this is of necessity an implicit inference (but compare not knowing inThd2 P10 with the notation that R is using here) 10 minutes after Peter's last post, R has addressed the question that both Help C and Peter had posed [Thd2 P11 & 12] See comment above about likely need for greater clarity
Thd2 P14 5.57pm	R: Assuming m can have multiple values: For m=0, n^2 will equal $25k^2$, so it will be divisible by 5. For m=2, n^2 will equal $25k^2+20k+4$. Therefore, $n^2+16=25k^2+20k+20$, so it will be divisible by 5. For m=3, n^2 will equal $25k^2+30k+9$. Therefore, $n^2+16=25k^2+30k+25$, so it will be divisible by 5. Yes, so n can be expressed as 5k+m, where k is a positive non-zero integer, and m equals 0, 2, or 3.	R has answered his own question (assuming 'can have') from the end of Thd2 P13, taking just 9 minutes to write this all out!

Post Number Day/time	Message extract	Commentary on Social Actions
Thd2 P15 6.02pm	R: Got it! n is divisible by 6, so n^2 is divisible by *36*. n^2 is divisible by 4, so n^2+16 **must** also be divisible by *4*. Either n^2 or n^2+16 must be divisible by *5*, from the last post. *36*5*4*=**720**	Five minutes after writing out the three possible cases, the 'Got it!' Post arrives Having been ensconced in the modular proof R returns to link the findings above to the 'numbers' involved In the course of 23 hours, R has the solution and conveys with 'Got it!' a sense of pleasure at having done so. There is no indication that it will be written up as a formal proof (and it could be argued that there is no need to be) During day 2 on the problem R and Peter have been communicating for an hour and a half to arrive at the solution – the day having started with R asking for others to help (Help C gave one hint) as had not understood Peter, but Peter persisted and R worked towards the end worked at the problem sending three consecutive messages whilst involved in finding the solution
Thd2 P16 6.05pm	Peter: yes well done this completes the proof. i remember fondly this question. this was my first bmo question i completed. arrr memories ... yes anyway. well done.	Peter concludes with praise and a personal comment about when he first solved it
Whilst there has been some lack of clarity in Peter's explanation there is a sense that R has been fully engaged in solving the problem. Moreover, unaided, Peter made a judgment call as to how much R knew (about modular arithmetic) in order to be able to do this problem. If Peter and R were in a classroom f2f, this 'messy' and at times lacking clear explanations is how, without teacher intervention, 'pupils' might work.		

Table C: Thread Three

The same question is started on a third thread so 9 months later (and a different academic year). S is in year 12. Help G (who first appears in post 3 is in year 11, the same year as Peter).

Post Number Day/time	Message extract	Commentary on Social Actions
Thd3 P1 Thursday 4.33pm	S: n be an integer greater than 6 where n - 1 and n + 1 are both prime. Prove that $n^2(n^2 +16)$ is divisible by 720. I've written 720 as prime factors, and I've proved that the expression is divisible by all of the prime factors except 5. Can anyone give me a hint?	This is S's 36th post so is a relatively new poster but had been involved more than R in thread 2
Thd3 P2 Thursday 4.43pm	[Help from Peter]: consider n mod5, what can it/cant it be?	Peter has taken 10 minutes to reply and responding to 'except 5' immediately hinted about working in mod5 (see Thd2 above). This time he has not initially asked what form do prime numbers take as in Thd2 above. Peter's hint is succinct and does not ask whether S is familiar with modular arithmetic (a difficulty in Thd2 when R was not conversant with the form and which, it might be argued, that an experienced teacher would have stored as a potential problem and brought it to the fore the next time the same question arose)
Thd3 P3 Thursday 4.45pm	Help G: Write n as 5k+a and consider which numbers a can be given n-1 and n+1 are both prime (so not divisible by 5) and substitute these possible expressions for n into $n^2(n^2+16)$. (Alternatively/equivalently if you know about modular arithmetic consider possible values of n mod 5) You said you showed it is divisible by the other prime factors of 720 - did you make sure you showed that it was divisible by them to the right power (ie that 3^2 and 2^4)?	Help G repeats Peter's instructions to R in Thd2 P12 As Peter's comment immediately above Offering additional advice though 3^2 and 2^4 (prime factors) as 9 and 16 respectively rather than 36 and 4 in R's work
Thd3 P4 & P5 Thursday 5.03pm	S: sorry about the formatting, I am putting the backslashes in, but they don't seem to be working [To Help G] This is what I've done - we know that n must be divisible by 2 and 3 for the numbers on either side of it to be prime. $n^2(n^2+16) = n^4+ 16n^2$. n^4 is divisible by 2^4 and 3^4 (so is also divisible by 3^2). $16n^2$ is divisible by 2^4x2^2 (so is also divisible by 2^4). Therefore, $n^2(n^2+16)$ must be divisible by 3^2 and 2^4	A simultaneous need to 'master' the board's mark up of mathematical elements Addressing the last part of Help G's post immediately above and arriving at the solution (apparently) far more quickly than R in thread 2

Post Number Day/time	Message extract	Commentary on Social Actions
	n-1 and n+1 can't be 0 mod 5, therefore n can't be 1 or 4 mod 5. This means it must be 2 or 3 mod five, so it's square must be 4 mod5. Therefore n^2 + 16 is 4+1 mod 5 = 0 mod 5. Thanks everyone. I can't believe I didn't see that...	Throughout this post S is 'correct' [see Help G's next post immediately below] and clearly understands what they are doing. The succinctness e.g. conflates the explanation and some others might consider including immediate steps. This is not a 'show your working out' issue as so often required or suggested in answering public examination questions. It is S's personal solution to the problem and at the moment sufficient for the situation
5.08pm	Sorry again, I've just realized that you need to put + in ….	Formatting problem solved – but still apologising if it has caused (assumingly) any annoyance or difficulty to helpers
Thd3 P6 Thursday 5.08pm	Help G: One small thing you've missed - n can be 0 mod 5, but that gives you n^2 is 0 mod 5 so you're still fine. Have you got the second bit about the converse as well?	The 'correctness' above is tempered by Help G as S omits one of the three cases (0 mod 5). This could be interpreted as a lack of rigour on S's behalf whilst simultaneously recognizing S's high performance at this subject This was Peter's initial query that started Thd1 P1
Thd3 P7 Thursday 6.58pm	S: The converse is false because 720 is divisible by 720, but 720+1=721 has a factor of 7	S has found a counterexample without further posting and indeed it is the same one suggested by Help C [Thd1 P7] thread one that became 'blindingly obvious' to Peter [Thd1 P8]
At this stage the problem has been fully resolved (or so it appears). Two other AskNRICH posters (ANP(1) and ANP(2)) join in the 'conversation'.		
Thd3 P8 & P9 Thursday 7.41pm 7.44pm	ANP(1): If the converse were true, then it would be a really, really fast way to find big prime numbers! ANP(2): Not to mention being a proof of the twin prime conjecture!	Two further posters continue the conversation broadening the mathematical connections

Post Number Day/time	Message extract	Commentary on Social Actions
Thd3 P10 Thursday 8.46pm	S: I've just realised that my counter example is exceedingly wrong as while 720 is divisible by 720, $720^2(720^2+16)$ isn't. So is seems that converse is false, but finding a counter example might be hard. Maybe by trying to solve $n^4 + 16n^2 - 720n=0$, but I feel there should be something like a proof by contradiction. (By the way, for it to be a proof of the twin prime conjecture you'd also have to prove that there are infinitely many multiples of 720 that can by written as the product of a square number and that number +16.)	This post is a real surprise! It is difficult to think why S went back to this – what was nagging away in her head? Having solved it all correctly, S decides (incorrectly) that they have made a giant error. In fact they go backward considering that it would be hard to find a counterexample. Though the 'I feel' illustrates intuition. Is this a 'howler'? S is also able to show that she is (now?) cognisant with the twin prime conjecture and does not simply state it but is able to relate it (to some extent) specifically to this problem. The 'howler' now seems all the more surprising for S's work
Thd3 P11 Thursday 8.56pm	Help C: [copies S's post from above] **quote:** I've just realised that my counter example is exceedingly wrong as while 720 is divisible by 720, $720^2(720^2+16)$ isn't. Yes it is! $720^2(720^2+16)/720 = 720(720^2+16)$.	Help C is the same person who originally posted 720 in the first thread (Thd1 P7) – now some three months later is contributing to this thread with the same fact) 'Yes it is' to relate to the original counterexample, rather than the error is exceedingly wrong!
Thd3 P12 Friday 5.05pm	S [To Help C]: For some reason I was thinking of $720^2 + (720^2+16)$. Lets just hope I didn't make a mistake like that on the BMO today...	Indeed!

Thread One

In the first thread Peter, known to be male and aged 14 to 15, arrives and asks for help of others aware that some within the group will be more experienced and will know the solution. Having first been asked to follow their intuition (with possibly a hint within the wording of the question that the converse will not be true), a second helper suggests looking for a counterexample. Already the more experienced members have offered two key strategies that are part of the mathematician's toolkit: intuition, the 'hunch' present at the outset of the problem, and the common method of looking for a counterexample as a starting point in proving something is not true, if that coincides with the 'hunch'. From the responses that follow, three people other than Peter, think about a possible counterexample i.e. Help B's comment will eventually involve Peter, and Help A, C & D. Having found a counterexample, Peter shares it with the group. This could have been the end of the problem but Help A, obviously still reading the thread, suggests that 24 is another counterexample. Being a counterexample less than 48, coupled with a smiley face, there is a feeling of 'one-up-man-ship' i.e. that this is a 'smarter' counterexample. Here there is the first instance of camaraderie. But then Help C (known to be a male first year mathematics undergraduate) joins with a much larger number (720) and once suggested, to anyone who had not thought of it this is a 'clever' solution – simple (blindingly obvious) because of the division by 720 and 'I wish I had thought of that' (kicking one self for not seeing it) choice. Peter obviously appreciates/admires (lol - laughing out loud - the choice of 120 and complements the 'owner' on their choice; immediately another indication of friendliness between the group, (if the way that Help A offer his choice, is felt to be friendly). The spirit of friendly banter is also evident when nearly seven hours after the thread has 'died', a new respondent (Help D) who must have decided to read through the thread, sees that A's counterexample of 24 is incorrect. On the surface D's comments '*Not to be a spoil sport, but I don't think 24 quite cuts it as a counterexample* ☺' appears to be the most subtle of ways to publicly announce the error. Indeed the let down '*not to be a spoil sport*' is gentle: '*I think*', allows for any error on D's part to be correct (thought there is also the feeling that D really thinks that they (and not Help A) are correct: '*not quite cuts it*' replaces the glare of mistake, and the friendly smiling emoticon at the end both further softens any put down and replicates A's emoticon. But this interpretation might be 'overkill' because both A and D are veteran posters and will have got to know each other (in at least a virtual sense) and thus are engaged in friendly banter rather than one trying to let the other person, who has made the error, down gently. In this respect D could be either laughing *with* or *at* the person who has made the error and whilst laughing at would be neither good nor acceptable practice to a newcomer, it would 'amongst friends' be precisely how an error (howler) would be pointed out. A seems only too happy to laugh at his own error (even if this is saving face) and explain why he was being 'so silly' (even if one could infer that this is really conveying some sort of superiority). Whatever was really going on between A and D in this virtual world, the analysis offered here it articulates the actions of one person 'humourously' pointing out an error to another face-to-face where offence may or may not be taken.

Thread Two

The second thread was started by R (believed to be a year younger than Peter) some two months later and Peter, the originator of the first thread, the first to respond. When Peter asked for help in the first thread he explained that he had been able to solve the first part of the question using a known fact about prime numbers. As R is stuck on the first part, Peter immediately offers a hint based on his own solution. With two new helpers (E and F) offering suggestions, R is able to move part way through the problem, apparently ignoring Peter's slightly minimal (and potentially more perplexing hint). However, Peter returns persisting with his method even though R had not taken this up the first time. R obviously does not understand what Peter is 'driving at', apologising to Peter for not '*really understand[ing]*' and asking if there is someone else who can either show them an alternative way or (implying more clearly) explain Peter's hint. At this juncture Help C, (the 720 person from the first thread) responds but does little more than repeat a previous hint (from E) but combined with Peter's afterthought of mod 5. This after thought is posted one minute after Peter's post working with mod 60 and is indicative of although statements have been made, the mind is still mulling over initial ideas are still being considered and simplification (from 60 to just 5) becomes clear. Peter returns having read that R cannot understand his method and judging the obstacle to be that R is not yet familiar with modular arithmetic. Again there is no put down about any perceived lack of knowledge as Peter writes: '*since i don think that you understand modular arithmetic (dont worry about this) i shall write in a basic algerbraic form*'. Rather it conveys warm support and care. (Although a colleague in a personal communication suggested that this phrase might be considered patronising, given the general tenure of the board's messages, I do not think that this was the intention. Indeed as R continues to communicate with Peter for the remainder of the thread, R is perhaps unlikely to have taken offence). Peter's working here is substantial and given that it is only 31 minutes after R has politely asked others to help, Peter has remained unfazed. It is impossible to know what happened in the intervening 10 minutes, but R appears to have gained familiarity of, and developed their mathematical experience with, this algebraic form of modular arithmetic. Although R asks another question as to whether there can be multiple values, the next posting indicates that the question has been self-answered. There are in fact three consecutive posts from R (9 and 5 minutes apart) by when R has the solution. **'Got it!'** - one can only imagine the relief and satisfaction (a là punching one's fist in the air) that the two words, plus the exclamation mark, convey. Peter's conclusion to the thread contains a personal reminisce (as would occur in conversation) as well as offering praise and acknowledgement of R's success. Although at the half way stage R was seeking for alternative helpers to Peter, from this point until the end it was only Peter who was working with R. The 'textbook' written out explanation from Peter was sufficient for R to solve the problem.

Thread Three

The same problem appears for a third time eight months later and in a different academic year. S, the person making the plea (believed to be a year older than Peter and thus two years older than R), is like R, stuck on the first part of the question, though has managed to get to the point (unlike R) where there is a need to show divisibility by 5. Peter is again the first person to respond and repeats the minimal mod 5 hint. There is almost simultaneously (i.e. 2 minutes later) more detailed help from G that S responds to by writing out the solution. Whilst the text appears cumbersome as S is unable to work out how to mark up the mathematical text, and for which she apologies and later can rectify, the solution is succinct and personal to S – it may not be in the form that a teacher would expect the working to be shown. However although S has the solution, G returns to gently tell S ('*just one small thing*') that they have missed the zero case and ask about whether S has done the second part, i.e. the part that Peter was stuck on in thread one. When S replies again it is to give 720, the same 'simple' counterexample as C had done in the first thread. At this stage, the solution is complete and one is impressed that a second person has come up with 720. But then two more events occur. The first involves a new participant who has not posted in either this thread or the previous two 'throws in' a statement about the problems if indeed the converse had been proved to be true (a conversational remark) to which yet another new participant adds a comment about an important, well known conjecture. Both these statements can be viewed as both comments to S and to other members who participate in Ask NRICH. The second event I consider to be 'startling'. Whilst at the same time replying to the conjecture statement, S, having obtained 720 as the counterexample, suddenly and unexpectedly announces that '*my counter example is exceedingly wrong*'. Not only this, but S has returned to the problem and tried a different approach. In bringing the three threads together, Help C (the original 720 person) returns to reassure that 720 is correct! Looking at the exchange, S's '*isn't*' receives C's pantomime-like response '*it is*'. Just as before, errors and misunderstandings are treated with kindness and humour. Like A, S explains how they came to make their error and concludes with a personal remark, hoping that they hadn't made a similar (i.e. implied silly) mistake earlier in the day when taking a test that this particular problem has been a practice for.

Key		
	DR:	Direct Response to poster
	FR:	Follow Up response to earlier message
	MR:	My/Mine Response/Solution from (as a result of) earlier message/posting
	OR:	Open Response/Message (to all AskNRICHers)
	PUR	an open response that is Picked Up by specific poster(s)

Table A Thread One

Post	Day/time	Time Gap	Poster	Message	Response Type & Persons Involved	Synopsis of interaction / comments
Thd1 P1	Saturday 11.13am	-	Peter	Peter: Let n be an integer greater than 6. Prove that if n - 1 and n + 1 are both prime, then $n^2(n^2 +16)$ is divisible by 720. Is the converse true? i have managed to prove the first part of the question using the fact that all primes are of the form 6n-1 and 6n+1. when i tried to prove the converse i cant do it. i know that 2 and 3 divide n and n is of the form 2mod5 3mod5 or mod5. from here where do i go? thanks	OR PUR by A & B	Completed first part of question but cannot do second part in finding if converse is true
Thd1 P2	Saturday 11.14am	1min	Help A	Help A: Do you think the converse is true?	DR to Peter	Suggests starting with an intuitive approach – 'feeling' whether it is true or not true
Thd1 P3	Saturday 11.37am	23 mins	Peter	Peter: i presume that it isn't but im not very sure	DR to A	Responds by saying that he assumes that it not true, but is not sure
Thd1 P4	Saturday 11.39am	2 mins	Help B	Help B: If you look back over your proof, you used the fact that ALL primes are 6n-1 and 6n+1. However, is the converse of *this* true? Are all 6n-1 and 6n+1 prime? Using this, you can construct a counterexample.	DR to Peter	Connects Peter's solution from the first part of the problem and suggests looking for a counterexample
Thd1 P5	Saturday 11.58am	19 mins	Peter	Peter: thanks i ve got it now, for anyone who's interested one counter example is 48.	DR to B OR PUR by A & C	Has found, and shares, 48 as a counterexample

Post	Day/time	Time Gap	Poster	Message	Response Type & Persons Involved	Synopsis of interaction / comments
Thd1 P6	Saturday 12.02pm	4 mins	Help A	Help A: or 24 ☺	DR to Peter MR fr B4 OR PUR by C & D	'Smugly' (via emoticon) suggests 24 would also do (in fact it does not)
Thd1 P7	Saturday 12.07pm	5 mins	Help C	Help C: Or if you really want to do no work whatsoever when it comes to multiplication just use 720	DR to Peter MR fr B4 OR	Gives the 'blindingly-obvious-once-someone-has-pointed-it-out' solution of 720
Thd1 P8	Saturday 12.10pm	3 mins	Peter	Peter: lol i totally missed that	DR to C	Amused (lol - laughs out loud) at missing the obvious
Thd1 P9	Saturday 6.59pm	6 hrs 49	Help D	Help D: Not to be a spoil sport, but I don't think 24 quite cuts it as a counterexample ☺	FR to B4 DR to A	Politely suggests that 24 'does not quite cut' it as a counterexample
Thd1 P10	Saturday 7.11pm	12 mins	Help A	Help A: Sorry haha, I was thinking that all numbers 0 (mod 6) worked. Good job i didn't make that mistake when I took the paper last year!	DR to D OR	Laughs at own error and shares mistaken thoughts

Table B Thread Two (One and a half months later)

Post	Day/time	Time Gap	Poster	Message	Response Type	Synopsis of interaction / comments
Thd2 P1	Monday 7.05 pm	-	R	Let n be an integer greater than 6. Prove that if n-1 and n+1 are both prime, then $n^2(n^2+16)$ is divisible by 720. How would I start this question? I thought of trying to split 720 up into its prime factors, 2,2,2,2,3,3,5. However, this didn't get me anywhere. Can anybody give me any hints?	OR PUR by P & E	Asks for help on the first part of the problem
Thd2 P2	Monday 7.07 pm	2 mins	Help Peter	what form do primes greater than 6 take. as soon as you see what to do this is very simple so i shall leave the hint at that	DR to R	Responds with key question (based on what he had used to answer the question mentioned in Thd 1 P1)
Thd2 P3	Monday 7.07 pm		Help E	If n-1 and n+1 are prime, what can you say about n (nothing too deep - just divisibility sorts of things)?	DR to R	Provides different, (probably) more manageable key question
Thd2 P4	Monday 7.14pm	7 minutes	R	R: My thoughts: • n must be even, because primes above 6 are all odd, so (n-1) must be odd, so n is even. • n must also be divisible by 3, because (n+1) and (n-1) are both **not** divisible by 3. • I think that there must be some way to work out the last digit of n by knowing that (n-1) and (n+1) are not divisible by 5, but either n^2 or n^2+16 must be divisible by 5 (so the product is divisible by 720). I can't work out how though.	DR to P & E OR PUR by C & F	Responds with his thoughts to Help E's questions. Feels that he needs to show that one of two specific terms must be divisible by 5
Thd2 P5	Monday 7.31pm	17 mins	Help F	Help F: The first two points suffice. n is even, and n is divisible by 3, can you make a slightly stronger statement about n and use this as the base of your argument?	FR to E3 DR to R	Responds to R thoughts using Help E's key questions
Thd2 P6	Monday 7.32pm	1 mins	R	I think n must end in 2 or 8. This is because n^2+16 must divide by 10, so n^2 must end in 4, so n must end in 2 or 8	DR to E	Although this message follows Help F, the time gap suggests that for P6, R may have continued to work on his thoughts (see P4).
Thd2 P7	Monday 7.34pm	2 mins		N must divide by 6, and end in 2 or 8, so n can be expressed as (some multiple of 30)+(12 or 18). I'm not sure how to write this algebraically though	DR to F	For P7 it is more likely that R could be responding to Help F's comment and suggestion

Post	Day/time	Time Gap	Poster	Message	Response Type	Synopsis of interaction / comments
Thd2 P8	Monday 7.38pm	4 mins	Help Peter	if you know modular arithmetic try putting multiples of 6 where 6kmod30+1 and 6kmod30-1 are both possibly prime as n. i think that there is a nicer way but this is still nice and simple and im tired at the moment. hope this helps. if you dont know modular arithmetic mod 30 is just a multiple of 30 and 6kmod30 is 6k + mod30	DR to R	Returns to revive own method of solution hinted at in P2 above. Mentions modular arithmetic for the first time. Personal feeling that there are other methods that could be employed but feels own choice is simple. Implies feeling tired is currently effecting own ideas
Thd2 P9	Monday 7.39pm	1 mins		sorry ive realised you can write this as mod5	DR to R	Afterthought – mulling over problem after posting
Thd2 P10	Tuesday 5.07pm	Next evening	R	(P10a) I don't really understand this Peter. Sorry. (P10b) Does anyone know a different way to give a hint/explain Peter's hint?	DR to P OR PUR by C	Politely responds to Peter to say that they have not understood his help and asks if anyone else can either explain Peter's hint or suggest an alternative way of obtaining the solution
Thd2 P11	Tuesday 5.27pm	20 mins	Help C	If n-1 and n+1 are prime, what are the possible remainders when you divide n by 5?	FR to E3 FR to P9 DR to R	(Same person as in Thd 1). Offers a hint which connects with Help E's questions, R's own message 4 and Peter's afterthought of mod 5
Thd2 P12	Tuesday 5.38pm	11mins	Help Peter	since i don think that you understand modular arithmetic (dont worry about this) i shall write in a basic algerbraic form 6k - cannot be prime because 6 divides it 6k+1 - maybe prime 6k+2 - cannot be prime because 2 divides it 6k+3 - cannot be prime because 3 divides it 6k+4 - cannot be prime because 2 divides it 6k+5 - maybe prime where k > 0 so for x to be prime x is in the form 6k-1 or 6k-1. so if n-1 and n+1 are both prime n = 6k so then this proves your statement earlier that 6 divides n. so now you must show that n^2 or n^2+16 is divisible by 5 now n is in the form 5k, 5k+1, 5k+2, 5k+3, 5k+4 for which of these is it true that n-1 and n+1 is prime?	DR to R	Decided that R is unfamiliar with modular arithmetic (though should not worry about this) and provides a detailed algebraic solution that will take R part way there – thus leaving R to complete the final part.

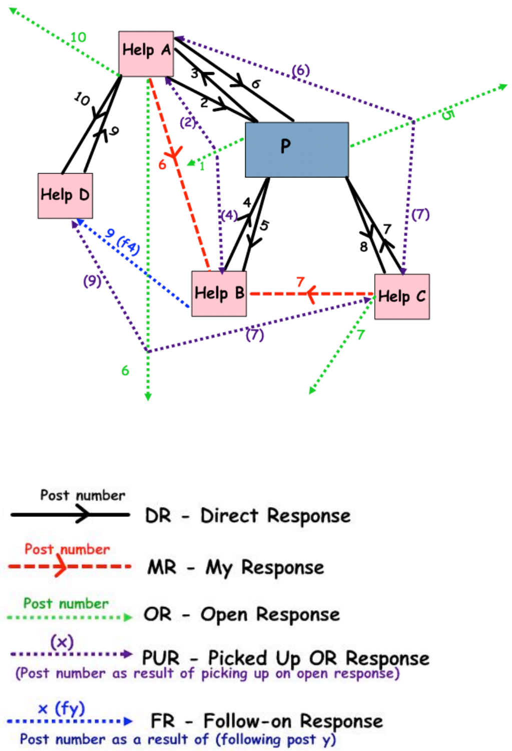
Post	Day/time	Time Gap	Poster	Message	Response Type	Synopsis of interaction / comments
				now square n [contd.] and sub in $n^2 = 5k + m$ where you know m and then prove what that n^2+16 or n^2 are divisible by 5 i hope this makes sense.		
Thd2 P13	Tuesday 5.48pm	10 mins	R	OK. 5k+1, because n-1 would divide by 5 and 5k+4, because n+1 would divide by 5. Is there a way to narrow it down further? Or can m have multiple values.	DR to P	Responds to Peter's work by continuing to move some way towards the final solution, but not quite completed yet.
Thd2 P14	Tuesday 5.57pm	9 mins		Assuming m can have multiple values: For m=0, n^2 will equal $25k^2$, so it will be divisible by 5. For m=2, n^2 will equal $25k^2+20k+4$. Therefore, $n^2+16=25k^2+20k+20$, so it will be divisible by 5. For m=3, n^2 will equal $25k^2+30k+9$. Therefore, $n^2+16=25k^2+30k+25$, so it will be divisible by 5. Yes, so n can be expressed as 5k+m, where k is a positive non-zero integer, and m equals 0, 2, or 3	DR to P	Peter provides further detailed help which allows R to reply …
Thd2 P15	Tuesday 6.02pm	5 mins		Got it! n is divisible by 6, so n^2 is divisible by *36*. n^2 is divisible by 4, so n^2+16 **must** also be divisible by *4*. Either n^2 or n^2+16 must be divisible by *5*, from the last post. _36*5*4_=**720**	DR to P OR	**'Got it!'** Elation/Relief in succeeding. R completes the solution having shown that one of the two specific terms must be divisible by 5 (as R had predicted in P9 above)
Thd2 P16	Tuesday 6.05pm	3 min	Help Peter	yes well done this completes the proof. i remember fondly this question. this was my first bmo question i completed. arrr memories ... yes anyway. well done	DR to R	Congratulates R and adds a personal memory

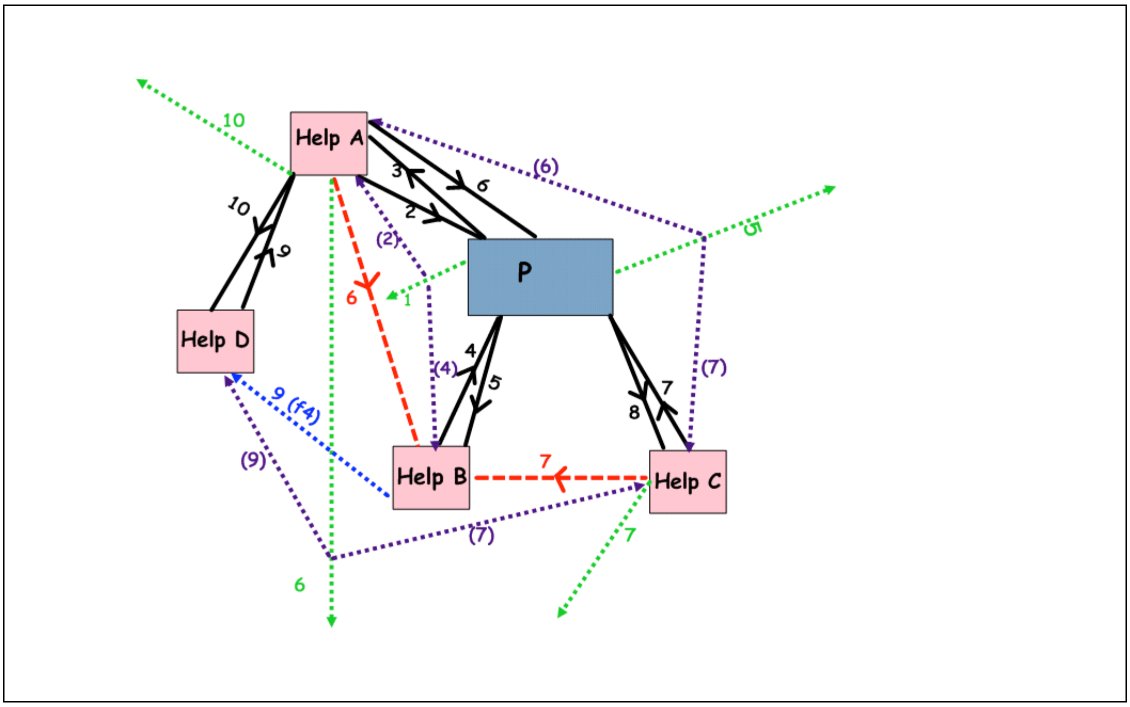
Table C Thread Three (Eight months later – the next academic year)

Post	Day/time	Time Gap	Poster	Message	Response Type	Synopsis of interaction / comments
Thd3 P1	Thursday 4.33pm	-	S	n be an integer greater than 6 where n - 1 and n + 1 are both prime. Prove that $n^2(n^2 +16)$ is divisible by 720. I've written 720 as prime factors, and I've proved that the expression is divisible by all of the prime factors except 5. Can anyone give me a hint?	OR PUR by G & P	Like R above is stuck on the first part of the question though has already worked to the stage where there is a need to show divisibility by five
Thd3 P2	Thursday 4.43pm	10 mins	Help P	Peter: consider n mod5, what can it/cant it be?	DR to S	First to respond (as with R) with a repeat of the mod 5 hint.
Thd3 P3	Thursday 4.45pm	2 mins	Help G	Write n as 5k+a and consider which numbers a can be given n-1 and n+1 are both prime (so not divisible by 5) and substitute these possible expressions for n into $n^2(n^2+16)$. (Alternatively/equivalently if you know about modular arithmetic consider possible values of n mod 5) You said you showed it is divisible by the other prime factors of 720 - did you make sure you showed that it was divisible by them to the right power (ie that 3^2 and 2^4)?	DR to S	Provides a more detailed response and help which includes within other ideas the same hint as Peter's
Thd3 P4	Thursday 5.03pm	18 mins	S	P4a: sorry about the formatting, I am putting the backslashes in, but they don't seem to be working P4b: This is what I've done - we know that n must be divisible by 2 and 3 for the numbers on either side of it to be prime. $n^2(n^2+16) = n^4+ 16n^2$. n^4 is divisible by 2^4 and 3^4 (so is also divisible by 3^2). $16n^2$ is divisible by 2^4x2^2 (so is also divisible by 2^4). Therefore, $n^2(n^2+16)$ must be divisible by 3^2 and 2^4 n-1 and n+1 can't be 0 mod 5, therefore n can't be 1 or 4 mod 5. This means it must be 2 or 3 mod five, so it's square must be 4 mod5. Therefore $n^2 + 16$ is 4+1 mod 5 = 0 mod 5. P4c: Thanks everyone. I can't believe I didn't see that...	OR DR to G OR OR	Apologises for not knowing how to format mathematical text to appear properly on the board. Shares working to show that problem has been solved (actually, not quite completely solved)
Thd3 P5	Thursday 5.08pm	5 mins		Sorry again, I've just realized that you need to put + in …	OR	Apologies again for lack of formatting but explains what they had been doing wrong

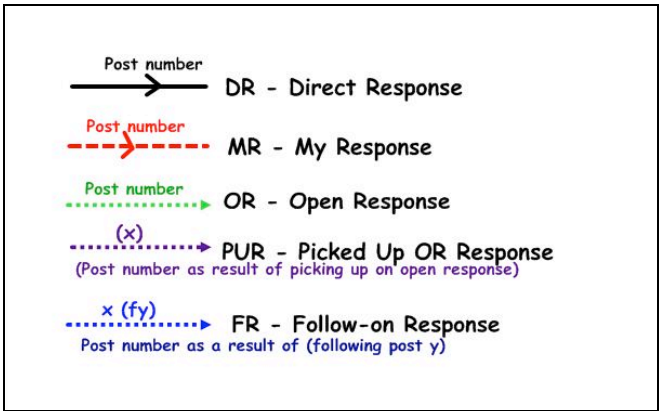
Post	Day/time	Time Gap	Poster	Message	Response Type	Synopsis of interaction / comments
Thd3 P6	Thursday 5.08pm	5 mins	Help G	One small thing you've missed - n can be 0 mod 5, but that gives you n^2 is 0 mod 5 so you're still fine. Have you got the second bit about the converse as well?	DR to S	Responds to highlight the missing part that will complete S's solution and enquiries of S whether they have solved the second part of the question
Thd3 P7	Thursday 6.58pm	1 hr 50 mins	S	The converse is false because 720 is divisible by 720, but 720+1=721 has a factor of 7	DR to G	Responds with the same 'blindingly-obvious' counterexample of 720 that Help C had given in Thd 1 and to which Peter had 'lol'
At this stage the problem has been fully resolved (or so it would appear) Two posters (ANP1 & 2) new to the thread join in the discussion						
Thd3 P8	Thursday 7.41pm	43 mins	ANP (1)	If the converse were true, then it would be a really, really fast way to find big prime numbers!	MR fr G6 DR to S OR PUR by ANP (2)	Adds an extra 'conversational' comment about why it is 'useful' that the converse is not true
Thd3 P9	Thursday 7.44pm	2 mins	ANP (2)	Not to mention being a proof of the twin prime conjecture!	DR to S & ANP (1)	Responds to comment above by mentioning twin prime conjecture
Thd3 P10	Thursday 8.46pm	1 hr 2 mins	S	P10a: I've just realised that my counter example is exceedingly wrong as while 720 is divisible by 720, $720^2(720^2+16)$ isn't. So is seems that converse is false, but finding a counter example might be hard. Maybe by trying to solve $n^4 + 16n^2 - 720n=0$, but I feel there should be something like a proof by contradiction. P10b: By the way, for it to be a proof of the twin prime conjecture you'd also have to prove that there are infinitely many multiples of 720 that can by written as the product of a square number and that number +16.)	OR PUR by C DR to ANP (1) & ANP (2)	Returns to announce that the 720 previously mentioned was 'exceedingly wrong' and does not work as a counterexample [though from Thread One it obviously does work]. S shows new thoughts in trying to find the solution and has complicated the problem by obtaining a quartic equation and considering that perhaps proof by contradiction is now required. The message concludes with a response about the twin prime conjecture.

Post	Day/time	Time Gap	Poster	Message	Response Type	Synopsis of interaction / comments
Thd3 P11	Thursday 8.56pm	10 minutes	Help C	**quote:** I've just realised that my counter example is exceedingly wrong as while 720 is divisible by 720, $720^2(720^2+16)$ isn't. Yes it is! $720^2(720^2+16)/720 = 720(720^2+16)$	DR to S	Picks up S's exact words '… (720) isn't' and replies 'Yes it is!' Help C should know as the first person to suggest that it as a counterexample in thread one (which so amused Peter).
Thd3 P12	Friday 5.05pm	Next day	S	For some reason I was thinking of $720^2 + (720^2+16)$. Lets just hope I didn't make a mistake like that on the BMO today.	DR to C OR	Responds to explain how they had come to make the error (inextricably interpreting incorrect operation sign and concludes that they hope they did not do anything as silly as that in the test taken earlier in the day.

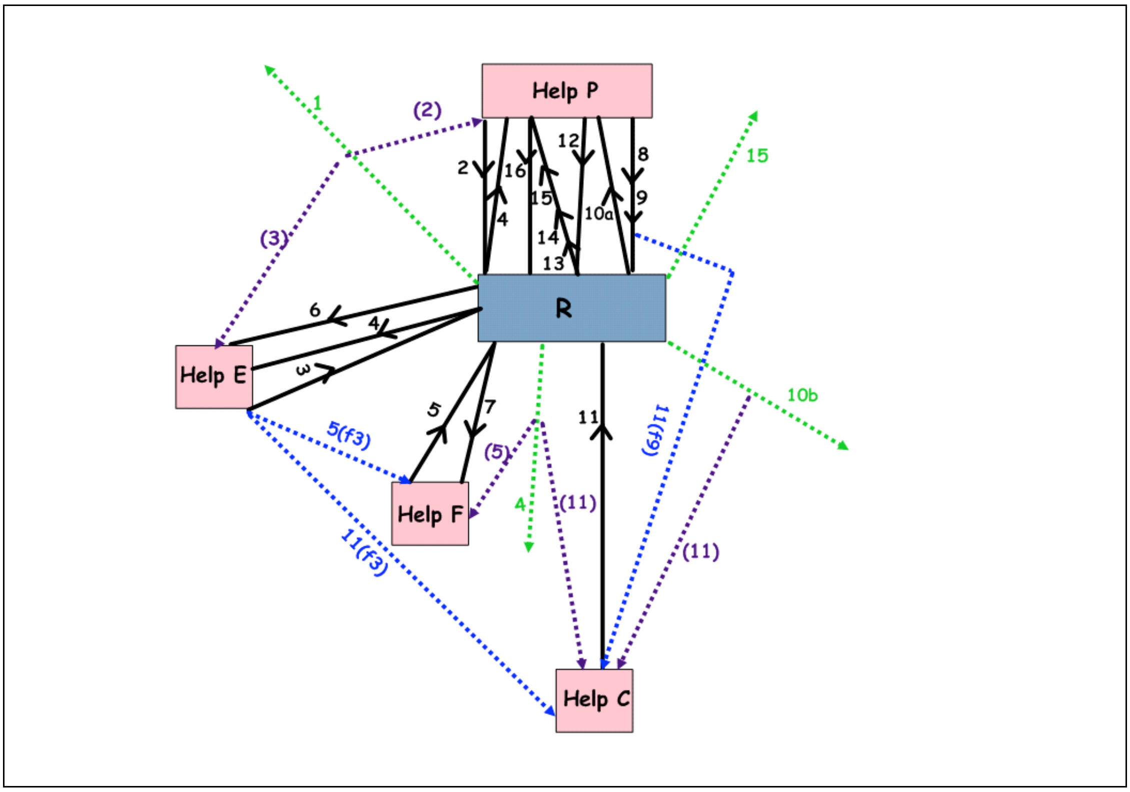
Post	Gap	Poster	Message text	Response Type	Synopsis of interaction / comments	Connection Diagram
P1		Peter	Let n be an integer greater than 6. Prove that if n - 1 and n + 1 are both prime, then $n^2(n^2 +16)$ is divisible by 720. Is the converse true? i have managed to prove the first part of the question using the fact that all primes are of the form 6n-1 and 6n+1. when i tried to prove the converse i cant do it. i know that 2 and 3 divide n and n is of the form 2mod5 3mod5 or mod5. from here where do i go? thanks	OR PU by A & B	Completed first part of question but cannot do second part in finding if converse is true	
P2	1min	Help A	Do you think the converse is true?	DR to Peter	Suggests starting with an intuitive approach – 'feeling' whether it is true or not true	
P3	23 mins	Peter	i presume that it isn't but im not very sure	DR to A	Responds by saying that he assumes that it not true, but is not sure	
P4	2 mins	HelpB	If you look back over your proof, you used the fact that ALL primes are 6n-1 and 6n+1. However, is the converse of *this* true? Are all 6n-1 and 6n+1 prime? Using this, you can construct a counterexample.	DR to Peter	Connects Peter's solution from the first part of the problem and suggests looking for a counterexample	
P5	19 mins	Peter	thanks i ve got it now, for anyone who's interested one counter example is 48.	DR to B OR PU by A & C	Has found, and shares, 48 as a counterexample	
P6	4 mins	HelpA	or 24 ☺	DR to Peter MR fr B4 OR PU by D	'Smugly' (via emoticon) suggests 24 would also do (in fact it does not)	
P7	5 mins	HelpC	Or if you really want to do no work whatsoever when it comes to multiplication just use 720	DR to Peter MR fr B4 OR	Gives the 'blindingly-obvious-once-someone-has-pointed-it-out' solution of 720	
P8	3 mins	Peter	lol i totally missed that	DR to C	Amused (lol - laughs out loud) at missing the obvious	
P9	6 hrs 49 min	HelpD	Not to be a spoil sport, but I don't think 24 quite cuts it as a counterexample ☺	FR to B4 DR to A	Politely suggests that 24 'does not quite cut' it as a counterexample	
P10	12 mins	HelpA	Sorry haha, I was thinking that all numbers 0 (mod 6) worked. Good job i didn't make that mistake when I took the paper last year!	DR to D OR	Laughs at own error and shares mistaken thoughts	



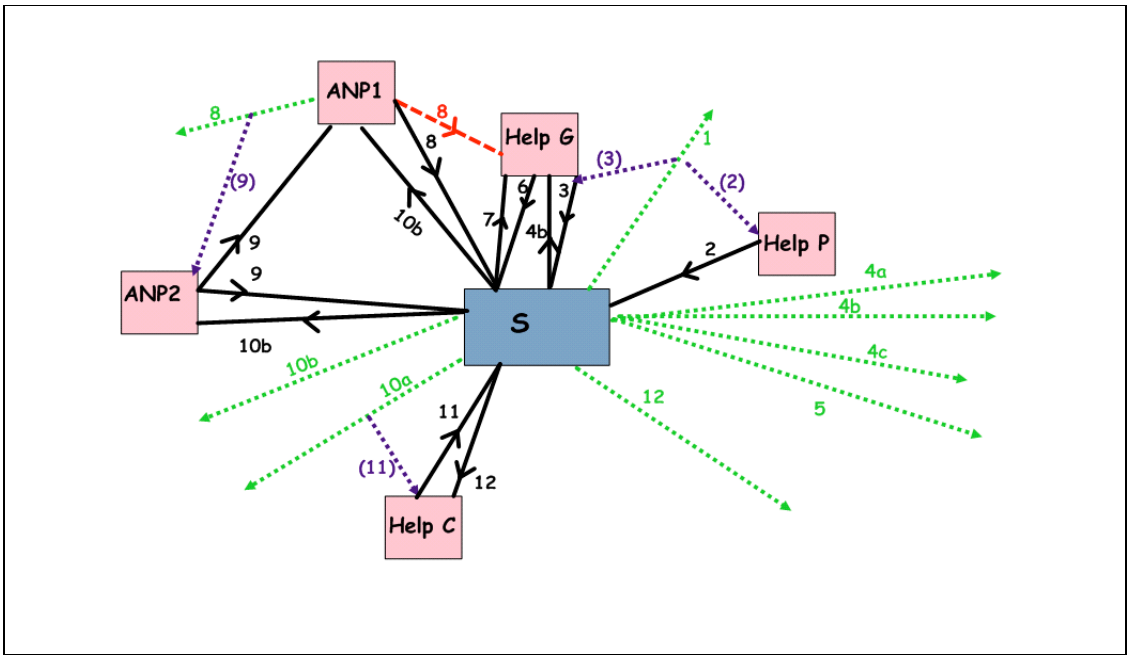
Connection Diagram for 3Thd: Thread1



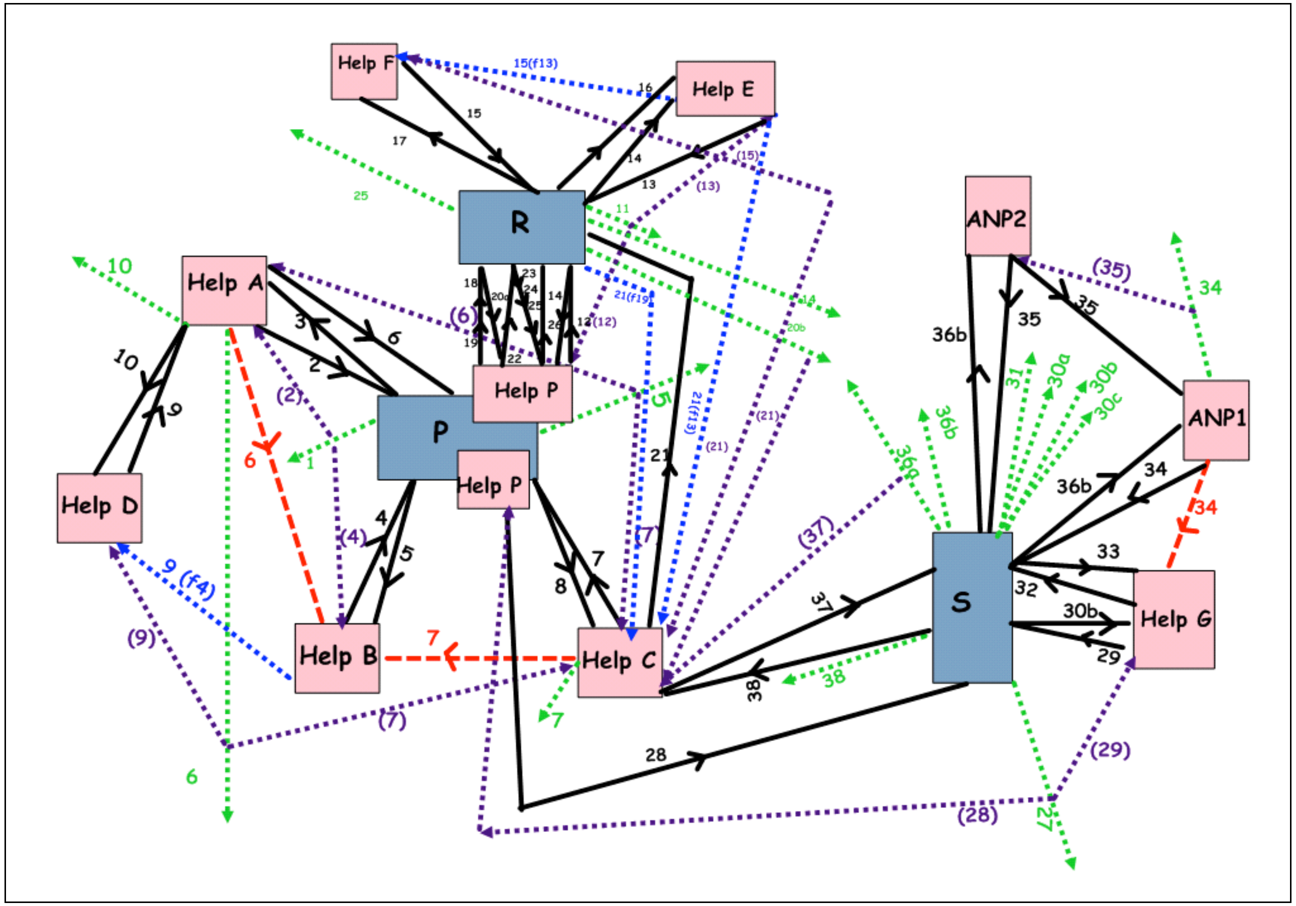
Key for Connection Diagrams



Connection Diagram for 3Thd: Thread2



Connection Diagram for 3Thd: Thread3



Combined Connection Diagram for all 3Thds

Date: 9 November 2010 18:05:21 +0000
From: Rex@cam.ac.uk
To: Tim@cam.ac.uk
Cc: Christine@cam.ac.uk, Libby@cam.ac.uk
Subject: dominoes

Dear (ex?) colleagues,

Time you had a little problem to stop you becoming bored...

I've only played a little with it, it may or may not be rather trivial in the end.

Suppose you are lonely and are playing dominoes on your own. You have the full set, of 28. Can you lay them end to end so that they 'connect' in the usual way, e.g. 2-5 could be followed by 5-1, then by 1-6, etc. Obviously the doubles such as 4-4 don't play any real part in this.

In fact I have a set of Victorian dominoes that go up to 9 (55 of them). Same question.

Generalise...

Off on holiday on Thurs for 9 days, we are coming to Sylvia's event soon after, may see some of you.

Rex (and Norma)

Date: 12 November 2010 09:36:19 +0000
From: Christine@cam.ac.uk>
To: Rex@cam.ac.uk>
Cc: Tim@cam.ac.uk>, Libby @cam.ac.uk>, James@xxx
Subject: Re: dominoes

A good breakfast time problem!

> Suppose you are lonely and are playing dominoes on your own. You have
> the full set, of 28. Can you lay them end to end so that they 'connect' in
> the usual way,

I am more of a loner than I thought. My first answer was empirically yes - I spent many hours of half term holidays attempting to do exactly that - and surely I succeeded at least once with all the dominoes present.

James' first answer was no - there are seven blanks and thats an odd number. So we have had a chat with a piece of paper and a pencil ....

and we have a recursive proof.

I remember that I also had an aesthetic rule about a fairly even distribution of the numbers which our recursive rule does not satisfy. C

Date: 12 November 2010 10:08:43 +0000
From: Christine@cam.ac.uk>
To: Rex@cam.ac.uk>
Cc: Tim@cam.ac.uk>, Libby @cam.ac.uk>
Subject: Re: dominoes [and -unrelated question]

James points out I may have changed the original question. Perhaps you only wanted a connection in a line and not in a loop (my reading). So - an extension?

Christine

Date: 13 November 2010 04:21:56 +0000
From: Tim@cam.ac.uk>
To: Rex@cam.ac.uk>
Cc: Christine@cam.ac.uk, Libby@cam.ac.uk
Subject: Re: dominoes

I had a chance to think about this on a train to Newcastle Friday evening
...

Isn't it the Konigsberg bridges problem?

Suppose we have n domino 'tokens', from blank (zero say) to n-1. Let them be the vertices of a graph. Any edge of that graph 'is' then one domino, with two distinct tokens. The full set of dominoes (omitting, as you say, duplicates) is then the 'handshakes' graph, with each pair of (distinct) vertices joined by a graph. The order of every vertex of the graph is n-1. A chain of dominoes with none omitted is a complete unicursal (Eulerian) circuit of the graph. Such a path exists if the number of vertices of odd order is zero (in which case the path can be closed, a 'loop') or two.

Thus Rex's chain (and, indeed, Christine's closed loop) exists iff n is odd.

Example: n=5

10 dominoes, with (one) loop 01-12-23-34-41-13-30-04-42-20

Tim
(sleepless in Newcastle ...)

Date: 17 November 2010 08:47:58 +0000
From: Christine@cam.ac.uk
To: Tim@cam.ac.uk
Cc: Rex@cam.ac.uk>, Libby Jared ecj20@cam.ac.uk
Subject: Re: dominoes

Hallo

> Isn't it the Konigsberg bridges problem?

Yes it is and I think the way I constructed a recursive proof is similar to
the proof that takes a basic chain (or loop) and then the remaining
dominoes/edges fall into loops and you add them in to the chain.

However if you do that by induction with merging just two loops you get
part loop with no n+1's and N+2's and the rest with all n+1s and n+2s.
That's not very pretty. So I was wondering if there is a more evenly spread method for listing the tiles.
Maybe I'll draw it as a K-graph and see.

C

Date: 17 November 2010 09:44:59 +0000
From: Tim@cam.ac.uk>
To: Christine@cam.ac.uk>
Cc: Rex@cam.ac.uk, Libbycam.ac.uk
Subject: Re: dominoes

--On 17 November 2010 08:47 +0000 Christine@cam.ac.uk>wrote:

> However if you do that by induction with merging just two loops you get
> part loop with no n+1's and N+2's and the rest with all n+1s and n+2s.
> That’s not very pretty. So I was wondering if there is a more evenly
> spread method for listing the tiles.

The usual algorithm to 'construct' a closed, complete unicursal path involves (some) choice while there is still more than one edge "away from" each vertex. I suppose there's a way of making it more deterministic, as you suggest.

Tim

Date: 18 November 2010 13:45:51 +0000
From: Tim@cam.ac.uk>
To: Christine@cam.ac.uk Rex@cam.ac.uk>
Cc: Libby @cam.ac
Subject: Re: dominoes and -unrelated question

I'm not an authority on [unrelated problem] but …I was in Heffers just now, and looked in Alan Beardon's 2005 book (in which the topic first appears very close to the end!). He defines …

Best wishes,
Tim

‘Plea1’s messages offering thanks in advance (P1, P4 ExThd1) and conveying progress and a ‘happy’ conclusion[1](P9, P10 ExThd1) is typical of being involved in a pleasant environment. Asking for politeness and respect does not automatically engender a sense of care, but nevertheless there is a tangible feeling of care within the threads, no doubt helped by the fact that active members have a common interest in mathematics. The amount of care and encouragement that ‘HelpA’ gives to ‘Plea2’ in ExThd2 is considerable. In ExThd1 there are three examples where care towards the person asking for help is shown: In the first message of help that ‘Plea1’ receives, (P2 ExThd1), whilst leaving ‘Plea1’ to work things out for himself, ‘Help1’ is at pains to make ‘Plea1’ feel comfortable by the invitation to post back if difficulties remain. ‘Help2’ continues in the supportive spirit by concluding the message with *‘Can you solve it now?’* (P5 ExThd1). The moderator’s response demonstrates a keenness to convey to ‘Plea1’ that the error is normal and many people cannot find mistakes in their own work (P7 ExThd1).

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[1] A bullet within the Posting Protocols for Asking Questions covers this.

RG1 To investigate pupils' general perceptions of doing mathematics in school and of using NRICH type problems in home/school settings	
Research Questions	**RQ1: What are the common practices of using NRICH problems in the home context?** *Why and with whom do these students do NRICH problems at home? To what extent do these students perceive their teacher knowing that they do mathematics problems at home? Why do these students not tell their teacher?* **RQ2: What views do students using an on-line mathematics resource (NRICH) have concerning their experience of school mathematics?** *What are students' perceptions about doing mathematics puzzles and problems in lessons? What are students' perceptions about the relative merits of rules, methods and understanding? How do these students seek help with school mathematics?*
Further comment	The work undertaken for this research goal provided background information to the exploration and characterisation of AskNRICH. As such no claims to new knowledge are being made though confirmation of the finding from earlier evaluative studies [Jared 1998]. This current study establishes the home and alone school student (predominantly not telling their teacher) interested in (independently) pursuing their mathematical studies
RG2 To formulate an analytical approach appropriate to the nature of AskNRICH	
Research Questions	**RQ3: Can existing methods / frameworks for analysing Computer Mediated Communication forums be employed in analysing AskNRICH?** *What different types of frameworks have been reported? What different methods/approaches already exist? What are the key methodological issues?* **RQ4: How should the exploration of AskNRICH threads be organized (planned, structured and executed)?** *Which threads should be selected for analysis? How should individual threads be analysed?*
Contribution to Claims (New Knowledge)	Claim 1: A set of techniques, that includes some new elements, has been formed that manage the complexities, size and nature of the task of analysing the AskNRICH web-board
	[contd]

RG3: To undertake the exploration of the AskNRICH artefact	
Research Questions	**RQ5: What does AskNRICH offer to participants to enable them to pursue their mathematical practices?** *Necessary background information about AskNRICH: What is it? What are the different sections of the web-board? What are the posting protocols on AskNRICH? Who are the participants? Why do they belong? How is AskNRICH typically used?* **RQ6: What are participants' common practices when using the AskNRICH web-board?** *What characteristics do participants of AskNRICH exhibit as they pursue their interest in mathematics? What mathematics teaching and mathematics learning roles are manifested within AskNRICH?* **RQ7: What results from participants' practices when using the AskNRICH web-board?** *What types of interactions are shown between the participants as they engage with mathematics? In what ways does the behaviour of AskNRICH participants emulate the working practices of professional mathematicians?*
Contribution to Claims (New Knowledge)	Claim 2: Analysis of teaching and learning aspects of exchanges within AskNRICH has demonstrated that the virtual world of AskNRICH and the behaviours of the AskNRICHers strongly promote opportunities to engage in a transformational pedagogy Claim 3: The AskNRICH environment (i) engenders a harmonious mathematical learning experience and (ii) provides an example of positive, Internet-based, learning benefits
Overarching Research Objective: **To characterise the network that constitutes AskNRICH, a virtual world that allows people to meet within it and engage in doing mathematics**	
Contribution to Claims (New Knowledge)	Claim 4: AskNRICH can be successfully characterised using a concept of 'place', based on a modification of Gee's model of an Affinity Space, through the introduction and definition of two new concepts, Pupil Learning Place [PLP] and Second Learning Place [SLP] Claim 5: The nature of AskNRICH as a learning place embodies qualities having the potential to complement learning in schools